Hysteresis Modelling of Pneumatic Artificial Muscle using General Cubic Equation and Factor Theorem Prediction Method

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ABSTRACT

Due to inherent hysteresis in a pneumatic artificial muscle, the accompanying control of this compliant actuator becomes more complicated. In literature, only a few implementations of the hysteresis modelling in the associated position control system could be found. In addition, the high complexity of the contraction-force and contraction-pressure models when accounting for hysteresis effect, the implementation of such model is limited in its use in the pneumatic muscle position control only. However, implementing a complicated control algorithm does not always indicate the best solution that could be used to control the pneumatic artificial muscles. There are arguments in the field of rehabilitation robotics regarding what was the best control system to the orthotic problem for rehabilitation. It is preferred that the control system should be simplified as much as possible; multiple sensors and impedances are only increase the complexity of the control system. Rather than using a very complicated algorithm for control system of the pneumatic artificial muscle, a simple and noble prediction method using general cubic equation and factor theorem is proposed for the hysteresis modelling at different loads. The methodology used to establish the hysteresis modelling and prediction method of the pneumatic artificial muscle are as follows; first is the characterization of the pneumatic artificial muscle at different loads and pressures; second is to develop a prediction

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method for generating constraint models of hysteresis data at a different loads using general cubic equation and factor theorem; third is to establish a simple theorem or algorithm to extract the hysteresis models (i.e., contraction and expansion) of a pneumatic artificial muscle at different loads based on the generated constraint models; and the final stage of the research is to obtain the hysteresis models at different loads. The generated hysteresis models and hysteresis data obtained from experimental study were compared to verify the reliability of the proposed hysteresis modelling prediction method. The simulation results shows that the hysteresis modelling was able to generate an apropriate constraint models at a different loads.

Keywords: *Hysteresis Modelling Prediction Method, Pneumatic Artificial Muscle, General Cubic Equation, Factor Theorem*

Introduction

Early work of this research study presents a survey on the lower-limb rehabilitation orthosis which implemented pneumatic artificial muscle type of actuators such as McKibben artificial muscle, rubbertuators, air muscle, pneumatic artificial muscle (PAM) or pneumatic muscle actuator (PMA). Generally, the type of actuators such as pneumatic artificial muscle plays an important role in development of assistive rehabilitation robotics system. In the last two decades, the development of rehabilitation orthosis which implement the pneumatic artificial muscle was rather slow compared to the other types of actuated rehabilitation orthosis using ac-motor, dc-motor, pneumatic cylinder, linear actuator, series elastic actuator (SEA), and brushless servomotor. However, in the past 10 years, the interest in this research field has grown exponentially mainly due to its clear advantages such as low weight, structural flexibility, compactness, and the most important is the inherent compliance compared to other types of artificial muscles. The exponential growth of rehabilitation orthosis system might also due to the challenges it proposed to resolve the nonlinearity behaviours and difficulties of the system. The inherent nonlinearity behaviours of pneumatic artificial muscle might be present because of the nonlinear relation between the contracting force to its internal pressure and length (i.e., contraction ratio). Other than these factors, it is also occurred due to the nonlinearity of the pressure build-up and the hysteresis which is related to its geometric construction.

Variety of different approaches and methods has been introduced and implemented in order to reduce the effect of the hysteresis during the pneumatic artificial muscle's position control system, where an accurate and precise position control of the contraction-force and contraction-pressure models were desired. Based on the article review, it could be concluded that, most of previous researches on the hysteresis modelling of pneumatic artificial muscle were only focusing on the development of the statics modelling of the hysteresis. The authors considered different approaches and methods to try and resolve the nonlinearity problems such as using the empirical model [1, 3, and 6], geometrical analysis [2], virtual work [4], energy modelling [5], and energy conservation [7]. However, for the dynamics modelling of the hysteresis, they only implement an offset to extract the hysteresis models for both contraction and expansion. Even though the research outputs were not sufficient to simulate the hysteresis behaviours of the pneumatic artificial muscle, these researches were still part of an on-going process in developing the hysteresis modelling prediction method. Additionally, it will contribute to the development of new approaches, techniques, algorithms and methods for resolving the hysteresis modelling problems.

On the other hand, most of the current researches were focusing on the development of dynamic modelling of the hysteresis, where a different methods were introduced and implemented such as the generalized Bouc-Wen model [15], Maxwell slip model [11, 12, and 14], Preisach model [10], and Prandtl–Ishlinskii model [19]. According to these proposed methods, the contraction and expansion of hysteresis were extracted based on the generated constraint model or the so called “virgin curve” of the hysteresis data. In general, all of the proposed models were implementing approximation methods, and for that reason, no model has yet to achieve a satisfactory output force or pressure prediction. Therefore, for the proposed hysteresis modelling prediction method using general cubic equation and factor theorem, there are various parameters which need to be considered in order to obtain satisfactory and precise prediction of hysteresis modelling such as the minimum contraction, maximum contraction, first real root of constraint model at minimum loading condition, tangency point, inflection point, regression factors (i.e., minimum and maximum contractions), growth factor (i.e., intersection point) etc.

