Wave-Based Substructuring Method for Dynamic Behaviour Investigation of Solid Meshing Based Finite Element Model of a Structure

Ahmad Burhani Bin Ahmad Basri
*Muhamad Norhisham Bin Abdul Rani**
Mohd Azmi Bin Yunus
Wan Imaan Izhan Bin Wan Iskandar Mirza
Structural Dynamic Analysis & Validation (SDAV)
Center of Excellence forDynamic and Control, Faculty of Mechanical Engineering, Universiti Teknologi MARA
40450 Shah Alam, Selangor,Malaysia

Siti Akmar Binti Abu Samah
Akademi Pengajian Bahasa
Universiti Teknologi MARA
40450 Shah Alam, Selangor, Malaysia

mnarani@salam.uitm.edu.my

ABSTRACT

The significant advantage of using solid meshing for the development of the FE model of a structure is that the structure can be modelled without any priori geometric simplification. This provides a higher accuracy of finite element model in comparison with the beam and shell meshing element. However, 3D or solid meshing usually creates an unfavourable way of modelling especially when it comes to modelling a large complex engineering structure. This paper puts forward the idea of using wave-based substructuring (WBS) to investigate the dynamic behaviour of the solid meshing based FE model of a structure with a large number of interface DOFs. The finite element method was used to construct the full finite element model of the structure and NASTRAN 103 was then used for the normal modes analysis. A new finite element model of the structure with reduced interface DOFs was constructed based on the WBS method. The measurement of the dynamic behaviour of the structure was carried out using free-free boundary conditions and an impact hammer test. The predicted results of the proposed method are then compared with those from the full

ISSN 1823- 5514, eISSN 2550-164X
© 2016 Faculty of Mechanical Engineering,
Universiti Teknologi MARA (UiTM), Malaysia.
Received for review: 2017-05-14
Accepted for publication: 2017-10-10
Published: 2018-04-15

finite element model and experimental counterparts. It was found that the recommended relative threshold value to be use for Singular Value Decomposition (SVD) in 3D modelling is 1e-6. The comparisons of the results between the full FE, WBS method and test models revealed that the use of WBS method has led to a dramatic reduction in the expenditure of computational time by 80% faster than the full FEM while still maintaining a satisfactory level of accuracy. This indicates that the WBS method can be used economically and efficiently for the determination of dynamic behaviour of a solid meshing based finite element model.

Keywords: *Wave-Based Substructuring, Finite Element Analysis, Reduced Order Method, Modal Testing, Normal Modes Analysis*

Introduction

The finite element method (FEM) is one of the most powerful approximation methods in the engineering community. The method has been widely used by engineers in the modelling and simulation of various engineering structures before the physical prototypes of the structures are developed. Since the need of the accuracy in the analysis results is extremely demanding, this situation has pushed the engineers to use fine meshes in the modelling of the structure. However, using fine meshes, solid meshing in particular, will usually lead to the high expenditure especially in terms of modelling, mesh preparation, computing and post-processing effort.

One way to reduce the expenditure it may be necessary to use coarse meshes. However, coarse meshes usually lead to degrade accuracy, negating the advantage of directness in geometric dimensional reduction effect[1]. Theoretically, a solid meshing based FE model consists of much larger number of degrees of freedom (DOFs) than the beam and shell meshing based FE models. Therefore, it gives rise to a huge increment for the expenditure of CPU time recorded in the prediction of the dynamic behaviour of the solid meshing based FE model, in comparison with that of beam and shell meshing based FE model. Consequently, the solid meshing based FE model will have more number of DOFs at the junction between the substructures and the residual structure which may result in the increment of CPU time.

