

**SOLVING SECOND ORDER ORDINARY DIFFERENTIAL  
EQUATION USING RUNGE-KUTTA METHOD**

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## DECLARATION BY CANDIDATE

We certify that this report and the project to which it refers is the product of our own work and that any idea or quotation from the work of other people, published or otherwise are fully acknowledge in accordance with the standard referring practices of the discipline.



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## ABSTRACT

Second order differential equation play a prominent role in many disciplines including engineering, physics, economics, and biology. Theoretically, solving second order differential equation is to reduce it to first order and zero order differential equation which lead to laborious work of calculation. To overcome this problem, researcher tends to use numerical method. Therefore, this research present Second Order (Modified Euler, Improved Euler), Fourth Order (Classical, Butcher-Johnson) and Sixth Order (Butcher-1, Butcher-2) Runge Kutta for solving differential equation problem numerically. To determine the most efficient numerical method, the numerical solutions are compared with the theoretical solutions in terms of accuracy and CPU time with different step sizes.

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