

**SOLVING FIRST ORDER ORDINARY DIFFERENTIAL
EQUATION USING ADAPTIVE RUNGE-KUTTA METHOD**

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DECLARATION BY CANDIDATE

I certify that this report and the report to which it refers is the product of my own work and that any idea or quotation from the work of other people, published or otherwise are fully acknowledge in accordance with the standard referring practices of the discipline.



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ABSTRACT

Runge-Kutta (RK) method can be used to solve first order ordinary differential equation problem in the form of numerical method. Recently, researchers have developed many versions of Runge-Kutta method to increase its accuracy and efficiency. In this research, fourth order adaptive Runge-Kutta methods of Bogacki-Shampine, Cash-Karp, Runge-Kutta of Order Four and Runge-Kutta Fehlberg are used to solve six different first order ordinary differential equation problems using different step size and final solution point. The numerical results are compared with theoretical solution in order to obtain accuracy and to determine the best method based on error and central processing unit (CPU) time.

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