HANDLING MIXED NUMBERS ON A NUMBER LINE – A PROBLEM AMONG LOWER SECONDARY STUDENTS

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Abstract
Primary school Mathematics curriculum dictates that students will be taught to look at fractions in three different forms: proper fractions, improper fractions and mixed numbers beginning Year 3 at primary schools. Based on their four-year exposure to fractions at primary school, students should be able to determine equal intervals on a structured number line. Using the part-whole approach to interpret fractions, students will normally break up the distance between two points on the number line into equal number of parts to represent equal intervals. In a previous study done to assess the conceptual understanding of fractions among secondary students, there were three items involving students handling fractions that attracted our attention, i) item involving naming proper fractions on a number line, given two reference points, 0 and 1; ii) item involving placing a mixed number on a number line, given two consecutive proper fractions as reference points and iii) item involving naming two fractions A and B placed an interval distance to the right of the first and second reference points, respectively. Estimation is a process that needed preliminary selection of simple numbers to be worked on mentally and this choice of number will help in approximating the results. There were few shortcomings on the part of the students in the estimation processes that they went through. As a result, the previous study indicated students facing difficulties as they progressed through from items i) to iii). There is a need to understand this phenomena, thus, these three items were included as items in the Probing Interview for the current study which was conducted on random samples of students from selected colleges in the North Zone of Malaysia. It is hoped that these findings will shed some information on fraction confusion that interfered with their estimation and computation abilities.

Keywords: *probing interview, structured number line, part-whole, mixed numbers*

1. INTRODUCTION

Mathematics educators have discussed fervently the significant role of estimation in the learning and use of mathematics. Being able to estimate helps to provide clarity of thoughts and clears the passage for discussions, facilitates problem solving, and develops a consistent attitude to procedural applications (Usiskin, 1986). However, the focus of today's school mathematics is centered more on computation in the number strand.

When a student does an estimation exercise, he selects simple numbers to operate on mentally and this selection will result in an approximate answer (Reys, 1984; Segovia & Castro, 2009); this indicates that there is a close relationship between estimation and mental computation. Primary school students are taught to do mental calculation as early as Year One (Mathematics Year 1, 2002). We conducted a study on 13-year old students from four selected colleges in the North Zone of Malaysia with the objective to compare the computation and estimation abilities of students in the Malaysian setting. These 13-year olds have undergone six years of learning at primary schools and were exposed to mental calculation as early as Year One and were introduced the estimation concept at Year Three.

They were asked to respond to a Computation Test and an Estimation Test, with 15 similar items covering four major areas in the curriculum, namely, numbers, decimals, money and fractions. Then six students, three males and three females, were selected to be interviewed using the Probing Interview instrument.

The Probing Interview instrument was developed to assess how students think when they estimate and compute. It consisted of six items based on the curriculum for Mathematics Year Three to Year Six on the topic of fractions and decimal numbers. This paper will discuss the problems students have in handling mixed numbers on a number line with two reference points.

1.1 Definition Of Estimation In This Study

According to Segovia & Castro (2009), one can look at estimation from two perspectives: as computational estimation or measurement estimation. Computational estimation deals with arithmetic operations and how to judge the meaning of its results. Measurement estimation refers to how one judge the results from taking measurements. The analysis of responses to the three instruments used in this study adapted these estimation concepts by Segovia and Castro.

1.2 Estimation And Number Sense In The Primary School Curriculum

Mathematics education stresses the importance of developing number sense. This importance has been integrated into the Primary School Mathematics curriculum. For example, in teaching Whole Numbers to Year One students, in teaching students to understand and use the vocabulary of comparing and arranging numbers of quantities, students will be taught to identify one more or one less. Therefore, teachers should emphasize the fact that a number following another number in the counting on sequence is larger (Mathematics Year 1, 2002). At Year Three, teachers need to take note that students should be made to know not only that a number following another number in a counting on sequence is larger but also a number following another number in a counting down sequence is smaller (Mathematics Year 3, 2003).

