Effects of Mean Stress on the Durability Assessment for an Automobile Crankshaft using Markov Chain Modelling Technique

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ABSTRACT

This paper presents the mean stress effect on durability assessment in terms of fatigue life prognosis under random loading using Markov Chain technique. Durability assessment proposes the use of an embedded Markov process by incorporating loading data samples to artificially generate a series of loading history to predict the fatigue life cycle of the automobile crankshaft. The Markov process is continuously updated in order to minimize the loading data point for fatigue life prediction. The accuracy of the durability assessment is modelled using the finite element analysis and validated through statistical correlation properties based on the random loading, material and design of the component. The durability assessment using the Markov Chain modelling technique provided a statistical accuracy between 95 - 97% with a mean squared error of less than 2% compared to sampling data. Therefore, the use of a non-deterministic method of Markov Chain model provides a basis for the development of an accurate and efficient durability assessment between the stochastic model and practice of the deterministic method.

Introduction

Durability prediction of mechanical automobile components is important as it is commonly used in fatigue life prediction for structures and components. Fatigue failure of mechanical components is unavoidable even it was designed to withstand operating loads [1]. Fatigue of automobile components is considered to be a non-deterministic due to the random loads[2]. In the automotive industry, fatigue failure of the crankshaft is considered to be severe. As this deals with the operating of the entire engine. Various test such as Scanning Electron Microscope (SEM), fractography, tensile test, hardness tests chemical analysis, x-ray analysis was done to determine the root cause for the failure of the crankshaft [3-4]. Based on the experimental investigation, it was concluded that the failure is due to operational loads of bending and torsion that causes a high cycle and low stress from.

In durability prediction, the mean stress effects will influence the fatigue behaviour component based on the captured loading signal. The mean stress models Morrow and Smith, Watson and Topper is used to determine the appropriate fatigue life curves under random loads [5]**.** The appropriate mean stress model should be compared with the test data from the experimental analysis of fatigue failure in predicting the durability of the component. In order to assess the S-N curve, for components like the crankshaft are based on the multi-axial load that has the effects of mean stress [6].

This aim of this study is to predict the durability of the crankshaft with the effects of mean stress under operating loads. The fatigue failure of the crankshaft is categorized as a non-deterministic failure due to the vibrational loads under operating condition. Due to the lack of loading data, the durability assessment will have accuracy which is lower for predicting the fatigue life of the component. Hence, the Markov Chain (MC) is proposed in generating loading history data that embeds actual loading data in predicting the durability in terms of fatigue life prediction. With generated data within the given boundary condition is increased to have minimal intervals, the credible error would be reduced. Hence, providing that the Markov Chain can be used for durability assessment using strain analysis with good accurate fatigue life prediction.

Methodology

The proposed process flow for durability predication for the crankshaft using the *Rainflow* counting technique for assessing the mean stress effects of the crankshaft as illustrated in Figure 1. The MC modelled was used to model the failure in two state condition of bending and torsion. Fatigue life prediction is essential for durability assessment, where the operating loads with consideration of geometry design and fatigue material are associated with the

life of the structure. The framework provides the durability prediction when there are constraints of missing data during experimental setups.
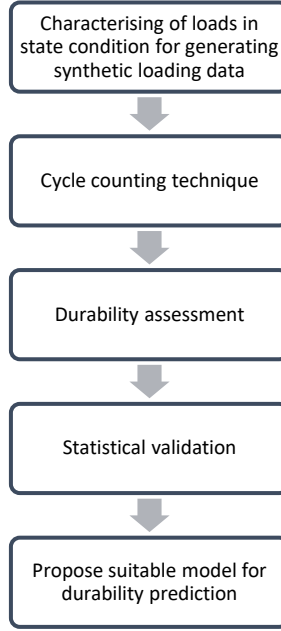