Furthermore, according to the review analysis on the recent trends of the pneumatic muscle actuated rehabilitation orthosis [21, 22, and 23], it could be understood that the suitable control schemes and strategies for this type of orthosis have yet to be found. Albeit that, this only suggested that the space available for the rehabilitation orthosis improvement and enhancement in either mechanical design or control strategies are still boundless. This opportunity will attract the researcher’s interest in proposing different new ideas and strategies to rectify previous methods or to discover new methods in pneumatic artificial muscle’s hysteresis modelling and its control scheme and strategies.

There are arguments in the field of rehabilitation robotics regarding what was the best control scheme and strategy to the pneumatic artificial muscle actuated orthosis problem. It is preferred that control systems should be simplified as much as possible; multiple sensors and impedances will only increase the complexity of control systems [21]. Rather than using a very complicated algorithm for control system of the pneumatic artificial muscle, a simple and noble method using general cubic equation based on factor theorem is proposed for the prediction of the hysteresis model at different loads. This research study was proposed in order to resolve two major problems in pneumatic muscle actuated rehabilitation orthosis which including; first is the characterization of the inherent hysteresis data to rectify the statics and dynamics behaviours of pneumatic muscle; and second is the introduction and implementation of hysteresis modelling prediction method at different loading conditions.

Methodology

The hysteresis modelling of pneumatic artificial muscle using general cubic equation and factor theorem prediction method are divided into four stages. The first stage is the characterization of the hysteresis data at different loading conditions which were obtained through an experimental study. The McKibben artificial muscle was chosen because of its advantageous properties and inherent compliance. The critical parameters of the pneumatic artificial muscle’s hysteresis data (i.e., first real root, tangency point, inflexion point, maximum contraction, minumum contraction, growth and regression factors) were investigated and determined. The experimental study that was carried out for this research is based on the isotonic test (i.e., pressure vs. contraction) at different loads and maximum operating pressure. The second stage of this study is to develop and establish a suitable model equation representing the hysteresis data at minimum loading condition with general constraint model. Therefore a prediction method using general cubic equation and factor theorem was constructed to represent and generate a constraint model of the hysteresis data at different loads. In addition, monotonic graph was used to generates the constraint models, where the graph consists only one real root and the other two roots are complex conjugates. However, the graph still has an intersection point at the vertical axis. The third stage is the verification of the hysteresis data (i.e., contraction and expansion) at different loads with general ellipse equation not centered at origin. This is a simple theorem proposed for extracting hysteresis models of a pneumatic artificial muscle at different loads based on the generated constraint models. The final stage of the research is to obtain the hysteresis models at different loading conditions based on the general cubic equation and its factor theorem.

The physical configuration provides the pneumatic artificial muscle with some desirable characteristics such as a variable stiffness or compliance, inherent damping, structural flexibility, as well as high power to weight ratio [1 – 4]. However, it exhibits a nonlinear relationship between the contracting force, internal pressure and its contraction length. In addition, the structural materials of the pneumatic artificial muscle inherently lead to hysteresis during cyclic contraction and extension [4]. This nonlinearity behaviour is addressed as a difficult error source to be handled especially during a precise positioning control. The hysteresis loops can be visibly seen by using an isometric test (i.e., force and length hysteresis) or an isotonic test (i.e., pressure and length hysteresis).

Hysteresis data characterization

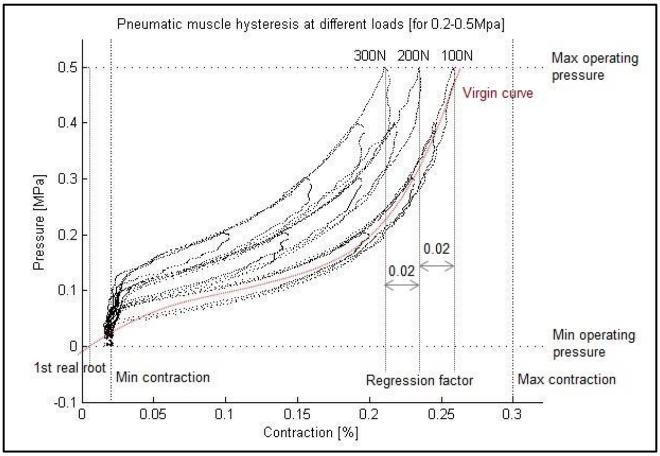

Figure 1: Hysteresis data at different loads of 100 - 300 N and pressure of 0.2 – 0.5 MPa

The first stage of this study is the characterization of pneumatic artificial muscle at different loads and pressures [23]. In this characterization stage, the hysteresis characteristics of the pneumatic artificial muscle at different loads such as its minimum contraction, maximum contraction, regression factor of minimum contraction, regression factor of maximum contraction, real root of the hysteresis's constraint models, tangency points, inflection points, growth factor for the graph stability, maximum operating pressure, and model equation for the hysteresis's offset were investigated. The identification of these parameter values and models are one of the crucial part of this research. Furthermore, these parameters are essential and critical for the development of a simple and noble model equation to represent the hysteresis data of the pneumatic artificial muscle. In addition, four parameters which determine the nonlinearity area of the hysteresis data are the minimum contraction, maximum contraction, maximum operating pressure, and constraint model at minimum loading condition. The experimental hysteresis data at different loads of 100 – 300 N and pressure of 0.2 – 0.5 MPa is shown in Figure 1.