The wave based structuring method (WBS) may be one of the efficient ways to alleviate the increment of CPU time as a result of using solid meshing based FE model. WBS is a substructuring approach in which the deformation of the coupling interface is written as a combination of a set of basis deformations called waves[2]. In this method, despite the structure having a bigger or larger interface nodes, the nodes are condensed into much smaller numbers. As the essential number of the waves is always much lower than the number of interface DOFs, faster assembly prediction is obtained[2]–[4]. The method has been successfully applied for the case of shell meshing element [2], [5]–[9] where a faster prediction can be obtained through the usage of the WBS method. However, to the best of authors' knowledge and open literature review, no information is available about the use of the WBS method in analysing the dynamic behaviour of a structure using solid meshing based FE model. It seems probable that the use of the WBS method maybe very helpful in increasing the efficiency and economics of the dynamic behaviour investigation of the structure. Therefore, this case study is a part of interesting investigation of the WBS method that should be looked into with close attention.

This paper puts forward the idea of using the wave-based substructuring (WBS) to investigate the dynamic behaviour of the solid meshing based FE model of a structure with a large number of interface DOFs. The use of the WBS method will lead to a dramatic reduction in the size of structure interfaces. As a result, the expenditure of computational time for the prediction of the dynamic behaviour of the structure which is usually very computational expensive becomes much less.

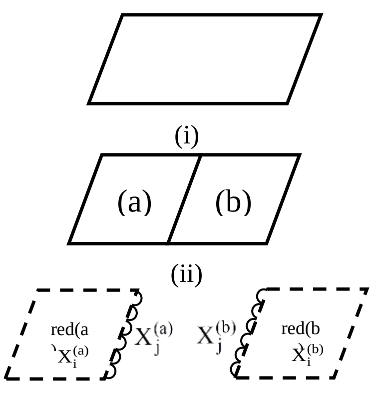


Figure 1: Reduction of structure in component mode synthesis (CMS) method

Theory of WBS

Consider an undamped structure with no external forces, the FE matrix form will be:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0 \tag{1}$$

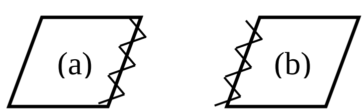
where **M** is the mass matrices and **K** is the stiffness matrices. The FE structure is divided into several non-overlapping substructures to obtain an assembled system as illustrated in Figure 1(ii).

In conventional CMS method, the full structure is decomposed into several substructures (Eg. (a) and (b)) which are then solved independently to obtain the reduced substructures before being reassembled and analysed to obtain an efficient iteration process. For each substructure, the DOFs, **x** are separated into interior DOFs, $\mathbf{x}_i$ and boundary DOFs or junction DOFs, $\mathbf{x}_\mathrm{j}$ at the boundary[6]. The system matrices for each substructure will be:

$$\begin{bmatrix} \mathbf{M}_{\mathrm{ii}} & \mathbf{M}_{\mathrm{ij}} \\ \mathbf{M}_{\mathrm{ji}} & \mathbf{M}_{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_{\mathrm{i}} \\ \ddot{\mathbf{x}}_{\mathrm{j}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{ii}} & \mathbf{K}_{\mathrm{ij}} \\ \mathbf{K}_{\mathrm{ji}} & \mathbf{K}_{\mathrm{jj}} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{i}} \\ \mathbf{x}_{\mathrm{j}} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{f}_{\mathrm{j}} \end{bmatrix} \tag{2}$$

In the WBS approach, the method will limit the junction DOFs, $\mathbf{x}_\mathrm{j}$ by reducing the interface description by introducing them to a set of basis function **W** (Figure 2), weighted with involvement factor **p**:

$$\mathbf{x}_{\mathrm{j}} = \mathbf{W} \times \mathbf{p} \tag{3}$$



$$\mathbf{x}_{\mathrm{j}}^{(a)} = \mathbf{W}^{(a)} \cdot \mathbf{p}^{(a)} \qquad \mathbf{x}_{\mathrm{j}}^{(b)} = \mathbf{W}^{(b)} \cdot \mathbf{p}^{(b)}$$

Figure 2: In WBS, the interface DOFs is represented in the form of waves which is much lower compared to interface DOFs used in CMS method