Figure 1 Flow process of durability prediction for the crankshaft

Step 1: The failure that occurs on the crankshaft is subjected to multi-axial loads modelled using MC. The MC states are modelled through the probability criterion as a steady state condition through time transition from the vibrational loads. Hence, the MC for the loading state condition can be modelled mathematically to predict the future state, with the understanding that the current state condition. Whereby the past state condition is independent. This is illustrated in Equation (1).

$$P\{X_{t+1} \mid X_0 = B, X_1 = X_i = i_1, ..., X_n = i_n\} = P\{X_{t+1} = T, X_t = B\}$$
$$P\{X_1 = T \mid X_0 = B\} = P\{X_{t+1} = T \mid X_t = B\} \tag{1}$$

where the probability P for the transition torsion, T and bending, B with respect to time t with non-negative integers of i and j. The failure for the crankshaft experiences quasi-static torsional loads of 10% or lesser compared bending load. The probabilistic model from Equation 2 is modelled as a probability matrix.

$$P^t = \begin{pmatrix} P_{aa} & P_{ab} \\ P_{ba} & P_{bb} \end{pmatrix}^a \tag{2}$$

where P^t is probability matrix transition over time. Likewise, synthetic loads can be generated by introducing μ as the probability vector, and the actual maximum and minimum loads.

$$E(t) = \begin{pmatrix} \mu_{11} & \mu_{12} \end{pmatrix} \times \begin{pmatrix} P_{aa} & P_{ab} \\ P_{ba} & P_{bb} \end{pmatrix}^t \times \begin{pmatrix} L_{max} \\ L_{min} \end{pmatrix} \tag{3}$$

where $E(t)$ is the expected failure that of the probability and loading matrix.

Step 2: The generated synthetic loading data obtained from Equation (3) is the calculated cycle counting technique in assessing the fatigue life of the component as illustrated in Equation (4). This is because the cycle counting method uses the technique of pairing the localised minimum and maximum with a small upward or downward excursion by moving backwards or forward from each cycle with minima and maxima value lesser and greater than i and j:

$$N_K^{rfc}(i,j) = \begin{cases} Rainflow\,cycle\,counting\,technique \\ minimum\,values < i\,and\,maximum > j \\ for\,x_k,\ t = 0,1,\ldots,K \end{cases} \tag{4}$$

Step 3: The durability of the crankshaft's material Grade ASTM 100-70-03 [7] was used as an input to the mean stress model. The strain-life is computed based on the total strain ε_t based on the properties for material fatigue:

$$\varepsilon_t = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}} \tag{5}$$

where K' and n' is the material coefficient. Effects of zero mean stress can be calculated from the Coffin-Manson equation as illustrated in Equation 6. Whereas, the presence of non-zero normal stress occurs in situations where the tensile mean stress will reduce the fatigue strength coefficient. In return, the compressive mean stress would increase the fatigue strength coefficient [8]. The Smith-Watson-Topper from Eq. 8 is commonly used grey cast materials [9] where the cyclic strain range and the maximum stress for loading mean stresses is in uniaxial loading conditions.

Coffin-Manson:

$$\varepsilon_{CM} = \frac{\sigma_f'}{E}\left(\frac{2}{D}\right)^b + \varepsilon'\left(\frac{2}{D}\right)^c$$

(6)

Morrow:

$$\varepsilon_{morrow} = \frac{\sigma_f' - \sigma_m}{E}\left(\frac{2}{D}\right)^b + \varepsilon_f'\left(\frac{2}{D}\right)^c$$

(7)

SWT:

$$\sigma_{max}\varepsilon_{SWT} = \frac{\sigma_f'}{E}\left(\frac{2}{D}\right)^{2b} + \sigma_f'\varepsilon_f'\left(\frac{2}{D}\right)^{b+c}$$

(8)

where b is the fatigue strength exponent; c is the fatigue ductility exponent; σ_f' is the fatigue strength coefficient; ε_f' is the fatigue ductility coefficient and N_f is the cycle life. Effects of mean stresses are used to determine an increasing or decreasing life for component or structure. This includes phases such as crack initiation and crack propagation based on load history. The selection of a suitable model using the statistical accuracy of the root mean square error when comparing against the materials fatigue data.