Constraint model prediction method

As for the second stage of the study, the prediction method for generating constraint model of a pneumatic artificial muscle's hysteresis data at different loads was designed. The purpose of the hysteresis modelling and prediction method is to increase the effectiveness and the accuracy of a pneumatic artificial muscle based on rehabilitation orthosis control system [11 – 12]. An experimental study was conducted to identify the dynamic characteristics of the pneumatic artificial muscle such as dimension (i.e., length and muscle contraction), pressure, and contraction force [11, 12, and 14]. The mathematical model proposed for the constraint model of hysteresis data was established using a 3rd order polynomial equation (i.e., general cubic equation and cubic equation by factor theorem). This particular method is used because the hysteresis data of pneumatic artificial muscle exhibit monotonic behaviours during isotonic test (pressure vs. contraction length).

$$f(x) = Ax^3 + Bx^2 + Cx + D \tag{1}$$

The general cubic equation has the form as shown in Equation (1); the term x^3 represents cubic equation (thus $a \neq 0$) as shown in Equation (4), but any or all of b, c and d can be zero. Just as a quadratic equation may have two real roots, a cubic equation possibly has three real roots. But unlike a quadratic equation which may have no real solution, a cubic equation always has at least one real root. If a cubic does had three roots, two or even all three of them may be repeated. This general cubic equation can be factorised to give a cubic equation with factor theorem as shown in Equation (2) and (3). From these characteristics, you can see why a cubic equation always has at least one real root. The cubic graph either start at large and negative or finish at large and positive (when the coefficient of x^3 is positive), or it start at large and positive and finish off at large and negative (when the coefficient of x^3 is negative). The graph of a cubic must cross the x-axis at least once giving you at least one real root. Thus, any problem that involves solving a cubic equation will have a real solution.

If the cubic equation $Ax^3 + Bx^2 + Cx + D = 0$ with integer coefficients has a rational root, it can be found using the rational root test: If the root $r = {}^{m}/_{n}$ is fully reduced, then m is a factor of D and n is a factor of A. All of the possible combinations of values for m and n (both positive and negative for one of them) can be checked for whether it satisfy the cubic equation. The rational root test may also be used for a cubic equation with rational coefficients using multiplication by the lowest common denominator of the coefficients. Thus, produces an equation with integer coefficients which have exactly the same roots. The rational root test is particularly useful when there are three real roots because of the algebraic solution unhelpfully expressed the real roots in terms of complex entities (if the test yields a rational root, it can be factored out and the remaining roots can be found by solving a quadratic). The rational root test is also helpful in the presence of one real and two complex roots because again, if it yields a rational root, it allows all of the roots to be written without the use of cube roots. If R is any root of the cubic, then factorizing out $x - R$ using polynomial long division to obtain the following equations;

$$f(x) = (x - R)(Ax^2 + (B + AR)x + C + BR + AR^2) \tag{2}$$
$$f(x) = (x - R)(ax^2 + bx + c) \tag{3}$$

where R is the first real root, $a = A$ is the constant that determine the amplitude of the cubic equation, b is the constant that determine the width of the graph and its gradient, and $c = {}^{D}\!/_{R}$ is a constant that determine the intersection of graph. Since the hysteresis data for the pneumatic muscle was using a monotonic graph, the value of parameter b is chosen to be a negative value as shown in equation (5). To implement the general cubic equation and its factor theorem, the value of the first real root cannot be zero.

$$R_i = R_i + eps \qquad (R_1 \neq 0) \tag{4}$$

The general cubic equation was implemented to fulfil the criteria of the isotonic test (i.e. pressure vs. contraction length) which exhibited monotonic behaviours. While, the factor theorem was used because, the hysteresis graph has a definite one real root, while the other roots are complex conjugates. Based on the general cubic equation, four critical points could be verified which are the first real root, inflexion point, tangency point and maximum contraction. However, the hysteresis graph has four identifiable limitations. The limitations are minimum contraction, maximum contraction, maximum operating pressure, and constraint model at minimum loading condition [22]. The minimum contraction of the pneumatic muscle's hysteresis data at different loads has the same value [23]. Unfortunately, the applied pressure will increased with an increment of loading condition. This behaviour shows that, the first real root for each constraint model at different loads will not be the same. There is an existing regression value which manipulates the value of the first real root of the general cubic equation (i.e., constraint model). In addition, the maximum contractions at different loads also decreasing with a certain regression value when the loading condition was increasing.

$$f(x) = (x - R)(ax^2 - bx + c) \quad (5)$$
$$f(x) = ax^3 - (b + aR_1)x^2 + (c + bR_1)x - cR_1 \quad (6)$$
$$f(x) = Ax^3 - Bx^2 + Cx + D \quad (7)$$