Equation (3) is substituted into Equation (2):

$$\begin{bmatrix} \mathbf{M}_{\mathrm{ii}} & \mathbf{M}_{\mathrm{ij}}\mathbf{W} \\ \mathbf{W}^{\mathrm{T}}\mathbf{M}_{\mathrm{ji}} & \mathbf{W}^{\mathrm{T}}\mathbf{M}_{\mathrm{jj}}\mathbf{W} \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{x}}_{\mathrm{i}} \\ \ddot{\mathbf{p}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_{\mathrm{ii}} & \mathbf{K}_{\mathrm{ij}}\mathbf{W} \\ \mathbf{W}^{\mathrm{T}}\mathbf{K}_{\mathrm{ji}} & \mathbf{W}^{\mathrm{T}}\mathbf{K}_{\mathrm{jj}}\mathbf{W} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{\mathrm{i}} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{W}^{\mathrm{T}}\mathbf{f}_{\mathrm{j}} \end{bmatrix} \tag{4}$$

At this point, the set of basis functions is known as "waves". As the number of waves is lower than the junction DOFs, $\mathbf{x}_\mathrm{j}$, this substitution will reduce the size of the interface FE matrix equation.

Continuity and equilibrium condition must be translated into interface basis functions to perform the assembly for the substructures. For a rigid connection[7] between *a* and *b* substructures, the continuity and equilibrium conditions are as follow:

$$\mathbf{x}_\mathrm{j}^{(\mathrm{a})} = \mathbf{x}_\mathrm{j}^{(\mathrm{b})} \text{ and } \mathbf{f}_\mathrm{j}^{(\mathrm{a})} + \mathbf{f}_\mathrm{j}^{(\mathrm{b})} = 0 \tag{5}$$

In WBS, the assembly of two substructures, *a* and *b* will have the junction DOFs expressed as $\mathbf{x}_\mathrm{j}^{(\mathrm{a})} = \mathbf{W}^{(\mathrm{a})} \times \mathbf{p}^{(\mathrm{a})}, \mathbf{x}_\mathrm{j}^{(\mathrm{b})} = \mathbf{W}^{(\mathrm{b})} \times \mathbf{p}^{(\mathrm{b})}$ respectively. The same condition is applied to the involvement factor *p*:

$$\mathbf{p}^{(\mathrm{a})} = \mathbf{p}^{(\mathrm{b})} \text{ and } \mathbf{f}_\mathrm{j}^{(\mathrm{a})} + \mathbf{f}_\mathrm{j}^{(\mathrm{b})} = 0 \tag{6}$$

As for the wave calculation that will be used to span the vector space at the interface, SVD orthonormalization is used. Since the wave calculation plays the most important aspect in determining the success of the WBS method applied, the correct practice is to apply a value of $T < 1.0e^{-5}$ for the relative threshold *T* for a 2D element FE model[10]. Using this value could give the most accurate required number of waves to be used for the analysis. However, using less value of relative threshold *T* will increase the number of waves which will also increase the computational (CPU) time. Therefore, the recommended practice is to apply the least acceptance relative threshold *T*.

Academic Case

Solid Element Modelling

In this case study, aluminium 6mm block plate with $401\text{mm} \times 401\text{mm}$ dimensional size having the material properties of the structure as follows was used:

- The Young's Modulus, $\mathrm{E} = 70\text{GPa}$
- The Shear modulus, $\mathrm{G} = 26.31\text{GPa}$
- Poisson Ratio, $\nu = 0.3$
- Mass density, $\rho = 2900\text{kg/m}^3$

As shown in Figure 3, the FE model was used to investigate the capability of the WBS method in predicting the dynamic behaviour of the 3D meshing based FE model of the aluminium block plate. The structure was discretised with 1mm meshing size and was divided into two parts, one is called residual and the other one is substructure. The meshing specifications of the structures are as follows:

- Residual structure consists of 81608 nodes and 60300 CQUAD elements.
- Substructure consists of 82416 nodes and 60903 CQUAD elements.

