Respondent Based Fuzzy Numbers in Fuzzy Decision Making

Fairuz Shohaimay
Nazirah Ramli
Siti Rosiah Mohamed
Ainun Hafizah Mohd

ABSTRACT

There are many studies on fuzzy decision making that incorporated fixed fuzzy numbers from the literature. However, these fuzzy numbers do not explain the actual respondents' opinions which will affect the overall decision making results. Therefore, fuzzy numbers based on respondents should be developed beforehand to be integrated into the existing fuzzy decision making tool. This paper aims to develop triangular fuzzy numbers based on respondents' opinions. Then, these fuzzy numbers were adopted into fuzzy evaluation method used in a supplier selection problem. The ranking results were analysed using seven different sets of fuzzy numbers. It was found that there is a variation in the ranking result when using fuzzy numbers based on different group of respondents, particularly the ranking of supplier based on main criterion. Hence, future studies in fuzzy decision making should include fuzzy numbers built based on respondents as they provide more reliable outcomes.

Keywords: *respondent based fuzzy numbers, fuzzy decision making, fuzzy evaluation, supplier selection*

Introduction

Decision making involves evaluation that is dependent on human judgements, prior knowledge, feelings and intuitions. It is somewhat unnatural to measure these feelings using real or crisp values. Hence, to correctly define the ambiguity and imprecise data, many decision making problems were modelled using fuzzy approach that is presented in linguistic terms (Zadeh, 1965).

Predominantly in fuzzy decision making, the linguistic terms are usually represented by fixed triangular or trapezoidal fuzzy numbers. These numbers were usually taken from the previous literature. Human intuition and judgement are subjective which varies between individuals. Respondents may have different perception on the representation of these fuzzy numbers (Benítez, Martín and Román, 2007). Hence, it can be argued that fuzzy numbers taken from the literature do not represent actual respondents or experts' opinions.

Previously, Yeh (2002) mentioned the option of user defined value range for the linguistic terms to be used in the assessment process. However, default values were assumed if they have no personal preference on the values to be used. Also, Tolosa and Guadarrama (2010) proposed a new method in collecting data and developing fuzzy numbers from non-expert users based on surveys or observations. In this regard, not many works have been found that actually develop fuzzy numbers based on respondents' opinions.

It is important to consider the use of fuzzy numbers that are constructed according to respondents' opinions or judgements as it may cause variations in the assessment outcomes. Hence, this paper aims to develop triangular fuzzy numbers based on different groups of respondents. Next, these fuzzy numbers were integrated into a fuzzy evaluation method used in a supplier selection problem. The ranking results of suppliers were analysed according to different groups of respondents.

Preliminaries

This section presents the fuzzy evaluation method proposed by Shohaimay, Ramli and Mohamed (2012).

Review on Fuzzy Evaluation Method by Shohaimay et al. (2012)

Step 1: For K decision makers, the fuzzy weight $\tilde{w}_j$, of each criterion is calculated using aggregated fuzzy assessment which is defined as

$$\tilde{w}_j = \frac{\sum_{k=1}^{K} \tilde{w}_j^k}{K}, \tag{1}$$

where $\tilde{w}_j^k$ is the importance weight of the k-th decision maker. The fuzzy weighted vector criteria can be represented as $\tilde{W} = \left[\tilde{w}_1 \quad \tilde{w}_2 \quad \cdots \quad \tilde{w}_j\right]^T$.

Step 2: The fuzzy weight $\tilde{g}_{ij}$, of each alternative is calculated using aggregated fuzzy assessment which is defined as

$$\tilde{g}_{ij} = \frac{\sum_{k=1}^{K} \tilde{x}_i^k j}{K}, \tag{2}$$

where $\tilde{x}_i^k j$ is the rating of the k-th decision maker.