At Year Six, when students are taught to divide fractions with a whole number and a fraction, teachers are required to model the division of fraction with another fraction as sharing. For example, in explaining that 1/2 when divided by 2 equals 1/4, teachers explain that half a vessel of liquid poured into a quarter-vessel makes two full quarter-vessels (Mathematics Year 6, 2006).

When students develop number sense, they show an improvement in their understanding of numbers, ways to represent numbers, ways to relate numbers and number systems; their understanding of operations and how they relate to one another; and their ability to compute fluently and to estimate reasonably (National Council of Teachers of Mathematics, 2000). Estimation is introduced as early as Year Three at primary schools and needs to be taught by teachers with the objective of developing a) an awareness of estimation, b) number sense, c) number concepts and d) estimation strategies, so that students can be more appreciative of estimation (Reys, 1986). In order for students to be appreciative of the concept, estimation cannot be taught as an isolated topic.

1.3 Developing Estimation And Mental Computation Abilities Of School Students

Approximate answers results from the selection of simple numbers to operation on in an estimation exercise (Reys, 1984; Segovia & Castro, 2009), thus there exists a close relationship between estimation and mental computation. In particular, Year One students are taught to do mental computation in the topic on Numbers (Mathematics Year 1, 2002).

Mental computation is not an inborn process; it is a process that has to be developed. However, the development does not necessarily be in the form of a test, it may take place in many ways (Heirdsfield, 2002). Mental computation produces exact answers and does not depend on external aids like pencil and paper (Reys, 1984). The experiences and practices one goes through can help develop more sophisticated strategies than traditional written methods (McIntosh, 2002; Asplin, Frid and Sparrow, 2006).

1.4 Estimation And Number Sense Computation And Polya's Four Step Algorithm

When students are able to estimate, they become more proficient at problem solving and are more consistent in their procedural applications (Usiskin, 1986). In particular, in order to be more proficient at problem solving, at Year Four students are taught to approach problem solving by using Polya's Four-Step Algorithm of understanding the problem, devising a plan, implementing the plan and looking back (Mathematics Year 4, 2006).

1.5 Estimation Helps Develop Understanding Of Number Size In Primary School Students

Behr & Post (1986) stated that in order to be able to estimate numbers, students will have to understand the size of the numbers, and likewise, estimation can help develop an understanding of number size. This can be observed in the teaching of the approximation concept, which is synonymous to the estimation concept.

Primary school students are exposed to the terms "approximate" or "approximation" (Mathematics Year 3, 2003; Mathematics Year 4, 2006; Mathematics Year 5, 2006, Mathematics Year 6, 2006). To approximate means finding a result which is not quite exact, but only slightly more or less in number or quantity and the proximity of the approximated value to the exact value can be controlled to a certain extent (Segovia & Castro, 2009), thus, approximation is similar to estimation in that it provides closeness to the exact value.

1.6 Computational Estimation And Measurement Estimation In The Primary School Mathematics Curriculum

Examples of applications of computation estimation can be seen in the Mathematics curriculum for Year Three and Year Four. Year Three students are taught to recognize one whole, one half, one quarter and three quarters and teachers must emphasize that fractions as equaled size portions of a whole or equal shares of a whole set. They are also taught to understand that the number following another number in the counting on sequence is larger and likewise, the number following another number in the counting back sequence is smaller. If we sum both these objectives, Year Three students can not only do accuracy check of the position of the numbers but they should also be able to place proper fractions in between 0 and 1 on a number line (Mathematics Year 3, 2003). At Year Four, this knowledge is enhanced further and students are taught to use number lines to express equivalent fractions to its simplest form of a proper fraction (Mathematics Year 4, 2006).

Year Four students improve their ability to do problem solving when they are taught to use Polya's Four-Step Algorithm of understanding the problem, devising a plan, implementing the plan and checking the solution, for example, on questions involving subtraction of proper fractions. In addition, Year Four students are also taught to use number lines to solve problems involving subtraction of proper fractions (Mathematics Year 4, 2006).