Table 1 Error estimation for various rotational speed

Rotational speed RPM	Maximum Sampling Data MPa	Maximum Synthetic Data MPa	Difference %	Minimum Sampling Data MPa	Minimum Synthetic Data MPa	Difference %
2000	1.00	1.05	5	6.23	6.20	2
3000	0.70	0.75	5	6.96	6.90	6
4000	0.94	1.00	6	7.63	7.58	5
5000	0.80	0.84	4	7.82	7.78	4

Results and Discussion

Markov chain model analysis
Table 1 shows the comparison between the synthetic maximum and minimum loads using Markov Chain model against the actual loading when subjected to bending and torsion loads. The error percentage estimation technique is used to compare between the actual maximum and minimum loads towards the synthetic loads. The difference estimated is reported to be within a range of 2% to 6% for the rotational speed of 2000, 3000, 4000 and 5000 when compared against actual loads obtained from the automobile industry. This is illustrated in Figure 2 and 3.
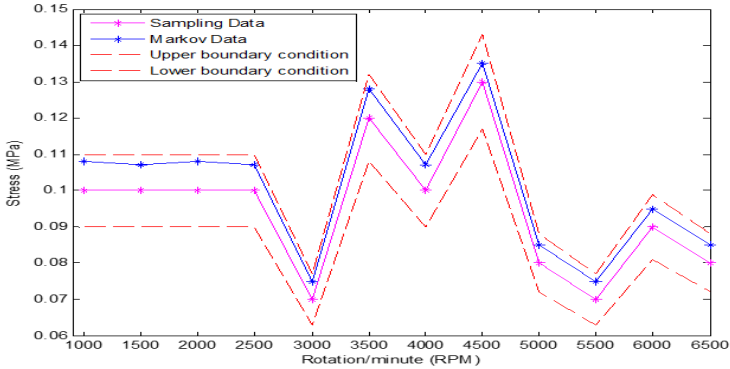
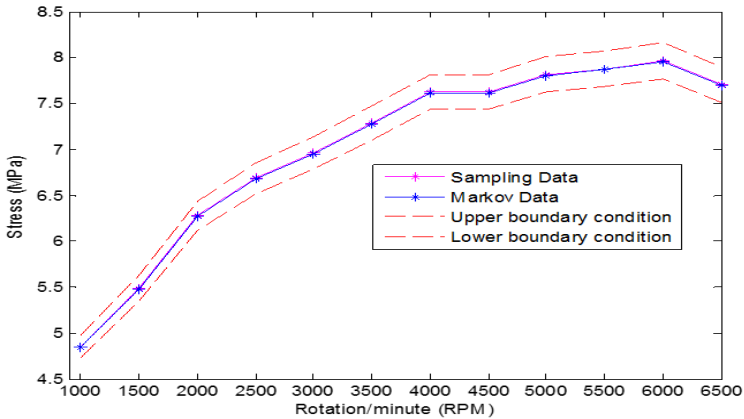Figure 2 Difference of minimum loading between actual and synthetic data.

Figure 3 Difference of minimum loading between actual and synthetic data.

Durability assessment

For predicting fatigue damage, the linear damage model was adopted based on the cycle-counting technique. The fatigue life is modelled using the strain-damage (ε-D) based on Coffin-Manson expression and Morrow and Smith-Watson-Topper model. It showed that the increased fatigue life for each rotational speed is due to the vibrational loads acting on the component during operating condition. The fatigue lifecycle for the component was computed to be between 6.10×10^3 to 1.76×10^2 cycles based on Table 2. This was obtained from uniaxial loading conditions with torsional loads being quasi-static. Hence, the fatigue life is observed to be highest between RPM4000 to RPM5000, whereby under rotational speed, the crankshaft is in optimal

working condition. The fatigue life for each rotational speed is illustrated as in Figures 4. These show the fatigue life behaviour at maximum stress for the crankshaft.