Step 3: The fuzzy grade matrix $\tilde{G}$ is build and defined as

$$\tilde{G} = \begin{pmatrix} \tilde{g}_{11} & \tilde{g}_{12} & \cdots & \tilde{g}_{1k} \\ \tilde{g}_{21} & \tilde{g}_{22} & \cdots & \tilde{g}_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{g}_{n1} & \tilde{g}_{n2} & \cdots & \tilde{g}_{nk} \end{pmatrix}, \tag{3}$$

where $\tilde{g}_{ij}$ denotes the fuzzy grade of the i-th alternative A_i with respect to the j-th criterion X_j, n denotes the number of alternatives and k denotes the number of criteria.

Step 4: The total fuzzy grade vector $\tilde{R}$ is calculated as

$$\tilde{R} = \tilde{G} \otimes \tilde{W} = \begin{pmatrix} \tilde{R}_1 \\ \tilde{R}_2 \\ \vdots \\ \tilde{R}_k \end{pmatrix}, \tag{4}$$

where $\tilde{R}_i$ denotes the total fuzzy grade of the i-th alternative A_i and $1 \le i \le n$.
Step 5: The ranking order of $\tilde{R}_i$ is calculated based on method of centroid point by Wang, Yang, Xu & Chin (2006) corresponding to a value of $\bar{x}$ defined as

$$\bar{x}(\tilde{R}_i) = \frac{\int_a^b x f_{\tilde{R}}^l dx + \int_b^c xw dx + \int_c^d x f_{\tilde{R}}^r dx}{\int_a^b f_{\tilde{R}}^l dx + \int_b^c w dx + \int_c^d f_{\tilde{R}}^r dx}, \tag{5}$$

where $f_{\tilde{R}}^r$ and $f_{\tilde{R}}^l$ are right and left membership function of $\tilde{R}_i$, respectively.

Methodology

The corresponding fuzzy numbers for the linguistic terms of importance weights and performance ratings are developed based on Abdolvand, Toloie and Taghiouryan (2008).

The linguistic terms for importance and performance levels are determined. For k respondents, the lower limit, modal and upper limit of the respective linguistic terms, denoted as a, b and d respectively, are given as

$$a = \min(L_1,\ L_2,\ L_3,\ \ldots,\ L_k), \tag{6}$$

$$d = \max(U_1,\ U_2,\ U_3,\ \ldots,\ U_k), \tag{7}$$

$$b = \frac{\sum_{i=1}^{k} M_i}{k}, \tag{8}$$

where

L_i is the lower limit of the range of the respective linguistic term for i-th respondent,

U_i is the upper limit of the range of the respective linguistic term of the i-th respondent, and

$M_i = \frac{1}{2}(L_i + U_i)$ of the respective linguistic term for i-th respondent, for $i = 1,\ 2,\ 3,\ \ldots,\ k$.

Results and Discussions

A total of 340 respondents were involved in this study. Respondents were asked to determine the appropriate scale of 0-100 for seven scale linguistic terms for importance weights and performance ratings. Triangular fuzzy numbers were developed based on equations (6), (7) and (8). Hence, two sets of fuzzy numbers corresponding to each linguistic term were obtained as shown in Tables 1 and 2.

TABLE 1. Linguistic Terms for Importance Weights based on Respondents (G).

Linguistic Terms	Fuzzy Numbers
Very Low	(0.00, 0.08, 0.60)
Low	(0.02, 0.24, 0.65)
Medium Low	(0.10, 0.38, 0.70)
Medium	(0.16, 0.51, 0.85)
Medium High	(0.25, 0.65, 0.95)
High	(0.40, 0.79, 0.99)
Very High	(0.60, 0.92, 1.00)

TABLE 2. Linguistic Terms for Performance Ratings based on Respondents (G).

Linguistic Terms	Fuzzy Numbers
Very Poor	(0.00, 0.87, 5.50)
Poor	(0.10, 2.42, 6.70)
Medium Poor	(1.00, 3.77, 7.80)
Fair	(1.50, 5.11, 9.10)
Medium Good	(2.50, 6.51, 9.50)
Good	(4.00, 7.87, 9.90)
Very Good	(6.00, 9.2.55, 10.00)

Figures 1 and 2 present the graphs of membership functions for the respective fuzzy numbers.