There are more instances of the use of the number lines in the Mathematics curriculum. For instance, besides learning how to convert fractions to decimals of tenths, hundredths, tenths and hundredths, and vice versa, Year Four and Year Five students are taught to recognize the place values of the tenths, hundredths, tenths and hundredths and so on as well as use the number line to represent these decimal numbers (Mathematics Year 4, 2006; Mathematics Year 5, 2006).

Clear usage of estimation is observed at Year Five in this topic when students are taught to round off decimal numbers to the nearest tenths or hundredths. Teachers are encouraged to use overlapping slides to compare decimal values of tenths, hundredths and thousandths (Mathematics Year 5, 2006).

When Year Six students learn to solve problems involving addition of mixed numbers, students are again required to employ the Polya's Four-Step algorithm for problem solving. When they get to the topic on problem solving involving subtraction of mixed numbers, they are again exposed to the use of number lines. At Year Six, students learn to use divide fractions with a whole numbers (Mathematics Year 6, 2006).

Some application of measurement estimation is observed in the curriculum for Year Five and Year Six Mathematics. For example, students are required to apply the four-step algorithms to the topics related to money, length, time, mass and volumes of liquid.

2. Methodology

385 students from the four selected colleges in the North Zone of Malaysia were required to sit for a 15-item Computation Test and a 15-item Estimation Test. Both tests have similar stem items covering four topics in the Mathematics Year Three to Year Six curriculum: whole numbers, fractions, decimals and money. The multiple-choice format was chosen for the Estimation Test to safeguard against students doing precise calculations (Bana & Dolma, 2006). The responses to both tests were analyzed using Rasch Measurement Model. At the end of the test time, six selected students (3 boys and 3 girls) were handpicked to be involved in the Probing Interviews.

Table 1 summarizes the item reliability, person reliability, item raw score-to-measure correlation and person raw score-to-measure correlation of responses to the Computation Test and Estimation Test.

TABLE 1
SUMMARY STATISTICS OF MEASURES OF RELIABILITY AND CORRELATION

ITEMS	TYPE OF TEST	
	Computation Test	Estimation Test
Item Reliability	0.98	0.98
Person Reliability	0.44	0.64
Item Raw Score-To-Measure Correlation	-0.96	-0.94
Person Raw Score-To-Measure Correlation	0.97	0.98

3. RESULTS AND DISCUSSIONS

Sekaran (2003) stated that reliability indicates stability and consistency of instrument to measure the concept and assess goodness of measure. To Zikmund (2003), reliability indicates the degree of freedom from error and yields consistent results. Specifically, person reliability index indicates the likelihood that person ordering is replicable if the sample of persons were given another similar instrument measuring the same construct (Bond & Fox, 2007). Likewise, the item reliability index indicates the likelihood of item ordering that is replicable given another sample of the same size with the same mode of behavior (Bond & Fox, 2007).

Table 1 reports the same index of 0.98 for item reliability of both tests. Item reliability is not dependent on the length of the test, this value simply implies these tests have a wide difficulty range and the sample is large and consistency can be expected of these inferences. This also means that the item ordering has a very high probability of being replicated if these same items are given to a different group of students (Bond & Fox, 2007).

However, the person reliability index is 0.44 for the Computation Test and 0.64 for the Estimation Test and both these values are considered low. Person reliability is not dependent on sample ability variance, thus, this low value may imply there is not much difference between their abilities, thus making it impossible for the samples to be discriminated into different levels. Therefore, this sample is not able to demonstrate a hierarchy of ability (Bond & Fox, 2007).

A reliability index lower than about 0.60 implies that one cannot confidently distinguish the top measure from the bottom one (Fisher Jr., Elbaum & Coulter, 2010). When reliability increases, the number of ranges in the scale that can be distinguished with confidence across samples also increases. Measures with reliabilities of 0.67 will tend to vary within two groups that can be separated with 95% confidence, measures of reliabilities of 0.80 will vary within three groups; of 0.90, four groups; 0.94, five groups; 0.96, six groups; 0.97, seven groups, and so on (Fisher Jr. et al, 2010). Since person reliability is not dependent on sample size, low person reliability may also mean that the test is not long enough, or there are not many categories per item (Winsteps, 2011).