Substituting all of the initial conditions into equation (7);

$$x = R_1, f(x) = 0;$$
$$x = Rx_1, f(x) = P_{max};$$
$$x = x_{tan}, f(x) = y_{tan};$$
$$x_{tan} = \frac{b}{2a} \qquad \alpha = \tan^{-1}\frac{y_{tan}}{x_{tan}}$$
$$x = x_{inf}, f(x) = y_{inf};$$
$$x_{inf} = \frac{b+aR_1}{3a} \qquad \beta = \tan^{-1}\frac{y_{inf}}{x_{inf}}$$
$$y_{inf} = y_{tan}, x_{inf} = x_{tan} - h;$$
$$h = \frac{\sqrt{b^2-4ac}}{2a} \qquad x_{inf} = x_{tan} - \frac{\sqrt{b^2-4ac}}{2a}$$

Solving all the equations;

$$D = -AR_1{}^3 + BR_1{}^2 - CR_1$$
$$A\left(Rx_1{}^3 - R_1{}^3\right) - B\left(Rx_1{}^2 - R_1{}^2\right) + C(Rx_1 - R_1) = P_{max}$$
$$A\left(x_{tan}{}^3 - R_1{}^3\right) - B\left(x_{tan}{}^2 - R_1{}^2\right) + C(x_{tan} - R_1) = y_{tan}$$
$$A\left(x_{inf}{}^3 - R_1{}^3\right) - B\left(x_{inf}{}^2 - R_1{}^2\right) + C\left(x_{inf} - R_1\right) = y_{inf}$$

Substitute parameter A into the equations;

$$CF_2 = \left(\frac{E_2}{E_1}\right)P_{max} - \left(\frac{E_2}{E_1}\right)G_1 + \left(\frac{E_2}{E_1}\right)CF_1 - y_{tan} + G_2$$
$$C\left(F_2 - \left(\frac{E_2}{E_1}\right)F_1\right) = \left(\frac{E_2}{E_1}\right)P_{max} - \left(\frac{E_2}{E_1}\right)G_1 - y_{tan} + G_2$$

Where;

$$E_1 = \left(\frac{Rx_1{}^3 - R_1{}^3}{x_{inf}{}^3 - R_1{}^3}\right)\left(x_{inf}{}^2 - R_1{}^2\right) - \left(Rx_1{}^2 - R_1{}^2\right)$$
$$E_2 = \left(\frac{x_{tan}{}^3 - R_1{}^3}{x_{inf}{}^3 - R_1{}^3}\right)\left(x_{inf}{}^2 - R_1{}^2\right) - \left(x_{tan}{}^2 - R_1{}^2\right)$$
$$F_1 = \left(\frac{Rx_1{}^3 - R_1{}^3}{x_{inf}{}^3 - R_1{}^3}\right)\left(x_{inf} - R_1\right) + (Rx_1 - R_1)$$

$$F_2 = \left(\frac{{x_{tan}}^3 - {R_1}^3}{{x_{inf}}^3 - {R_1}^3}\right)(x_{inf} - R_1) + (x_{tan} - R_1)$$
$$G_1 = \left(\frac{{Rx_1}^3 - {R_1}^3}{{x_{inf}}^3 - {R_1}^3}\right) y_{inf}$$
$$G_2 = \left(\frac{{x_{tan}}^3 - {R_1}^3}{{x_{inf}}^3 - {R_1}^3}\right) y_{inf}$$

$$BE_1 - CF_1 = P_{max} - G_1$$
$$BE_2 - CF_2 = x_{tan} - G_2$$
$$B = \frac{P_{max} - G_1 + CF_1}{E_1}$$
$$C = \frac{\left(\frac{E_2}{E_1}\right) P_{max} - \left(\frac{E_2}{E_1}\right) G_1 - y_{tan} + G_2}{\left(F_2 - \left(\frac{E_2}{E_1}\right) F_1\right)}$$

Obtaining all parameters for the general cubic equation;

$$f(x) = Ax^3 - Bx^2 + Cx + D$$
$$A = \frac{y_{inf} + B\left({x_{inf}}^2 - {R_1}^2\right) - C\left(x_{inf} - R_1\right)}{\left({x_{inf}}^3 - {R_1}^3\right)} \tag{8}$$
$$B = \frac{P_{max} - G_1 + CF_1}{E_1} \tag{9}$$
$$C = \frac{\left(\frac{E_2}{E_1}\right) P_{max} - \left(\frac{E_2}{E_1}\right) G_1 - y_{tan} + G_2}{\left(F_2 - \left(\frac{E_2}{E_1}\right) F_1\right)} \tag{10}$$
$$D = -A{R_1}^3 + B{R_1}^2 - CR_1 \tag{11}$$

Obtaining all parameters for the factor theorem cubic equation;

$$a = A \tag{12}$$
$$b = B - aR_1 \tag{13}$$
$$c = C - bR_1 \tag{14}$$

The obtained general cubic model equation based on developed equations (1) - (14) was then implemented into a developed general user interface (GUI) which generates a constraint model of the hysteresis data at minimum loading condition using the MATLAB language programming software. The GUI was shown in Figure 2. By specifying the locations of the tangency and inflexion points on the graph based on the hysteresis data analysis (experimental data), the parameters (i.e., a, b, c, R_1, A, B, C, and D)

for the general cubic equation and factor theorem cubic equation can be generated.