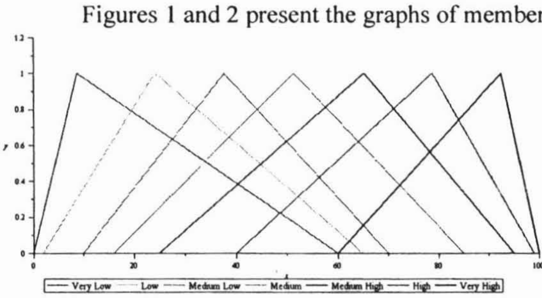

Figure 1: Fuzzy numbers for importance weights

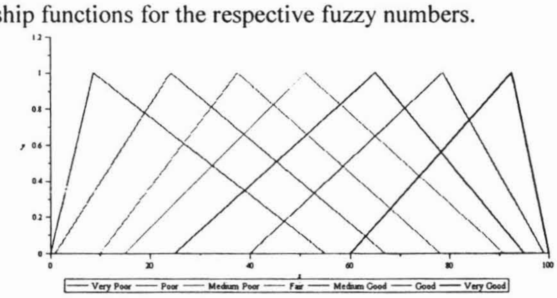

Figure 2: Fuzzy numbers for performance ratings

The respondents were further categorised into six smaller groups based on their academic background to produce 12 different sets of fuzzy numbers, as shown in Tables 3 to 8.

TABLE 3. Linguistic Terms for Importance Weights and Performance Ratings based on Respondents (G_1).

Importance Weights		Performance Ratings	
Linguistic Terms	**Fuzzy Numbers**	**Linguistic Terms**	**Fuzzy Numbers**
Very Low	(0.00, 0.09, 0.50)	Very Poor	(0.00, 0.87, 5.00)
Low	(0.09, 0.25, 0.60)	Poor	(0.50, 2.43, 6.00)
Medium Low	(0.15, 0.39, 0.70)	Medium Poor	(1.50, 3.83, 7.00)
Medium	(0.30, 0.53, 0.80)	Fair	(2.50, 5.23, 8.00)
Medium High	(0.40, 0.67, 0.90)	Medium Good	(4.00, 6.68, 9.00)
High	(0.60, 0.80, 0.97)	Good	(6.00, 8.07, 9.60)
Very High	(0.71, 0.93, 1.00)	Very Good	(7.00, 9.36, 10.00)

TABLE 4. Linguistic Terms for Importance Weights and Performance Ratings based on Respondents (G_2).

Importance Weights		Performance Ratings	
Linguistic Terms	**Fuzzy Numbers**	**Linguistic Terms**	**Fuzzy Numbers**
Very Low	(0.00, 0.09, 0.50)	Very Poor	(0.00, 0.91, 5.50)
Low	(0.05, 0.25, 0.55)	Poor	(0.50, 2.47, 6.00)
Medium Low	(0.10,).38, 0.65)	Medium Poor	(1.00, 3.87, 6.50)
Medium	(0.20, 0.52, 0.80)	Fair	(1.50, 5.21, 8.00)
Medium High	(0.35, 0.66, 0.85)	Medium Good	(4.00, 6.58, 8.90)
High	(0.60, 0.79, 0.98)	Good	(6.00, 7.90, 9.50)
Very High	(0.70, 0.93, 1.00)	Very Good	(7.00, 9.26, 10.00)

TABLE 5. Linguistic Terms for Importance Weights and Performance Ratings based on Respondents (G_3).

Importance Weights		Performance Ratings	
Linguistic Terms	**Fuzzy Numbers**	**Linguistic Terms**	**Fuzzy Numbers**
Very Low	(0.00, 0.07, 0.30)	Very Poor	(0.00, 0.77, 3.00)
Low	(0.08, 0.20, 0.40)	Poor	(0.90, 2.14, 4.50)
Medium Low	(0.15, 0.32, 0.50)	Medium Poor	(1.50, 3.35, 5.50)
Medium	(0.25, 0.46, 0.70)	Fair	(2.50, 4.64, 6.50)
Medium High	(0.40, 0.61, 0.80)	Medium Good	(4.00, 6.12, 8.00)
High	(0.50, 0.77, 0.99)	Good	(5.00, 7.66, 9.00)
Very High	(0.60, 0.91, 1.00)	Very Good	(7.00, 9.20, 10.00)

TABLE 6. Linguistic Terms for Importance Weights and Performance Ratings based on Respondents (G_4).