As can be seen from Table 1, the person raw score-to-measure correlation is reported as 0.97 for the Computation Test and 0.98 for the Estimation Test. Table 1 also reports the item raw score-to-measure correlation as -0.96 for the Computation Test and -0.94 for the Estimation Test. For both these values indicate that the proportion of very high and very low scores is low (Winsteps, 2011), implying that the selected students from these colleges have a small ability difference between them. This paper will discuss the problems these students have in handling mixed numbers on a number line with two reference points. We will focus our discussions on three items, Items 4 to 6 in the Probing Interview.

There were unanswered questions from a previous study by Noordin, Abdol Razak, Dollah and Alias (2009) which was undertaken to assess the conceptual understanding of fractions among secondary students. We will analyze Items 4 to 6 in the Probing Interview to help us further understand the problems students face when dealing with fractions on a number line. Items 4 and 5 are the same in both studies. Item 6 have a slight difference. Instead of 1 1/4, Item 6 in the current study required students to place 1.25 which is the equivalent value to 1 1/4 on a number line.

Noordin et al (2009) found that students were able to name proper fractions on a number line with two reference points, 0 and 1. The findings from the current study agree with the previous findings by Noordin et al. To elaborate further, we will discuss the responses to Item 4 of the Probing Interview, as displayed in Figure 1.

Item 4: **Diagram 1** shows a number line. What fraction must be written in the box?

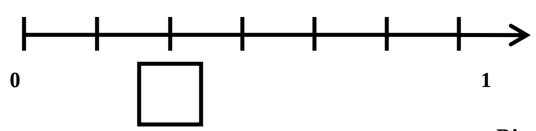

Diagram 1

Figure 1. Item 4 of the Probing Interview

Majority of the students who were interviewed were able to name the fraction that must be written in the box in Diagram 1 as 2/5. They supported their responses by explaining that there were five intervals of size 1/5 between 0 and 1. However, there was one student who stated that the number of intervals between 0 and 1 is 6, hence giving his answer as 1/3. The following paragraph discloses the contents of this particular interview session.

Student: “Diagram 1 shows a number line. What fraction must be written in the box?” 1/3.
Teacher: This is 1/3. How did you get 1/6 down here?
Student: Because, it line have a six.
Teacher: You have 6 lines. Ok, beginning from?
Student: Beginning from zero.
Teacher: Ok, zero. Kemudian (translated as then), 1/6 then…
Student: 3/6, 5/6…eh, 4/6, 5/6, 7/6
Teacher: Ok, then this is should be?
Student: 1/3

The highlighted portion of the transcript of the interview details out the error that was made. To this student, the intervals are determined by how many marks there are on the line; if there are six marks then the interval should begin with 1/6. This implies that the student did not have the prior knowledge that an interval is defined by the value of the distance between two marks on a number line.

In the previous study, only about 50% of the respondents were able to mark 1 1/4 on the number line and only about 32% of the respondents were able to name A and B in Item 5. Items 5 and 6 are displayed in Figure 2 below.

Item 5: **Diagram 2** shows a number line.

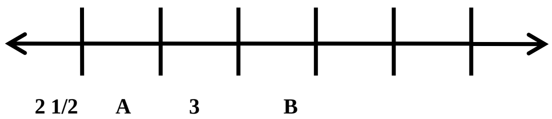

Diagram 2

Determine the value of A and B.

Answer: A = ________ and B = __________

Item 6: **Diagram 3** shows a number line. Show **1.25** by marking the number line.

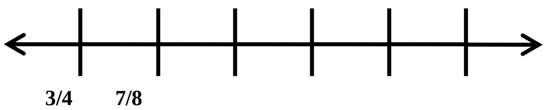

Diagram 3

Figure 2: Item 6 of the Probing Interview

It was discovered that out of five selected responses to Item 6, four explained the process of getting the size of the interval between 3/4 and 7/8 correctly and four out of five responses managed to conclude that the sixth mark on the number line was 1.25. Below are two excerpts from two correct responses from the interview sessions.