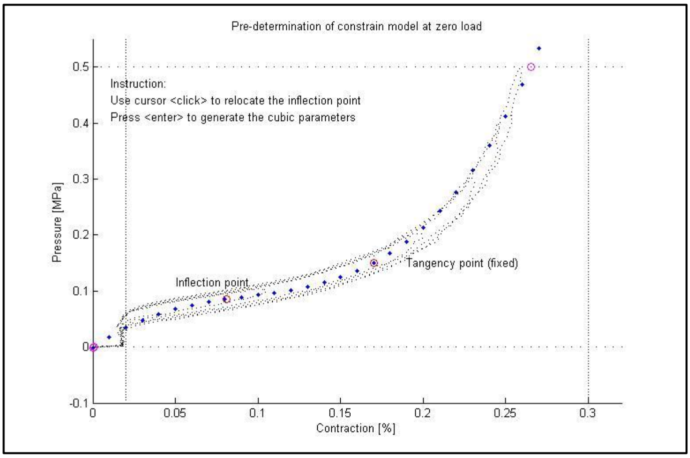

Figure 2: Constraint model using general cubic equation and its factor theorem prediction method at minimum loading condition.

Constraint model prediction at different loads

In this stage, a suitable model equation representing the constraint model of the hysteresis data at different loads was established. Thus, a suitable aproximation and prediction method for the constraint model of the hysteresis data (black dotted lines) was first established using the developed general cubic equation and factor theorem. The constaint model of hysteresis data at minimum loading condition was obtained by identifying all the critical parameters of the hysteresis data virgin curve at minimum loading condition such as the first root, contraction length at maximum operating pressure, tengency point, and inflexion point. Together with the data gained from the characterization, the constraint model at different loads could be predicted and generated by specifying the growth factor of graph stability, and regression factors of the maximum and minimum contractions. The generated constraint models of a hysteresis data at different load conditions using the proposed general cubic equation and factor theorem prediction method is shown in Figure 3 (blue dotted lines).

Constraint model equation;

$$CM_{ij} = A_i x_j{}^3 - B_i x_j{}^2 + C_i x_j + D_i \tag{15}$$

$$a_i = \frac{1}{(Rx_i - R_i)}\left(\frac{P_{max} + c_i R_i}{Rx_i{}^2} - \left(\frac{c_i + bR_i}{Rx_i}\right) + b\right) \tag{16}$$

$$R_1 \neq 0 \approx eps$$
$$R_i = R_i + eps$$

First real root regression factors;

$$R_i = R_{i-1} - RG_L \tag{17}$$

Where, the parameter $R_{i=1,2,3\ldots}$ is the first real root of the general cubic equation at zero pressure value intersection, and RG_L is the rate of regression constant of the minimum contraction value.

Maximum contraction regression factors;

$$Rx_i = Rx_{i-1} - RG_U \tag{18}$$

Where, $Rx_{i=1,2,3\ldots}$ is the maximum contraction of pneumatic artificial muscle at the maximum operating pressure, and RG_U is the rate of regression constant of the maximum contraction value.

Quadratic's intersection constant growth factors, refer to equation (3);

$$c_i = c_{i-1} + GR_c \tag{19}$$
$$c_1 = \frac{D_1}{R_1} \tag{20}$$

Where, $c_{i=1,2,3\ldots}$ is the quadratic's intersection constant of the general cubic equation, and GR_c is the growth parameter constant of the graph stability (i.e., generated constraint models at different loading conditions).
Constraint model at different loads;

$$CM_{ij} = \left(x_j - R_i\right)\left(a_i x_j{}^2 - b x_j + c_i\right) \tag{21}$$

The determination of the first real root, tangency point, and intersection point of constraint model at minimum loading condition from the hysteresis data are significant and important. If these parameters could be obtained, the general cubic equation and its factor theorem at minimum loading condition can be partially determined. Therefore, the maximum contraction of hysteresis data at minimum loading condition and maximum operating pressure values is need to be specified before the general cubic equation and its factor theorem could be fully determined and verified. With these parameters obtained, the two most critical parameters which are the amplitude and gradient of the general cubic equation could be identified. The amplitude and gradient of the general cubic equation are what actually differentiate each of the generated constraint models of the hysteresis data at different loading conditions. With the addition of the regression and growth factors, the constraint model of hysteresis data at different loads became much stable and reliable. Figure 3 shows the critical parameters (i.e., first real roots, tangency points, inflexion points, and maximum contractions) of the constraint model at different loading condition of 100 N, 200 N and 300 N using general cubic equation and factor theorem. With the verification and identification of these parameters value from the hysteresis data at different loads obtained from the experimental study, the prediction of constraint models can be generated.