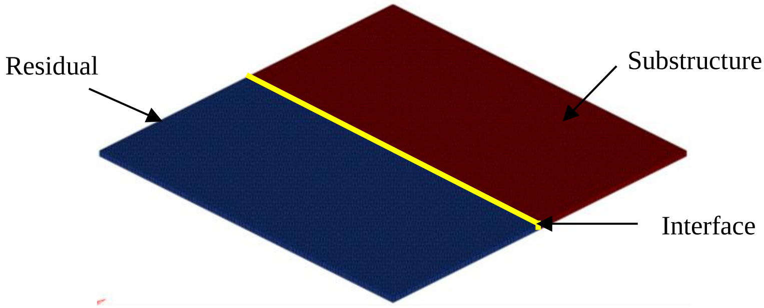


Figure 3: The aluminium 6mm plate

The interface nodes were defined as 808 coincident nodes and connected through rigid connectors as shown in Figure 3 (yellow line). Free-free boundary conditions were applied for the analysis. A 2D meshing based FE model for the same structure was performed in advance using the WBS method. The results obtained from the 2D meshing based FE model were used to compare with those of 3D (Solid) meshing based FE model . The 2D meshing based FE model used the same material properties as the 3D meshing based FE model.

Results and Discussion

For the 3D (solid) meshing based FE model analysis, the suitable threshold value required to be used might differ from that of the 2D meshing based FE model. Therefore, in determining the most suitable threshold value for singular value decomposition (SVD), different threshold values ranging from $1.0e^{-3}$ to $1.0e^{-6}$ were used for the test. The accuracy of the predicted natural frequencies in comparison with the full FE results by using different threshold value for SVD is shown in Figure 4. It was found that by using a threshold value of $1.0e^{-6}$, a more accurate result can be obtained. This differs from the value that was used for the 2D meshing based FE model which was $1.0e^{-5}$. The reason of this changing might be due to the increasing number of the interface DOFs in solid element FE model where a higher number of waves were needed to fully span the waves along the interfaces between the residual and substructure.

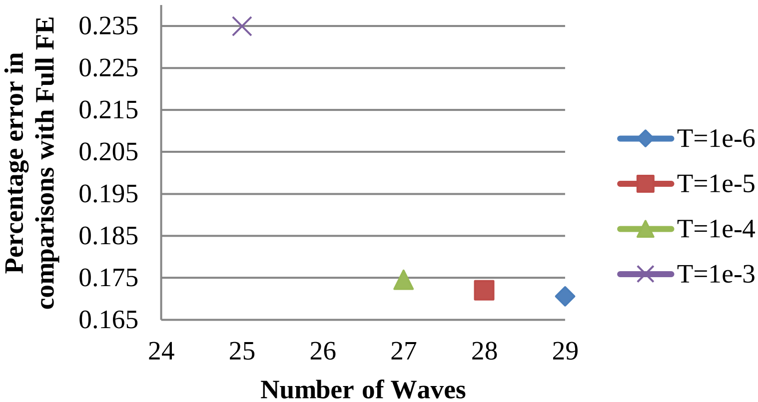


Figure 4: The accuracy of the predicted natural frequencies in comparison with full FE results by using different threshold value for SVD

As for the WBS reduction of the substructure, the residual attachment modes were requested for each wave participation factor. The results of the analysis with and without the residual attachment modes in comparison with the full FE results are illustrated in Figure 5. The results of the eigenvalues for the analysis with residual vector (RESVEC) show the same 100% trend line as the full FE, while the eigenvalues for the analysis without RESVEC show significant difference and are unstable in the trend line. By including the residual attachment modes for the reduction process, an accurate prediction of the dynamic behaviour of the large span structure can be obtained. Furthermore, the frequency of interest for the substructure reduction must be 1.5 times higher than the frequency of interest of the assembled structure. The residual attachment modes must be included and sufficient in the reduction process because it is a powerful technique that can be used to mitigate against the effect of mode truncation in the analysis[11].