Table 2 Fatigue life of the crankshaft based on the synthetic data

Rotational speed (RPM)	Coffin-Manson (cycles)	Morrow (cycles)	SWT (cycles)
2000	6.10×10^3	1.87×10^3	1.46×10^3
3000	3.11×10^2	1.09×10^3	8.58×10^2
4000	1.76×10^2	6.42×10^2	4.02×10^2
5000	1.77×10^2	5.63×10^2	4.53×10^2

Table 3. Ductile cast iron grade 100-70-03 [7].

Strain Life Fatigue Database	Ductile cast iron Grade 100-70-03
Cyclic Strain Hardening Exponent (n')	0.087
Cyclic Strength Coefficient (K')	945
Fatigue Ductility Coefficient (ε'_f)	0.5855
Fatigue Ductility Exponent (c)	-0.7198
Fatigue Strength Coefficient (σ'_f)	926
Fatigue Strength Exponent (b)	-0.0669

For durability prediction, the synthetic loads were converted from stress to strain using Equation (5) for each cycle by including material properties as shown in Table 3.

The modelling of the Markov chain was statistically compared between synthetic and actual data to determine an appropriate model to be used in the evaluation of the durability of the component. The ε-N models were assessed against [7] to obtain the suitable model for fatigue life prediction. Since the component is subjected to the random loss that illustrates a fatigue behaviour as a decreasing life based on mean loads. The life prediction from cycle counting technique was statistically validated against Grade ASTM 100-70-03. The accuracy of the synthetic data generated was more than 95% which was modelled in the 95% confidence intervals. This is illustrated in Figure 5, 6 and 7. The fatigue life was proportional in behaviour where the notch has high plastic deformation. The *SWT* model is appropriate to be used for the crankshaft due to the total fatigue that is directly related to cyclic strain and the maximum stress during vibrational loads.
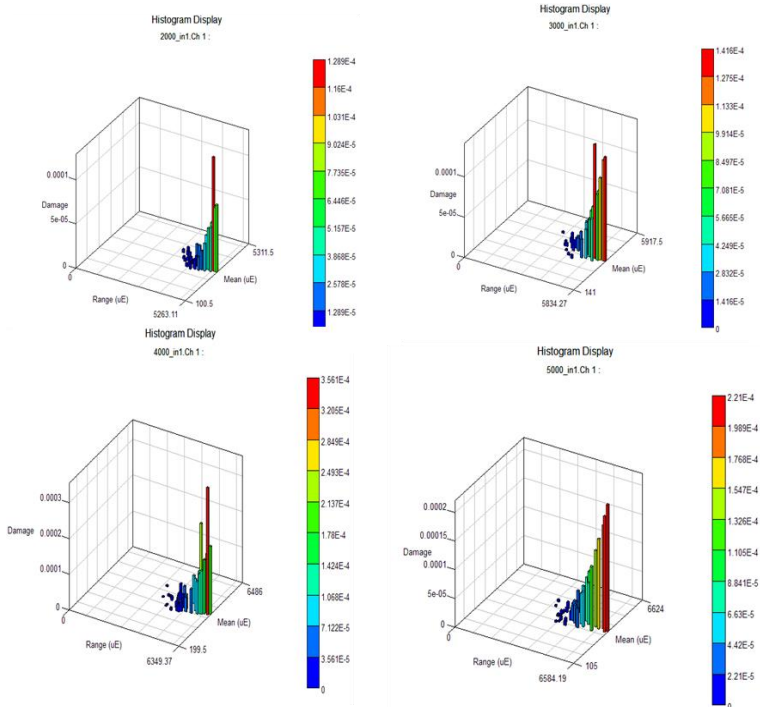
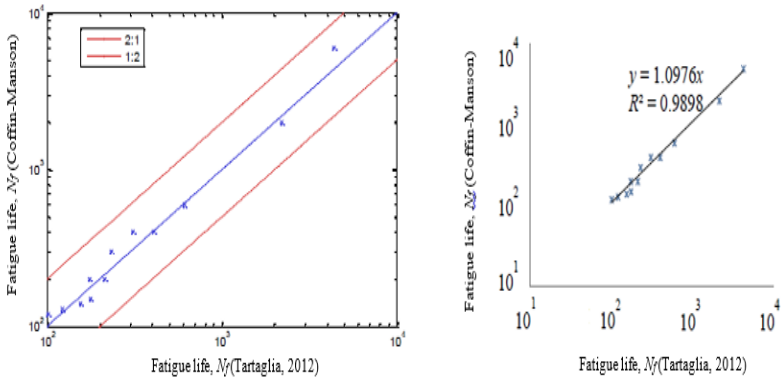Figure 4 Predicted fatigue damage using the synthetic data

