

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

**COEFFICIENT ESTIMATES FOR A SUBCLASS
OF CLOSE-TO-CONVEX FUNCTIONS
ASSOCIATED WITH TILTED KOEBE
FUNCTIONS**

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TABLE OF CONTENTS

ACKNOWLEDGEMENTS	I
TABLE OF CONTENTS	II
LIST OF SYMBOLS	III
LIST OF FIGURES	III
ABSTRACT	IV
1. INTRODUCTION	1
1.1 Problem Statement	3
1.2 Objectives	3
1.3 Significance of Study	4
1.4 Scope and Limitation	4
2. BACKGROUND THEORY AND LITERATURE REVIEW	5
2.1 Background Theory	5
2.2 Literature Review	8
3. METHODOLOGY	12
3.1 Reference Model and Approach	12
3.2 Implementation Step	13
4. RESULTS AND DISCUSSION	18
4.1 Representation Theorem	18
4.2 Coefficient Estimate	28
4.3 Validation of Results	40
5. CONCLUSION AND RECOMMENDATION	42
6. REFERENCES	43

LIST OF SYMBOLS

Symbol	Description
C	Set of complex number
D	Domain
Y	Open unit disc $Y = \{s : s < 1\}$
N	Class of normalized univalent function
S	Class of starlike function
K	Class of convex function
P	Positive real part
L	Class of close-to-convex function
$k(\varepsilon, s)$	Generalized Koebe Function

LIST OF FIGURES

Figure 3.1: Implementation Steps	13
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ABSTRACT

Every class of function has different extremal properties such as coefficient estimates, arc length, distortion theorem, radius properties, growth theorem, center and radius and area. Furthermore, it becomes a huge problem to define a suitable method in finding the extremal properties for this new class of function and the coefficient estimates. In this research, we define a subclass of close-to-convex functions associated with generalized Koebe functions and find extremal properties for this generalized Koebe functions.

In this project, we focus on subclass of close-to-convex functions associated with generalized Koebe functions which satisfies the condition $M(\lambda, \varepsilon, \delta)$ denoted as

$$\operatorname{Re} \left\{ e^{i\lambda} \frac{sf'(s)}{k(\varepsilon, s)} \right\} > \delta \quad (s \in Y),$$

in open unit disc $Y = \{s : |s| < 1\}$, where $|\lambda| < \pi, \cos \lambda > \delta, 0 \leq \delta < 1, -1 < \varepsilon \leq 1$. The representation theorem and coefficient estimate as their extremal properties are achieved for the function of this generalized Koebe functions.

The result of the coefficient estimates that we obtain is

$$|a_n| = \left(\frac{\varepsilon^n + (1 - 2g_{\lambda\delta})\varepsilon^{n-1}}{(1 + \varepsilon)} \right) (-1)^{n-1} + \left(\frac{2\varepsilon g_{\lambda\delta}}{(1 + \varepsilon)^2} \right) (-1)^{n-1} + \left(\frac{2g_{\lambda\delta}}{n(1 + \varepsilon)^2} \right).$$

The aim of this project will be on some extremal properties of subclass of close-to-convex functions associated with generalized Koebe functions based on Herglotz representation theorem, as used by Rathi, Soh, & Akbarally (2018) and Mohamad (2000). The class of function and method that we used can help other researcher to understand well from the result that we obtained. Moreover, other researchers can use this project to find new extremal properties of different class of function as their reference.