Importance Weights		Performance Ratings	
Linguistic Terms	**Fuzzy Numbers**	**Linguistic Terms**	**Fuzzy Numbers**
Very Low	(0.00, 0.09, 0.60)	Very Poor	(0.00, 0.85, 4.00)
Low	(0.02, 0.23, 0.65)	Poor	(0.50, 2.30, 5.00)
Medium Low	(0.11, 0.36, 0.70)	Medium Poor	(1.10, 3.62, 6.00)
Medium	(0.24, 0.50, 0.75)	Fair	(2.40, 5.00, 7.50)
Medium High	(0.40, 0.65, 0.88)	Medium Good	(4.00, 6.39, 8.80)
High	(0.45, 0.79, 0.92)	Good	(4.50, 7.72, 9.30)
Very High	(0.60, 0.92, 1.00)	Very Good	(6.00, 9.13, 10.00)

TABLE 7. Linguistic Terms for Importance Weights and Performance Ratings based on Respondents (G_5).

Importance Weights		Performance Ratings	
Linguistic Terms	**Fuzzy Numbers**	**Linguistic Terms**	**Fuzzy Numbers**
Very Low	(0.00, 0.08, 0.40)	Very Poor	(0.00, 0.83, 4.00)
Low	(0.05, 0.24, 0.60)	Poor	(0.50, 2.35, 5.00)
Medium Low	(0.10, 0.38, 0.70)	Medium Poor	(1.00, 3.73, 6.00)
Medium	(0.20, 0.52, 0.80)	Fair	(2.00, 5.15, 8.00)
Medium High	(0.40, 0.66, 0.90)	Medium Good	(4.00, 6.59, 9.00)

High	(0.60, 0.79, 0.95)	Good	(6.00, 7.95, 9.50)
Very High	(0.75, 0.93, 1.00)	Very Good	(7.00, 9.31, 10.00)

TABLE 8. Linguistic Terms for Importance Weights and Performance Ratings based on Respondents (G_6).

Importance Weights		Performance Ratings	
Linguistic Terms	**Fuzzy Numbers**	**Linguistic Terms**	**Fuzzy Numbers**
Very Low	(0.00, 0.06, 0.30)	Very Poor	(0.00, 0.46, 2.10)
Low	(0.05, 0.20, 0.40)	Poor	(0.10, 1.64, 3.50)
Medium Low	(0.12, 0.34, 0.54)	Medium Poor	(1.00, 3.10, 6.00)
Medium	(0.22, 0.48, 0.70)	Fair	(2.00, 4.70, 7.00)
Medium High	(0.35, 0.61, 0.80)	Medium Good	(4.00, 6.31, 8.00)
High	(0.50, 0.76, 0.95)	Good	(6.00, 7.83, 9.50)
Very High	(0.70, 0.91, 1.00)	Very Good	(7.90, 9.18, 10.00)

Next, the developed fuzzy numbers were adopted into a supplier selection problem presented in Shohaimay et al. (2012) for selecting the best Information Technology (IT) supplier based on certain criteria using fuzzy linguistic terms. It is based on the evaluation of *n* supplier, S_n by four decision makers (DMs). Nine items of their existing evaluation form are categorised into three main criteria which are background of supplier (X_1), product performance (X_2) and service performance (X_3). The linguistic values given by the DMs are shown in Tables 9 and 10.

TABLE 9. Importance Weights of Criteria by Each DM.

Criteria	**Decision Maker**			
	D_1	D_2	D_3	D_4
X_1	H	H	VH	MH
X_2	H	VH	VH	MH
X_3	H	VH	VH	MH

Source: Shohaimay et al. (2012)

TABLE 10. Performance Ratings of Suppliers based on each sub-criterion by Each DM.

