Closed Mode Separation of the Symmetrical Structure

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ABSTRACT

Structural modifications based on vibration analysis are very important in structural optimization of the system in order to modify the dynamics behaviour of the structure. Symmetrical shape structures are prone to close natural frequencies and it is vitally important to identify the mode shapes structure that are near to the desired operating frequency by moving the unwanted modes away from the concern region. The identification of mode shapes that are close in natural frequencies are very difficult to distinguish and do not correspond closely to the numerical prediction. In order to identify modes of the close frequencies, the experimental method of mass modifications will be used to modify dynamic properties of the structure. The objective of this project is to study the closed mode of the I-shape structure and to change the dynamic characteristics by mass modifications so that the mode shapes can be distinguished. The finite element model of the symmetrical shape structure is constructed using FE software packages and compared with the experimental data from LMS SCADAS software. The measured dynamic behaviour is obtained using an impact hammer testing and roving accelerometers under free-free boundary conditions. The results obtained from the predicted and experimental show that mass modification on the nodal point can be used to alter the natural frequencies without changing the mode shapes of the structure. It is believed that the dynamic characteristics of the symmetrical structure can be change by manipulating the mass of the structure.

Keywords: *closed mode, symmetrical shape, I-Shape, finite element analysis.*

Introduction

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The study of close or repeated modes in vibration analysis has received much attention for the past few decades. Symmetrical shape structures are prone to produce close mode [1], [2]. The close mode frequencies are very difficult to distinguish. Furthermore, at high frequency levels most structure are subjected to a large number of close natural frequencies that representing a verity of vibration modes. Therefore, structural modification can be used to identify or to split the close mode of the symmetrical shape structure. This technique is used to study the effect of physical parameter changes of a structural system on its dynamic properties which are in the form of natural frequencies and mode shapes. On top of that, M. Leszek et al. [3] stated that structural modification also can be used to alter the natural frequencies away from the excitation frequency or to change the position of the vibration mode. The dynamic properties of a structure are determined by its mass, stiffness and damping distribution [4]. Therefore by adding a mass is a simple modification that can be easily carrying out by determination of the proper reacceptance in the point where the additional element is added [5].

There are various structural modification methods available for the dynamic analyses of a structural system, such as sensitivity analyses based on eigenvalue and eigenvector derivatives [6], [7] and based on Rayleigh quotient iteration [8]. However, despite of these impressive developments, research in the structural modification of the symmetrical structure is relatively unexplored. Hence, a contribution of knowledge and research is needed to study the effect on structural modification of the symmetrical structure as it is prone to produce close mode frequencies.

Inspired by previous work by researchers, the focus of this research is mass modification on I-shape symmetrical structure and validation of model has been carried out by comparing the measured and predicted result. The idea of manipulating the additional mass to the structure has been carried out in order to split the close mode of the symmetrical shape structure.

Finite Element Analysis Method

In this paper asymmetrical structure (I-Shape) that is made from a thin plate structure shown in Figure 1 with the material properties as shown in table 1.There are two stages involved in completing this research project namely 1) finite element analysis (FEA) method and 2) experimental modal analysis (EMA) method.

In the numerical analysis, natural frequencies and mode shapes are obtained via OptiStruct from HyperWorks. There are three main steps involve which are pre-processing, processing and post-processing.

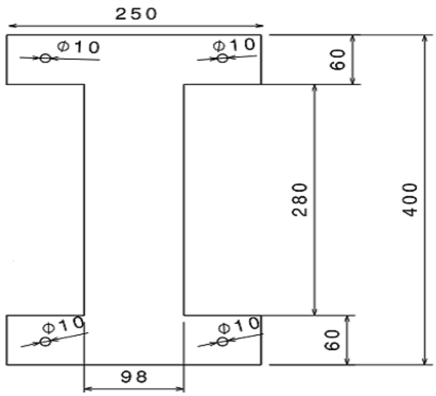

Figure 1: Dimension of the I-shape structure constructed using CATIA software

The material properties for I-shape structure were set as shown in the Table 1.