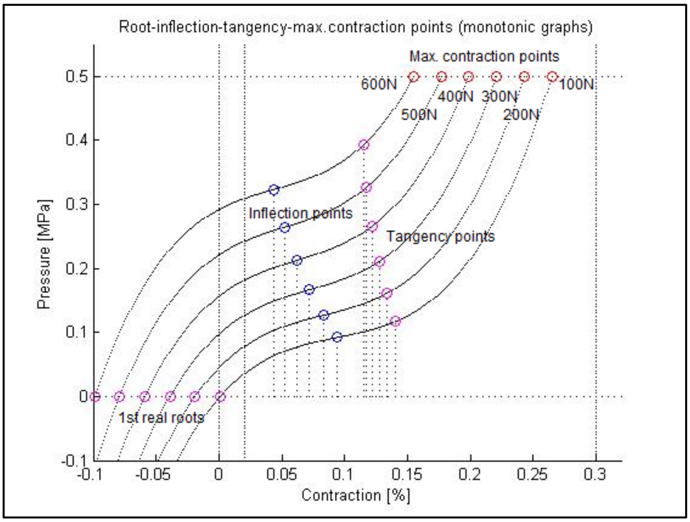

Figure 3: The critical parameters of the constraint model at different loads using general cubic equation.

Hysteresis model at different loads

The third stage is the establishment of a simple theorem to extract the hysteresis models (i.e., contraction and expansion modes) of a pneumatic artificial muscle's hysteresis data at different loads based on the generated constraint models. For this purpose, a verification model needs to be analysed and extracted from the hysteresis data patterns during contraction and expansion modes. The method to obtain this verification model is to find the contraction value difference in-between the contraction and expansion mode's data with constraint model's data at different loads. The hysteresis data first need to be separated and differentiate between the contraction and expansion mode, then compared it with the obtained constraint model of the data. With this method, the model derivation for the hysteresis offset should be able to be obtained and verified.

Based on the verification, it is found that the hysteresis offset during contraction and expansion resembles general cubic equation at all different loading conditions. The correlation coefficient values obtained between the separated hysteresis data and general ellipse equation approximately above $r = 0.8$ correlation value at different loading conditions and input pressures. Figure 7 shows the correlation coefficient value of the separated hysteresis data and general ellipse equation. Therefore, the general cubic equation not centered at origin was selected to be used as an offset to obtain the hysteresis model at different loads. General ellipse equation not centered at origin;

$$\frac{(x-h)^2}{ae^2} + \frac{(y-k)^2}{be^2} = 1 \tag{22}$$

$$y = k \pm be\sqrt{1 - \frac{(x-h)^2}{ae^2}} \tag{23}$$

Similar with the circle equations, the offsets was subtracted from the x and y terms to translate (or "move") the ellipse back to the origin. Thus, the full form of the equation is defined as equation (22) and modified to become equation (23), where the parameters a is the radius along the x-axis, b is the radius along the y-axis, (h, k) are the (x, y) center coordinates of the ellipse. By implementing the modified ellipse model into the generated constraint models (i.e., general cubic equation and factor theorem) at different loading conditions, it generates the desired hysteresis models for both contraction and expansion modes as shown in equation (25) and equation (26).

$$y_{ij}^{hys} = k_i^{hys} \pm be_i^{hys}\sqrt{1 - \frac{\left(x_j - h_i^{hys}\right)^2}{\left(ae_i^{hys}\right)^2}} \tag{24}$$

$$k_i^{hys} = 0$$
$$be_i^{hys} = 0.02$$
$$ae_i^{hys} = \frac{Rx_i - R_i}{2}$$
$$h_i^{hys} = R_1 + ae_i^{hys}$$

The hysteresis model for contraction mode;

$$HMC_{ij} = CM_{ij} + y_{ij}^{hys}$$
$$HMC_{ij} = (x_j - R_i)(a_i x_j^2 - bx_j + c_i) + y_{ij}^{hys} \quad (25)$$

The hysteresis model for expansion mode;

$$HMC_{ij} = CM_{ij} - y_{ij}^{hys}$$
$$HME_{ij} = (x_j - R_i)(a_i x_j^2 - bx_j + c_i) - y_{ij}^{hys} \quad (26)$$

The hysteresis model at different loads based on equations (25) and (26) using the general cubic equation and its factor theorem prediction method is shown in Figure 4. The generated hysteresis models used the general ellipse equation (24) to extract the contraction and expansion data from the generated constraint models at different loads. Figure 4 show the hysteresis model with load increment of 100 N, however, the developed hysteresis model was able to demonstrate an accuracy up to increment of 1 N.

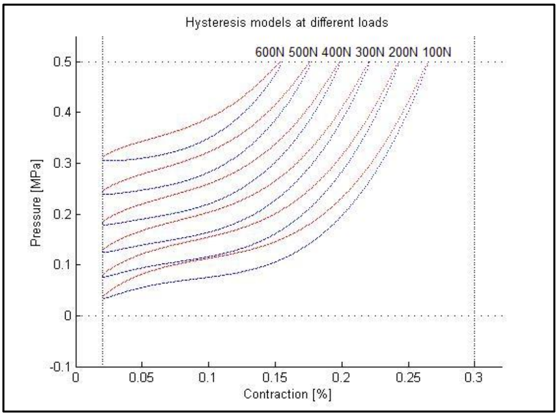

Figure 4: The hysteresis model at different loads using the general cubic equation and its factor theorem prediction method.