Criteria	**Sub-Criteria**	D_1				D_2				D_3				D_4			
		S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4	S_1	S_2	S_3	S_4
X_1	X_{11}	G	G	G	G	VG	G	MG	MG	G	MG	MG	G	G	MG	MG	G
	X_{12}	G	G	G	G	M	MG	MG	F	VG	VG	G	G	G	MG	MG	MG
	X_{13}	G	G	G	G	G	MG	MG	F	VG	G	G	G	MG	MG	MG	G
X_2	X_{21}	F	F	F	F	F	F	F	MG	G	MG	MG	G	G	MG	MG	MG
	X_{22}	G	G	G	G	G	MG	MG	MG	G	G	G	VG	G	MG	MG	MG
	X_{23}	G	G	G	G	G	G	MG	MG	G	G	G	G	G	MG	MG	G
X_3	X_{31}	G	G	G	G	VG	MG	MG	MG	G	F	G	G	G	MG	MG	MG
	X_{32}	G	G	G	G	G	F	MG	MG	G	MG	G	VG	G	MG	MG	G
	X_{33}	G	G	G	G	G	MG	MG	G	VG	G	G	VG	G	MG	MG	G

Source: Shohaimay et al. (2012)

By incorporating the developed fuzzy numbers, the ranking results obtained are mostly the same, as presented in Table 11. For criterion X_2 (product performance), the ranking order obtained as $S_1 \succ S_4 \succ S_2 \succ S_3$. While for criterion X_3 (supplier performance), the ranking order yields $S_1 \succ S_4 \succ S_3 \succ S_2$.

TABLE 11. Ranking Order by Incorporating Fuzzy Numbers based on Different Groups of Respondents

Criterion / Group	**Background of Supplier** X_1	**Product Performance** X_2	**Supplier Performance** X_3	**Final Ranking**
G	$S_1 \succ S_2 \succ S_4 \succ S_3$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$
G_1	$S_1 \succ S_2 \succ S_4 \succ S_3$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$
G_2	$S_1 \succ S_2 \succ S_4 \succ S_3$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$

G_3	$S_1 \succ S_2 \succ S_4 \succ S_3$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$
G_4	$S_1 \succ S_2 \succ S_3 \succ S_4$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$
G_5	$S_1 \succ S_2 \succ S_4 \succ S_3$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$
G_6	$S_1 \succ S_2 \succ S_4 \succ S_3$	$S_1 \succ S_4 \succ S_2 \succ S_3$	$S_1 \succ S_4 \succ S_3 \succ S_2$	$S_1 \succ S_4 \succ S_3 \succ S_2$

However, it is important to note that the ranking result varies for criterion X_1 (background of supplier), particularly for fuzzy numbers based on G_4, which produced the ranking result $S_1 \succ S_2 \succ S_3 \succ S_4$, which is different with the rest of the ranking order: $S_1 \succ S_2 \succ S_4 \succ S_3$. This indicates that different outcomes may be obtained when using fuzzy numbers based on different groups of respondents.

Conclusion

This study proposed to develop triangular fuzzy numbers based on respondents' opinions. The developed fuzzy numbers were then adopted into an existing fuzzy evaluation method in IT supplier selection problem. Comparison was made between the ranking results using fuzzy numbers based on different groups of respondents. The results showed that there is a difference in adopting different sets of fuzzy numbers. Hence, this indicates the importance of considering fuzzy numbers based on respondents during decision making process, as it may affect the final evaluation. It is suggested that fuzzy numbers should be developed based on respondents, rather than assuming fixed as appears in previous studies.

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FAIRUZ SHOHAIMAY, NAZIRAH RAMLI, SITI ROSIAH MOHAMED, AINUN HAFIZAH MOHD.
Universiti Teknologi MARA (Pahang).
fairuzshohaimay@pahang.uitm.edu.my,nazirahr@pahang.uitm.edu.my,siti_rosie@pahang.uitm.edu.my, ainunhafizah@pahang.uitm.edu.my.