Table 1: Material properties of the I-shape structure [9].

Properties	Values
Thickness, t (mm)	3.24
Density, ρ (Kg/m^3)	2700
Young Modulus, E (GPa)	69
Poisson's Ratio, ν	0.33

As shown in Figure 2, the element size is set up to be 10mm and type of mesh is "mixed" (combination of triangle and quadrilateral). The bandwidth or frequency of interest was set up from 1 Hz to 1000 Hz and the number of mode is set to be 10.

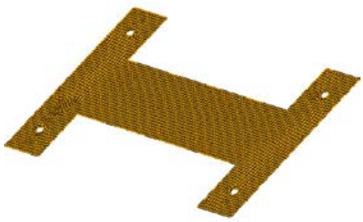

Figure 2: Meshing of the I-shape structure

The “spider web” with added mass at the centre of the hole was created by using RBE. Mass 0.12kg was added at the middle of the hole. The Figure 3 below represents the design of the “spider web”.

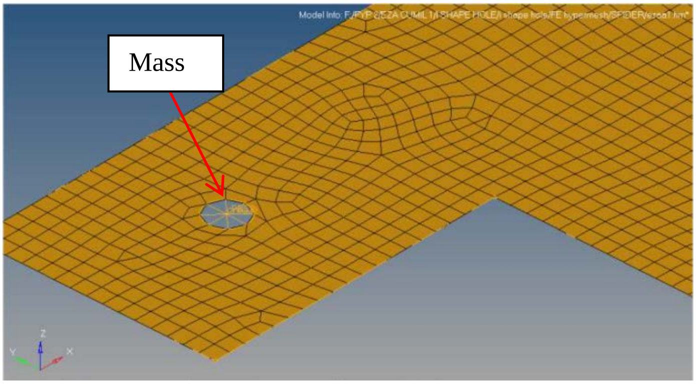

Figure 3: Spider web for the additional mass of I-shape structure

Experimental Modal Analysis Method

Experimental modal analysis has been performed on the structure that is fabricated from thin metal sheets with the nominal thickness 3.24 mm. The test structure was set-up in free-free boundary conditions by applying impact hammer testing. In order to illustrate free-free boundary conditions, both springs and nylon strings used to hanged the structure at every edge. The initial prediction of dynamic properties of the test structure is firstly performed to the test structure and the calculated natural frequencies and mode shapes is then used for the selection of the reference excitation points and the locations of measuring points of the test structure. As shown in Figure 4, this experimental set-up is vital to ensure the excitation point is able to excite all modes of interest and also to obtain reliable mode shapes of the structure [10], [11]. Meanwhile, the method of roving accelerometers is preferable and used in the experiment because the test structure is flexible and to avoid mass loading issue to the structure [12].

The experimental work was conducted with variety position of additional mass on the structure. The load and response were analysed by LMS SCADAS analyser.

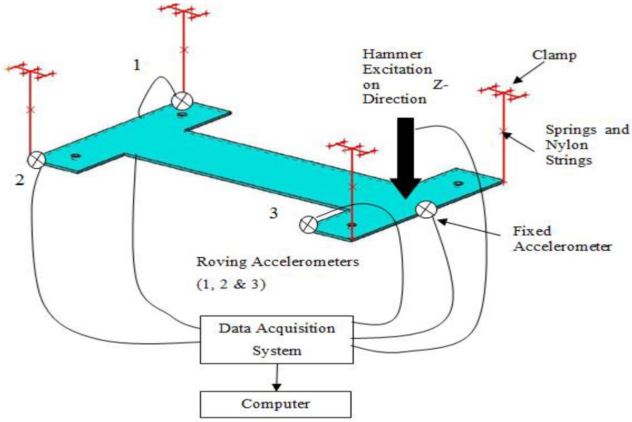

Figure 4: Schematic diagram of the I-shape structure test set up

Mathematical Modelling

The structure to be analysed was assumed as linear vibration system of *n*-degree-of-freedom. In this study, it's applied the differential equations of motion for undamped free vibration is