The width of the ellipse equation, be is determine from the isotonic test graph at minimum loading condition. There were two methods considered when extracting the hysteresis model from the constraint model equation. The first one is by finding the perpendicular distance in between the points, then verify the obtained contraction and expansion hysteresis data with the general ellipse model equation. The second method is by increasing and decreasing the constraint model at each points using general ellipse model equation. Based on the results, the second method shows a better result and easier to be implemented.

Results and discussion

The developed hysteresis models using the general cubic equation and factor theorem at different loads were compared with the hysteresis data obtained from the experimental study for every loading of 100 N, 200 N, and 300 N. Several parameters were first specified before the developed hysteresis modelling able to generate the desired hysteresis models at different loads such as the first real root, tangency point, minimum contraction, maximum contraction, and maximum operating pressure. These parameters were obtained at minimum loading condition of 100 N loads. Then, by implementing the measured regression factors of the minimum and maximum contraction as well as growth factor of the quadratic intersection point of the factorized general cubic equation, the hysteresis model at different loads was obtained. Based on the results, it shows that the developed hysteresis modelling prediction method was able to generates an appropriate constraint models and demonstrate comparable hysteresis models during cyclic compression and expression modes compared to the experimental hysteresis data. Figure 5 shows the hysteresis model verification at different loads using the general cubic equation and its factor theorem. The hysteresis model (i.e., virgin curve) of 100 N was based on the hysteresis data, and the remaining models were approximation model using the proposed method. The result shown that the obtained hysteresis models at different loads were able to represent the hysteresis data at 200 N and 300 N.

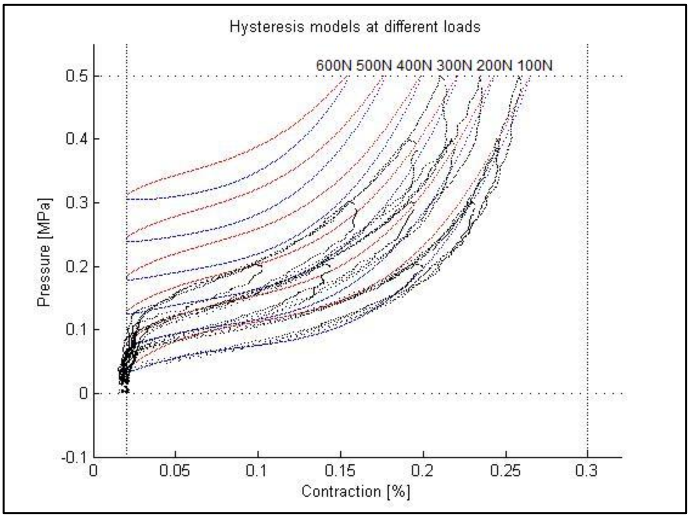

Figure 5: The hysteresis model verification at different loads using the general cubic equation and its factor theorem.

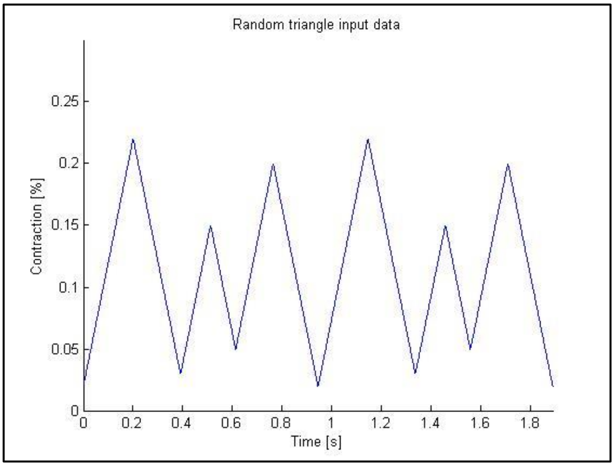

Figure 6: Random step input of loading condition to test the reliability of hysteresis model at different loads.

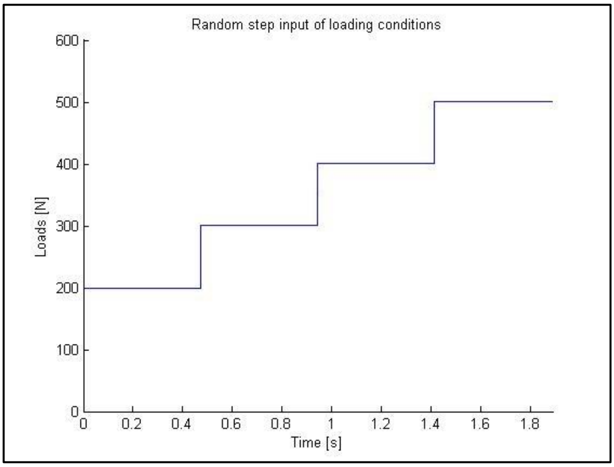

Figure 7: Random step input of loading condition to test the reliability of hysteresis model at different loads.