<u>Sample response 1:</u>

Teacher: Ok, where is 1.25 on the number line? How do you get that to be 1.25?
Student: Because 1/ 8… plus 8 multiply 1 and plus 2… can get 10/8
Teacher: 10/8, then… 10/8 is?
Student: 10 divided by 8… so, get…
Teacher: One point…
Student: One here… yeah
Teacher: Then, how do you get that to be 10/8?
Student: Because 7, ¾… 4 can make it, multiply by 2 and when 3 multiply by 2 can be 6/8. So, 7/8, 8/8, 9/8, 10/8

Sample response 2:

Student: "Diagram 3 shows a number line. Show 1.25 by marking the number line."
Teacher: Ok
Student: ι sama dengan 3/4 , ni saya darab dengan 2, 4 darab 2, 3 darab 2 sama dengan 6/8. So, sini 5/8, 7/8, 8/8, 9/8, 10/8. 10/8 ni sama dengan 1 1/4. So, 1/4 saya darab dengan 100 dapat 2/5. Saya tambah 1 kat depan tu… gerakkan titik perpuluhan 100, 1.25 lah.

(Translated as: 7/8 is equal to 3/4 , this I times 2, 4 times 2, 3 times 2 equals to 6/8. So, here 5/8, 7/8, 8/8, 9/8, 10/8. 10/8 is equal to 1 1/4. So, 1/4 I times 100 get 2/5. I add 1 in the front then I move the decimal point 100, 1.25 then).

Both these responses indicate students' abilities to find the equivalent fractions for 3/4 and then build an increasing sequence with an arithmetic difference of 1/8. This became the prior knowledge required before the students could decide that 10/8 was equivalent to 1.25.

We sometimes encounter instances when we cannot in any way find a reasonable explanation for the response students give. One such example is the following response to Item 6. We are not able to find an explanation for this response. It looked as if the student was trying to bring the both fractions 3/4 and 7/8 to a common denominator. It would an advantage if we understood what went amiss in this student's conceptual understanding of equivalent fraction.

Sample response 3:

Student: 3 point…eh, "diagram 3 shows a number line. Show 1.25 by marking the number line."
Teacher:
Student: ¾. I, firstly I times it with 5 and I divided with 2 and I get 7.1…
Teacher:
Student: 7/8. I time it with 5. Then I divided it by 4, and I got 8.3
Teacher:
Student: minus 7.1 equal to 1.2
Teacher:
Student: And the answer wants me to show, 1.25 by marking the number line
Teacher: vhy you got it down there?
Student: ıo…
Teacher: ıe 1.1 just now?
Student: ranslated as: Here … 1.2)
Teacher: Ok, that is 1.25? Ok, give it circle down there. Ok, sure? Very sure?
Student: ;!
Teacher: /ou.

Out of five responses to Item 5, three responded by giving A the correct value of 2 3/4 while the other two came up with the answer 2 2/2. When asked why they chose 2 3/4, one student answered "2 1/2, if converted to decimal will give 2.5 and in between 2.5 and 3, there must be a 2.75" (translated from Malay) while another answered "after 2 1/2, 3/4 is bigger than 1/2."

One of the two students who answered A equaled 2 2/2, gave B a value of 3 2/2 while the other gave a value of B as 3 2/3. The student who answered A equaled 2 2/2 explained that after 1 is 2, so 2 2/2. Using the same argument, he explained that B is 3 2/2. To the student who gave the value 2 2/2 to A, when asked why, he responded that after 1 is 2, so 2 2/2. This same boy explained that B is 3 2/2 because after 3 is 3 1/2. Surprisingly, the other student explained that since there must still be a number after 3, he used 3 as the denominator and so B is 3 2/3.

Behr & Post (1986) stated that students needed to be able to understand the size of numbers in order to be able to estimate numbers, and likewise, knowing how to estimate can help develop this understanding of number size. However, this did not take place in the students' estimation processes of Items 5 and 6.