$$M^0\ddot{x} + K^0 x = 0 \tag{1}$$

where M^0 is the mass matrix, K^0 the stiffness matrix and x, $\ddot{x}$ represent displacement and acceleration response vectors, respectively. Thus, the differential equations of motion of the modified structure corresponding to the (M^0, K^0) system are

$$M\ddot{x} + Kx = 0 \tag{2}$$

where $M = M^0 + \Delta M$, $K = K^0 + \Delta K$, ΔM denotes the changed magnitudes of the mass matrix M^0 and ΔK denotes the changed magnitudes of the stiffness matrix K^0. Next is by setting $\Phi = \psi\tilde{\Phi}$, and from (1) and (2) we obtain

$$\psi^t K^0 \psi = \psi^t M^0 \psi \Omega^2 \quad , \quad K_g^0 = M_g^0 \Omega^2 \tag{3}$$

$$\psi^t (K^0 + \Delta K)\psi\tilde{\Phi} = \psi^t (M^0 + \Delta M)\psi\tilde{\Phi}\Lambda^2 \tag{4}$$

then, we have

$$(K_g^0 + \Delta K_g)\tilde{\Phi} = (M_g^0 + \Delta M_g)\tilde{\Phi}\Lambda^2 \tag{5}$$

pre-multiplying (5) with $(M + \Delta M_g)^{-1}$ and using (3), we obtain

$$\mathrm{A}\tilde{\Phi} = \tilde{\Phi}\Lambda^2 \ , \tag{6}$$

where

$$\begin{aligned}\mathrm{A} &= (M_g^0 + \Delta M_g)^{-1}(K_g^0 + \Delta K_g) \\ &= (\mathrm{I} + (M_g^0)^{-1}\Delta M_g)^{-1}(M_g^0)^{-1}K_g^0(\mathrm{I} + (K_g^0)^{-1}\Delta K_g) \\ &= (\mathrm{I} + (M_g^0)^{-1}\Delta M_g)^{-1}\Omega^2(\mathrm{I} + (K_g^0)^{-1}\Delta K_g),\end{aligned} \tag{7}$$

$$\Delta M_g = \psi^t \Delta M \psi \ \text{ and } \ \Delta K_g = \psi^t \Delta K \psi,$$

where I is the identity matrix. The eigenvalue and eigenvector of the matrix *A*, Λ^2, $\tilde{\Phi}$ was retrieve from (6). From (4), (6) we know that Λ, ($\psi\tilde{\Phi}$) are natural circular frequency and the normal mode shape matrices of the (*M*, *K*) system, respectively. Hence

$$M_g = (\psi\tilde{\Phi})^t M(\psi\tilde{\Phi}), \tag{8}$$

$$\psi^t(M - \Delta M)\psi = M_g^0 \tag{9}$$

respectively pre-multiplying and post-multiplying (9) by $\tilde{\Phi}^t$ and $\tilde{\Phi}$, we obtain

$$(\psi\tilde{\Phi})^t M(\psi\tilde{\Phi}) - (\psi\tilde{\Phi})^t \Delta M(\psi\tilde{\Phi}) = \tilde{\Phi}^t M_g^0 \tilde{\Phi} \tag{10}$$

from (7), (8) and (10) we obtain the modal mass and stiffness matrices of the (*M*, *K*) system, respectively,

$$M_g = \tilde{\Phi}^t(M_g^0 + \Delta M_g)\tilde{\Phi}, \quad K_g = M_g\Lambda^2 \tag{11}$$

The natural frequencies and mode shapes of the finite element are compared with measured data in order to validate the accuracy of the finite element model. The modal assurance criterion (MAC) is normally used to quantify modes between finite element model and experiment. The modal assurance criterion (MAC) can be used to indicate the level of correlation of mode shapes between finite element model and experimental model. It is normally used to calculate experimental Φ_m and finite element modes Φ_a and presented in matrix form, which can be calculated from Eq. (12) [13].