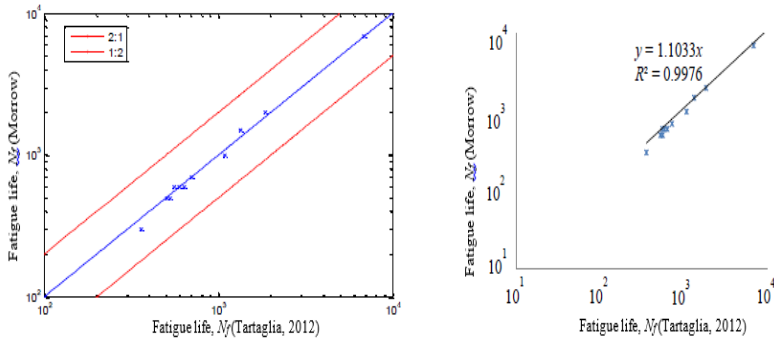Figure 5 Accuracy of the synthetic data using Coffin-Manson

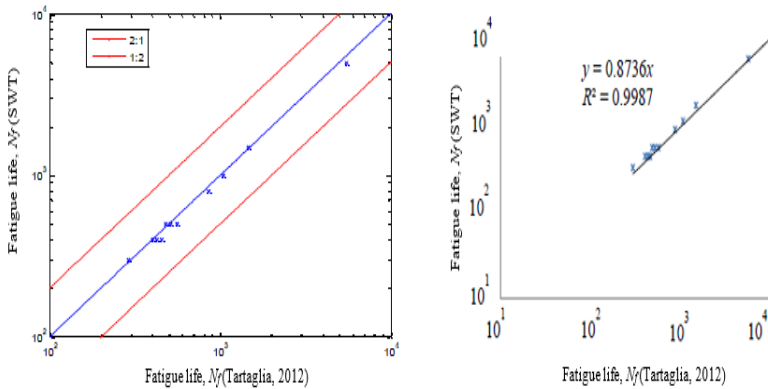Figure 6 Accuracy of the synthetic data using Morrow

Figure 7. The accuracy of the synthetic data using SWT

Conclusion

In this study, durability investigation of crankshaft using on the cycle counting for predicting ε-N is presented. The fatigue is modelled using the MC state condition. The accuracy of the MC model is within 3% - 6% against the actual load. This indicates there is less difference for MC in random loads modelling under upper and lower boundary loads for life prediction.

The synthetic vibration load data in stress was converted to the strain using Ramberg-Osgood in order to predict fatigue life cycle calculation using cycle counting approach and compared against the experimental analysis data. The Markov Chain algorithm reported that the predicted durability was within the given 95% confidence level. The Smith-Watson-Topper (*SWT*) fatigue mean stress indicated accuracy due to maximum tensile stress and fatigue

strain amplitude. Hence, the MC provides is computationally accurate in predicting durability under random loading.

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