ACKNOWLEDGMENT

We wish to express our gratitude to UiTM for funding this research work. Our sincere appreciation is also extended to Majlis Amanah Rakyat, the principals, teachers and students of all the colleges who took part in this study for their support in achieving the objectives of the study. Last but not least, thank you to all who participated directly and indirectly in making this study possible.

REFERENCES

Asplin, P., Frid, S., Sparrow, L. (2006). Game Playing to Develop Mental Computation: a case study. Merga conference Proceedings.

Bana, J. and Dolma, P. (2006). The relationship between Estimation and Computation abilities of Year 7 students. Merga27 Conference Proceedings.

Behr, M., & Post, T. (1986). Estimation and Children's Concept of Rational Number Size. In H. Schoen & M. Zweng (Eds.) Estimation and Mental Computation: 1986 NCTM Yearbook (pp. 103-111). Reston, VA: National Council of Teachers of Mathematics. Retrieved on 17 Jan 2010 from http://www.cehd.umn.edu/rational number pro ject/86_1.html

Bond, T. and Fox, C. M. (2007). Applying the Rasch Model: Fundamental Measurement in the Human Sciences. 2nd Edition. Lawrence Erlbaum Associates. Inc.

Fisher Jr., W.P., Elbaum, B. & Coulter, A. (2010). Reliability, Precision, and Measurement in the context of Data from Ability Tests, Surveys, and Assessments. Journal of Physics: Conference Series 238 (2010) 012036. doi:10.1088/1742-6596/238/1/012036. Retrieved on 13 Jan 2010 from iopscience.iop.org/1742-6596/238/1/.../1742-6596_238_1_012036.pdf

Heirdsfield, A. (2002). Mental methods moving along. In Asplin, P., Frid, S., Sparrow, L. Merga 2006 conference Proceedings.

Mathematics Year 1. (2002). Integrated Curriculum for Primary School. Curriculum Development Center. Ministry of Education Malaysia.

Mathematics Year 3. (2003). Integrated Curriculum for Primary School. Curriculum Development Center. Ministry of Education Malaysia.

Mathematics Year 4. (2006). Integrated Curriculum for Primary School. Curriculum Development Center. Ministry of Education Malaysia.

Mathematics Year 5. (2006). Integrated Curriculum for Primary School. Curriculum Development Center. Ministry of Education Malaysia.

Mathematics Year 6. (2006). Integrated Curriculum for Primary School. Curriculum Development Center. Ministry of Education Malaysia.

McIntosh, A. (2002). Common errors in mental computation of students in grades 3 - 10. In Asplin, P., Frid, S., Sparrow, L. Merga 2006 conference Proceedings.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM. In Bana, J. and Dolma, P. Merga27 Conference Proceedings.

Noordin, N., Abdol Razak, F., Dollah, R., and Alias, R. (2009). Primary School Students' Conceptual Understanding of Fractions. Research Management Institute, Universiti Teknologi Mara, Shah Alam.

Reys, R. (1984). Mental computation and estimation: Past, present and future. In Segovia, I. & Castro, E. Electronic Journal of Research in Educational Psychology.

Reys, B.J. (1986). Teaching Computational Estimation: Concepts and Strategies. Retrieved on 21 Dec 2010 from www.sde.ct.gov/...Estimation.../Teaching_Computational_Estimation.doc

Segovia, I. & Castro, E. (2009). Computational and Measurement estimation: Curriculum Foundations and Research carried out at the University of Granada, Mathematics Didactics Department. Electronic Journal of Research in Educational Psychology. 17, 7 (1) 2009, pp. 499-536.

Sekaran, U. (2003). Research Methods for Business – A Skill-Building Approach. 4th Edition. John Wiley & Sons, Inc.

Usiskin, Z. (1986). Reasons for estimating. In Segovia, I. & Castro, E. Electronic Journal of Research in Educational Psychology. 17, (1) 2009, pp. 499-536.
Winsteps Help for Rasch analysis. Retrieved from 99.236.93.8/winman/index.htm?disfile.htm
Zikmund, W. G. (2003). Business Research Methods – 7th Edition. South-Western, Thomson Learning.