$$\mathrm{MAC} = \tilde{\Phi}_m \tilde{\Phi}_a = \frac{\left| \tilde{\Phi}_m^T \tilde{\Phi}_a \right|^2}{(\tilde{\Phi}_a^T \tilde{\Phi}_a)(\tilde{\Phi}_m^T \tilde{\Phi}_m)} \tag{12}$$

Result and Discussion

Experimental and predicted data natural frequencies of the I-shape structure with relative error between both data for the structure with and without additional mass are shown in the table below.

Table 2: Comparison results of experimental I-plate between without additional mass and with additional mass

	I	**II**	**III**	**IV**
Modes	**Experiment Frequency without Mass (Hz)**	**Experiment Frequency with Mass (Hz)**	**Relative Error (%) [I-II/I]**	**MAC**
1	73.80	71.24	3.47	0.89
2	78.90	78.51	0.49	0.93
3	233.80	232.41	0.59	0.97
4	278.46	276.46	0.72	0.91
5	289.30	285.87	1.19	0.94
6	309.50	308.11	0.45	0.95
7	469.00	467.72	0.27	0.98
8	556.50	553.36	0.56	0.96
9	721.60	719.60	0.28	0.91
10	752.00	745.44	0.87	0.94
	Total Error (%)		8.90	

Table 3: Comparison results of finite element I-plate between without additional mass and with additional mass

	I	II	III	IV
Modes	**Finite Element without Mass (Hz)**	**Finite Element with Mass (Hz)**	**Relative Error (%) [I-II/I]**	**MAC**
1	80.08	62.08	22.48	0.87
2	82.82	82.22	0.72	0.95
3	246.51	237.15	3.80	0.96
4	297.72	284.07	4.58	0.91
5	304.67	302.58	0.69	0.94
6	332.56	329.75	0.84	0.99
7	493.35	492.93	0.09	0.96
8	595.40	595.04	0.06	0.97
9	727.53	727.67	0.02	0.94
10	729.88	730.21	0.05	0.96
	Total		33.32	

In this project, two result are shown in Table 2 and Table 3 which consist of two types of comparison namely experimental of I-plate between without additional mass and with additional mass (Table 2) and finite element I-plate between without additional mass and with additional mass (Table 3). By referring to the Table 2, the natural frequency values of first and second mode have been indicated as closed modes due to its value which are 73.80 Hz and 78.90 Hz. However, when adding the mass to the structure, the natural frequency of the first mode is largely reduce. Both in experimental and predicted, mode 1 have shown a large gap in term of it natural frequencies when the mass of structure been manipulated.

From the Table 2 and 3, it can be seen that, the natural frequencies values of mode 2 and mode 3 are not show much effect because it natural frequencies value are almost the same went comparing with before mass adding. This phenomenon has shown that, by adopting structural modification method, the close modes can be avoided. Experimentally, the natural frequency of mode 1 before adding the mass is 73.80 Hz, meanwhile the mode 2 is 78.90 Hz. However, the result shows that after adding the mass the natural frequency of mode 1 and mode are reduced 71.24 Hz and 78.51Hz respectively.

Meanwhile, the Table 3 clearly indicate that the same trend is also happened to the predicted result. This shown that, the structural modification method is reliable to be applied in finite element method. Furthermore, the value of natural frequencies between the structure with additional mass and without additional mass are differ. It is agreed that by manipulating the mass, it will shift the natural frequencies away from the excitation frequency.

Conclusion

The dynamic characteristic of a symmetrical structure with additional mass and without additional mass made from thin aluminium sheet was investigated experimentally and numerically. The results obtained from the predicted and experimental show that the symmetrical I-shape structure can prone to produce close mode. It is believed that the dynamic characteristics of the symmetrical structure can be change by manipulating the mass of the structure in which the close mode frequency can be distinguish and avoided.

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