UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

FOURTH HANKEL DETERMINANT FOR A SUBCLASS OF CLOSE-TO-CONVEX FUNCTION AND ITS VERIFICATION

P37S19

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Declaration

We hereby declare that this research, which we submit to lecturer involved the for evaluation of the final year project and the contents of this research are genuine and authentic. This research is our own work/investigation and does not infringe any copyright law which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements.

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Abstract

Complex analysis involves studying the properties of complex number function. Geometric function theory is one of the branches of complex analysis field. A new class of function is often introduced and it is common for researchers to find its properties. However, it is difficult for researchers to generalized existing subclasses of analytic functions with limited studies and inadequate sources of references on Hankel determinants. If a new class of function is successfully defined, then some of its properties discussed. In this project, we considered the class of close-to-convex function that has been introduced and presented by Silverman and Telage (1977), denoted by A that satisfies,

$$\Re \left\{ \frac{f'(z) + z f''(z)}{g'(z)} \right\} > 0.$$

This study will focus on Hankel determinant for this class of function. Maximization of function and triangle inequality are the crucial techniques that we used to prove the results. With the aid of some preliminary lemmas, the sharp results are successfully achieved for Fekete-Szegö functional and second Hankel determinant and it is also proved the upper bound for third Hankel determinant as well as fourth Hankel determinant. This study will be a major effort to contribute new results to all of the properties studied. Further studies can be done in finding the sharp upper bounds for third and fourth Hankel determinant in future by using appropriate approaches and considering new reliable properties that need to explore and determine further.

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