The results shown in Table 1 are the comparison between the full FE and WBS method in terms of the expenditure of CPU time taken for the analysis. Column I shows the results of the full FE and Column II shows the results of the WBS analysis. For the full FE model, the total number of nodes available and used for the analysis are 164024 nodes which has caused the CPU time for the analysis of the full FE model to 924 seconds. For the WBS model, instead of analysing the structure normal mode by using the full FE model or the interface nodes which are normally used in the convention reduced order such as Craig-Bampton method, the WBS method improve the efficiency of the analysis by reducing large interface nodes size. Therefore, the boundary nodes used between the residual and substructure consisting of 808 nodes were reduced to only 28 numbers of waves. As a result of a significant reduction in the size of interface DOFs, a dramatic decrease in the CPU time with only 188 seconds was recorded in comparison with the CPU time obtained from the full FE. In other words, the WBS model offers an efficient and economic solution with a satisfactory level of accuracy in the investigation of the dynamic behaviour of the structure with 80% faster than the full FE model.

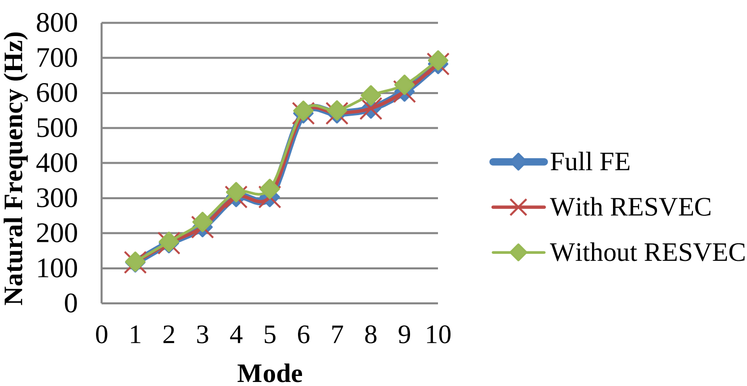


Figure 5: Results of the analysis with and without the residual attachment modes in comparison with the full FE results

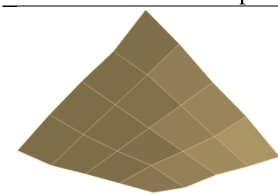
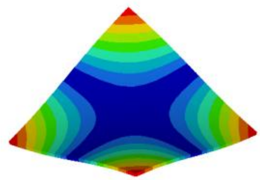
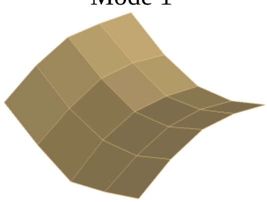
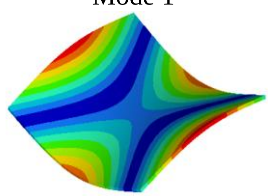
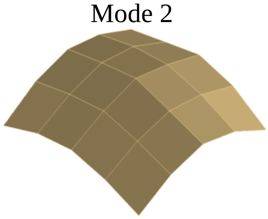
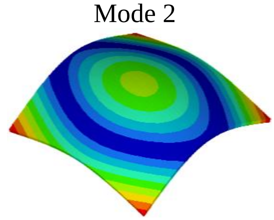
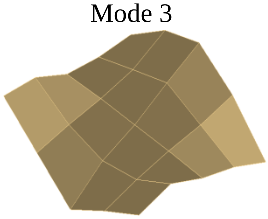
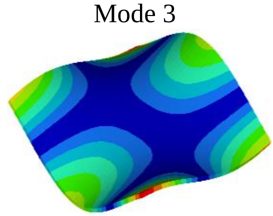
Table 1: Comparison of CPU time between the full FE and WBS models

	I Full FE	II WBS
Number of component normal modes	16	16
Number of Nodes	164024	28
CPU Time (s)	924	188
Time reduction (%)		80

Table 2 shows the numerical and measured mode shapes of the model. From the table, it can be observed that a clear and good agreement mode shapes between the predicted WBS and measured counterparts. This suggests that the use of the WBS method has no direct effect on the quality of the predicted mode shapes of the structure in particular.

From the results tabulated in Table 1 and Table 2 and the aforementioned made in the previous paragraph, the WBS is a method that is not only very effective to be used to reduce the computational time for the 2D (shell) meshing, but the method also offers the same capability to the 3D (solid) meshing based FE model which usually possesses a very large number of interface nodes in comparison with 2D meshing based FE model. Therefore, it would be very practical to use the WBS method for the reduction purposes if a 3D meshing based FE model was used in the dynamic behaviour analysis.