According to the review analysis and data verification, it is showed that the polynomial order of the virgin curves was reduced as the loading condition applied to the pneumatic artificial muscle was increased. Therefore, the generated constraint models based on general cubic equation were specified with the minimum contraction length of the pneumatic artificial muscle (i.e., 2% original length). Even though the prediction method of the hysteresis modelling was implementing general cubic equation and factor theorem to generate the constraint model for each loading condition, the introduction of minimum contraction value into the design algorithm reduced the complexity of the hysteresis model. Including the others boundary conditions or limitations that specify the nonlinearity area of the pneumatic artificial muscle, the developed hysteresis modelling was able to demonstrate a comparable simulation of the nonlinearity behaviours of the hysteresis data.

The triangle input data for the pneumatic artificial muscle contraction-expansion was used to test the design algorithm of the developed hysteresis modelling as shown in Figure 6. This data was used to alternate the input data with contraction and expansion modes. In addition, the random step input of loading conditions was also implemented to test the reliability of the hysteresis modelling with load disturbances as shown in Figure 7. Based on the results, it shows that the developed hysteresis model was able to instinctively redefine the present hysteresis model with a hysteresis model at different loads when the load difference was detected.

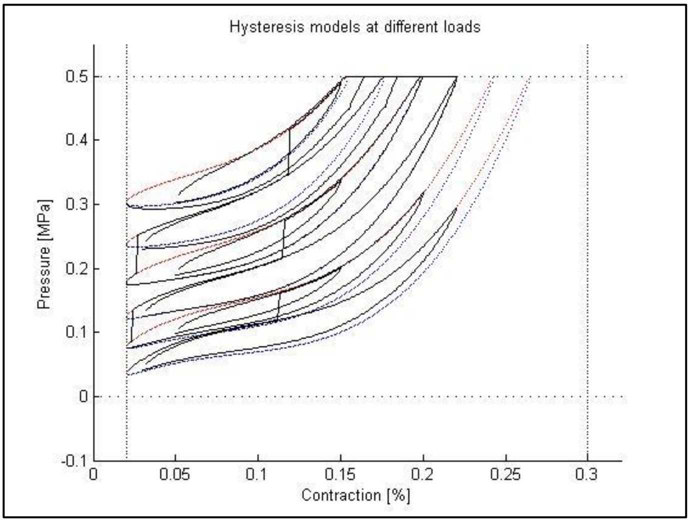

Figure 8: Hysteresis modelling simulation at different loads for cyclic contraction and expansion of pneumatic artificial muscle.

Figure 8 shows the hysteresis modelling simulation of the model at different loads for cyclic contraction and expansion of pneumatic artificial muscle. The results shows that hysteresis modelling was able to adapt with the change in load conditions (i.e., change of loads between 100 N up to 600 N). The hysteresis model was able to generate correction of parametric values to achieve or redefine its present hysteresis model when change in-between contraction and expansion modes occurred. Regarding the implementation of non-local memory, the design algorithm should be able to provide the generated constraint models with an increment value (during contraction) or decrement value (during expansion) of the hysteresis offset. Therefore, throughout the separated hysteresis data verification, modified general ellipse model was selected to extract the hysteresis models at different loads. With all of the achieved results, the design algorithm of the hysteresis modelling using general cubic equation and factor theorem prediction method was proved to be able to generate and simulate comparable hysteresis models of the pneumatic artificial muscle at different loads.

Conclusions

As a conclusion, a general cubic equation and factor theorem was used as a prediction method to design the hysteresis modelling for the pneumatic artificial muscle at different loading conditions. The development of this hysteresis modelling needs several boundary conditions or limitations which specify the nonlinearity area of the pneumatic artificial muscle during the cyclic contraction and expansion. In addition to this, regression factors of the minimum and maximum contractions at different loads as well as growth factor of the general cubic equation's intersection point for the graph stability needs to be identified throughout an experimental study of hysteresis data. With all of the critical parameters obtained, the hysteresis modelling prediction method of pneumatic artificial muscle at different loads was developed and verified. Based on the results, it shows that the developed algorithm was able to generate an appropriate constraint models to represents the virgin curves of the hysteresis data. The hysteresis models were then extracted from the generated constraint model by implementing the modified general ellipse equation as an offset during cyclic contraction and expansion. The results obtained shows that hysteresis models at different loads were comparable to the hysteresis data obtained from an experimental study. Furthermore, the pneumatic artificial muscle exhibits a non-local memory where it states that when the contraction or expansion of the pneumatic artificial muscle did not reach its maximum value, it will create a different path to return to its initial position. This non-local memory plays a major factor in hysteresis modelling. Without this factor, there will be a sudden increase in the pressure/force value (i.e., isotonic or isometric tests) of the hysteresis model when the modes (i.e., contraction and expansion) were shifted. Thus for future work, the effect of the non-local memory will be included into the design algorithm using general cubic equation and its factor theorem. This research finding will be important in improving lots of control system areas which implement pneumatic artificial muscle and equivalent actuators in the field of rehabilitation orthosis, robotics, biomedical etc.

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