UNIVERSITI TEKNOLOGI MARA

DISTANCE – BASED RANKING OF FUZZY NUMBERS

AHMAD SYAFADHLI BIN ABU BAKAR

Thesis submitted infulfillment
of the requirementsfor the degree of
Master of Science

Faculty of Computer and Mathematical Sciences

November 2012

AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any other degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

Name of Student	:	Ahmad Syafadhli Bin Abu Bakar
Student I.D. No.	:	2009696534
Programme	:	Master of Science (Information Technology and Quantitative Sciences)
Faculty	:	Faculty of Computer and Mathematical Sciences
Thesis Title	:	Distance – Based Ranking of Fuzzy Numbers
Signature of Candidate	:	.....
Date	:	November 2012

ABSTRACT

Ranking fuzzy numbers is one of the crucial areas of research in fuzzy set theory. Essentially, each presented ranking method is targeting on producing appropriate ranking order for a set of all types of fuzzy numbers including in ranking combinations of more than two fuzzy numbers properly. However, to rank fuzzy numbers appropriately may not be an easy task as fuzzy numbers are represented by possibility distribution hence indicates that they may or may not overlap with each other, having different shapes and distinct in terms of characteristics. Most presented ranking methods are able to rank fuzzy numbers with correct ranking order but there are certain situations where some ranking methods appear to have limitations particularly in ranking non – normal fuzzy numbers, non – overlapping fuzzy numbers and fuzzy numbers of different spreads. As to overcome these limitations, this research proposes a method of ranking fuzzy numbers using distance – based methods involving horizontal – x value, centroid, similarity measure, height and spread of fuzzy numbers. The proposed ranking method is applied to several benchmarking sets of fuzzy numbers for comparison purposes. As the outcomes, the proposed ranking method is found to have the ability on ranking trapezoidal, triangular and vertical line fuzzy numbers, producing correct ranking order for overlapping and non- overlapping case of fuzzy numbers, embedded fuzzy numbers, reflection case of fuzzy numbers and fuzzy numbers on the negative side. Furthermore, the proposed ranking method has also been applied on a real decision making problem faced by a company in Malaysia which is the Facilitators' evaluation in Social Security Organization (SOCSO). In this problem, the comparison between the original approach used by SOCSO and the proposed ranking method was made. The result shown that, using the proposed ranking method, a proper selection of facilitator has been made and this outcome does satisfy the committee members of SOCSO.

TABLE OF CONTENTS

AUTHOR'S DECLARATION ii

ABSTRACT iii

AKNOWLEDGEMENT iv

TABLE OF CONTENTS v

LIST OF TABLES vii

LIST OF FIGURES viii

LIST OF SYMBOLS ix

CHAPTER ONE: INTRODUCTION 1
1.1 Introduction 1
1.2 Ranking Fuzzy Numbers 2
1.3 Development of Distance – Based Ranking Method 5
1.3.1 Centroid – Based Ranking Method 5
1.3.2 Similarity Measure – Based Ranking Method 8
1.3.3 Spread – Based Ranking Method 10
1.4 Problem Statement 12
1.5 Research Question 14
1.6 Objective of the Study 14
1.7 Significance of the Study 14
1.8 Scope and Limitation 15
1.9 Organization of the Study 15
1.10 Summary 15

CHAPTER TWO: PRELIMINARIES 17
2.1 Basic Concepts of Fuzzy Set Theory 17
2.2 Basic Fuzzy Operations 18
2.3 Fuzzy Numbers and Its Operations 18
2.4 Centroid of Fuzzy Numbers 22
2.5 Similarity Measure 28
2.6 Summary 32

CHAPTER THREE: A DISTANCE – BASED RANKING OF FUZZY NUMBERS 33
3.1 Development of the Proposed Distance – Based Ranking Method 33
3.2 Implementation 37
3.3 Validation and Benchmarking 42
3.3.1 Trivial Case 42
3.3.1.1 Trivial Case 1 42
3.3.1.2 Trivial Case 2 46
3.3.2 Discussion on Trivial Cases 48
3.3.3 Embedded Cases 48
3.3.3.1 Embedded Case 1 48
3.3.3.2 Embedded Case 2 49

3.3.3.3 Embedded Case 3 50
3.3.4 Result and Validation for Embedded Fuzzy Numbers 50
3.3.5 Discussions on Embedded Cases 51
3.3.5.1 Embedded Case 1 51
3.3.5.2 Embedded Case 2 52
3.3.5.2 Embedded Case 3 52
3.3.6 Overlapping Cases 53
3.3.6.1 Overlapping Case 1 53
3.3.6.2 Overlapping Case 2 54
3.3.6.3 Overlapping Case 3 54
3.3.7 Result and Validation of Overlapping Fuzzy Numbers 55
3.3.8 Discussions on Overlapping Cases 56
3.3.8.1 Overlapping Case 1 56
3.3.8.2 Overlapping Case 2 57
3.3.8.3 Overlapping Case 3 57
3.3.9 Non – Overlapping Cases 58
3.3.9.1 Non – Overlapping Cases 1 58
3.3.9.2 Non – Overlapping Cases 2 58
3.3.10 Result and Validation for Non – Overlapping Fuzzy Numbers 59
3.3.11 Discussions on Non – Overlapping Cases 60
3.3.11.1 Non – Overlapping Case 1 60
3.3.11.2 Non – Overlapping Case 2 61
3.4 Summary 61

CHAPTER FOUR: APPLICATIONS 62
4.1 Overview on Decision Making Environment 62
4.2 Implementation of Ranking Fuzzy Numbers in Decision Making Problems 63
4.2.1 Decision Making 1: Risk Evaluation 1 63
4.2.2 Decision Making 2: Risk Evaluation 2 69
4.2.3 Decision Making 3: Fuzzy Game Profit and Loss 74
4.3 Discussions on Decision Making Examples 79
4.4 Case Study: Ranking Evaluation of Facilitators in SOCSO 80
4.4.1 Problem Description 81
4.4.2 Evaluation Form 81
4.4.3 Data 82
4.5 Methodology 82
4.6 Implementation 85
4.7 Summary 93

CHAPTER FIVE: CONCLUSIONS 94
5.1 Concluding Remarks 94
5.2 Contributions 96
5.3 Recommendation and Future Work 96

REFERENCES 98

APPENDICES 105