Table 2: Comparison between the measured and predicted mode shapes of the structure

Measured mode shape	Numerical mode shape
	
Mode 1	Mode 1
	
Mode 2	Mode 2
	
Mode 3	Mode 3
	
Mode 4	Mode 4

Conclusions

The use of the WBS method in the investigation of the dynamic behaviour of the 3D (solid) meshing based FE model of a structure is presented and discussed. It was found that the WBS method has been successfully used to predict the dynamic behaviour of the structure with a dramatic reduction in the expenditure of computational time. In addition, the method also recorded a satisfactory level of accuracy.

To achieve the efficiency and accuracy, firstly an adequate number of waves are required to fully span the waves along the interface DOFs of the substsructure and the residual. Therefore, the appropriate value of threshold must be used in order to calculate satisfactory minimum number of waves for the wave orthonormalisation solution. Secondly, it is recommended to include the residual attachment modes for the reduction of the solid meshing based FE model.

Acknowledgements

The authors gratefully acknowledge the Malaysian Ministry of Higher Education (MOHE) and Research Management Centre (RMC) of Universiti Teknologi MARA (UiTM) for providing financial support for this study through the fundamental research grant scheme (FRGS)-600-RMI/FRGS 5/3 (96/2016). The authors would also like to express their appreciation of helpful comments, suggestions and technical support provided by Mr. David Starbuck and Prof. Dr. Hadariah Bahron.

References

[1] U. of Colorado, “Chapter 8: SOLID ELEMENTS: OVERVIEW,” pp. 1–11, 2013.

[2] A. Maressa, D. Mundo, S. Donders, and W. Desmet, “A wave-based substructuring approach for concept modeling of vehicle joints,” *Comput. Struct.*, vol. 89, no. 23–24, pp. 2369–2376, 2011.

[3] J. M. Mencik and D. Duhamel, “A wave-based model reduction technique for the description of the dynamic behavior of periodic structures involving arbitrary-shaped substructures and large-sized finite element models,” *Finite Elem. Anal. Des.*, vol. 101, pp. 1–14, 2015.

[4] M. A. Tournour, N. Atalla, O. Chiello, and F. Sgard, “Validation, performance, convergence and application of free interface component mode synthesis,” *Comput. Struct.*, vol. 79, no. 20–21, pp. 1861–1876, 2001.

[5] A. B. Ahmad Basri, M. N. Abdul Rani, M. A. Yunus, L. Roslan, and W. I. I. Wan Iskandar Mirza, “The investigation of dynamic

behaviour of a structure using wave-based substructuring method," *MATEC Web Conf.*, vol. 90, p. 1011, Dec. 2017.

[6] S. Donders, R. Hadjit, M. Brughmans, L. Hermans, and W. Desmet, "Speeding up vibro-acoustic modification predictions using a Wave-Based Substructuring approach and ATV technology," *Forum Acusticum Budapest 2005 4th Eur. Congr. Acustics*, pp. 161–166, 2005.

[7] S. Donders, B. Pluymers, P. Ragnarsson, R. Hadjit, and W. Desmet, "The wave-based substructuring approach for the efficient description of interface dynamics in substructuring," *J. Sound Vib.*, vol. 329, no. 8, pp. 1062–1080, 2010.

[8] S. Donders, R. Hadjit, M. Brughmans, L. Hermans, and W. Desmet, "A wave-based sub-structuring approach for fast vehicle body optimisation," *Int. J. Veh. Des.*, vol. 43, no. 1/2/3/4, p. 100, 2007.

[9] P. Cermelj, B. Pluymers, S. Donders, W. Desmet, and M. Boltezar, "Basis Functions and Their Sensitivity in the Wave-Based Substructuring Approach," pp. 1491–1506, 2008.

[10] S. Donders, "Computer Aided Engineering Methodologies For Robust Automotive NVH Design," Katholieke Universiteit Leuven, 2008.

[11] M. N. Abdul Rani, "Structural Dynamic Analysis and Model Updating for a Welded Structure made from Thin Steel Sheets," University of Liverpool, 2012.