Fraction Decision Confusion – A Case Study among 13-Year Old Students

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Abstract— If one uses mental computation in an estimation procedure, there will be a previous selection of simple numbers to be operated on mentally. This choice of numbers will bring about approximate answers, thus, this implies that a close relationship exists between estimation and mental computation. A study was conducted in 2010 on 385 students from four selected colleges in the North Zone of Malaysia to assess the estimation and computation abilities of 13-year old students. These students had prior exposure to fractions at primary schools. Students were asked to respond to items on a Computation Test and an Estimation Test followed by a Probing Interview. The Computation and Estimation Tests have similar stem items. Analysis of the responses to the Computation and Estimation Tests was done using Rasch Measurement Model. Among issues investigated in the study was fraction confusion decision. This paper discusses the estimation problems student face when they compare fractions to another number. Items 2, 3 and 6 on the Computation Test were selected for analysis. Responses to all these three items demonstrated that majority of the students were able to convert fractions to decimal numbers, and vice versa. Majority of the selected students were also able to demonstrate their computational estimation ability by using prior knowledge on counting on or counting back sequence to decide which among the given decimal numbers in Item 3 was larger than 4 150/1000. However, responses to Items 2 and 6 indicated that students were confused. This impeded their judgments in deciding which among the given improper fractions in Item 2 was nearest to 10, and which among the given sums of fractions in Item 6 was nearest to 5. In terms of hierarchy of difficulty, Item 6 caused the most confusion to the students.

***Keywords*—Estimation, Computation, Fractions, Improper Fractions, Decimal Number**

INTRODUCTION

Estimation is making a "judgment of what results from a numeric operation or from the measurement of quantity ..." (Segovia and Castro, 2009). Thus, it requires mental computation, thinking and making sense of the computation and not so much of rules and mechanical procedures. Students are usually more successful on written computations than on number sense (Bana and Dolma, 2006), hence, there appears to be a significant gap between students' number sense and their computation ability.

We decided to compare the computation and estimation abilities of students in the Malaysian setting. They are exposed for four years to the estimation concept at primary schools beginning Year Three. We conducted a study in 2010 to assess the estimation and computation abilities of 13-year old students. Samples were taken from four selected colleges in the North Zone of Malaysia. Students were asked to respond to similar items on a Computation Test and an Estimation Test, which covered four major areas in the curriculum, namely, numbers, decimals, money and fractions. This was later followed by a Probing Interview.

This paper will discuss the problem of inability among students to decide whether a fraction is smaller or larger than another number. In order to evaluate how students compare values of fractions to another number, we will take a look at how students respond to three items, namely questions 2, 3 and 6 on the Computation test.

I. ESTIMATION

Estimation is very significant in the learning and use of mathematics. This importance has been stressed and emphasized by many educators. Reys (1992) suggested that more than 80% of all mathematical applications need the use of estimation over computation. Trafton (1986) stressed that curriculum developers should prioritize building a strong estimation strand into school programs. Usiskin (1986) stated that being able to estimate would help develop clarity in thinking and discussion, facilitate problem solving, and develop consistency in procedural applications. Then, students would not only be able to view mathematics as a distinct way of thinking instead of just a set of unconnected rules but they would also be able to give mathematics a place of importance in our technological society (Bana and Dolma, 2006). However, school mathematics is very much focused on computation in the number strand.

Reys (1986) stressed that estimation must be taught in a comprehensive manner; hence a teacher must aim a) to develop an awareness of estimation, b) to develop number sense, c) to develop number concepts and d) to develop estimation strategies. She pointed out that if it is taught as an isolated topic, then the effort would be counterproductive and would cause students to dislike the process.

Behr and Post (1986) stated that in order to be able to estimate numbers, students will have to understand the size of the numbers, and likewise, estimation can help develop an understanding of number size. According to Segovia and Castro (2009), estimation can either be computational estimation or measurement estimation. Computational estimation refers to arithmetic operations and how one judge the meaning of its results while measurement estimation refers to judgment made on results found after taking measurements. In this study, the instruments will adapt this estimation concept put forward by Segovia and Castro (2009). For clarity, as example, computational estimation takes place when a student tries to find an estimate of the value of 2367 multiplied by 45 while measurement estimate occurs when we want to find the estimated number of persons who participate in a parade.

II. Estimation And Number Sense

The development of number sense is important in mathematics education. National Council of Teachers of Mathematics (2000) pointed out that when students develop number sense, they will understand numbers, ways of representing numbers, relationships among numbers, and number system; understand meanings of operations and how they relate to one another; and compute fluently and make reasonable estimates. Tsao (2004) also appreciated this close association between estimation and number sense. He elaborated further by stating that estimation ability, computation ability, mental computation and affective issues are variables that affect development of number sense.

III. Estimation And Mental Computation

Development of mental computation is not inborn or inherent. Experiences and practice are required before one is able to develop strategies that are more sophisticated than traditional written methods (McIntosh, 2002; Asplin, Frid and Sparrow, 2007). This development need not necessarily be in the form of a test, it can be experienced in many other ways (Heirdsfield, 2002).

Mental computation can be characterized by its' ability to produce exact answers and its' independency of the need for external aids like pencil and paper (Reys, 1984). If one uses mental computation in an estimation procedure, there will be a previous selection of simple numbers to be operated on mentally and this choice of numbers will bring about approximate answers (Reys, 1984; Segovia & Castro, 2009), thus, this implies that a close relationship exists between estimation and mental computation. Therefore, students need to be taught how to do mental computation at school.

We can find the terms "approximate" or "approximation" in the Malaysian mathematics curriculum for primary schools (Mathematics Year 3, 2003; Mathematics Year 4, 2006; Mathematics Year 5, 2006, Mathematics Year 6, 2006). To approximate means finding a result which is sufficiently precise for a certain purpose, hence, emphasizing the fact of closeness to the exact value which can be controlled to a certain extent (Segovia and Castro, 2009). Although estimation does take error into consideration and there is no assurance of control, approximation can be considered as an estimation outcome in that it provides closeness to the exact value.

IV. Computational Estimation In The Primary School Mathematics Curriculum

Some applications of computational estimation are observed in the curriculum for Year Three and Year Four Mathematics. Year Three students are taught to understand that the number following another number in the counting on sequence is larger and likewise, the number following another number in the counting back sequence is smaller. By applying this knowledge, students can do accuracy check of the position of the numbers (Mathematics Year 3, 2003).

Year Four students are taught to determine the place values of digits in whole numbers up to 100000, thus enabling them to estimate quantities up to 100000 such as rounding off to the nearest tens, hundreds and thousands. The curriculum also encourage Year Four students to be allowed to estimate either before or after addition because "Estimating answers before adding builds confidence among pupils, while estimating after adding provides a check on operation performed" (Mathematics Year 4, 2006).

V. Measurement Estimation In The Primary School Mathematics Curriculum

Some application of measurement estimation is observed in the curriculum for Year Five and Year Six Mathematics. For example, students are required to apply the four-step algorithms to the topics related to money, length, time, mass and volumes of liquid.

VI. Methodology

The researchers in this study developed a 15-item Computation Test and a 15-item Estimation Test. Both tests have similar stem items. The stem items were chosen based on the topics in the curriculum for Mathematics Year Three to Year Six covering four areas: whole numbers, fractions, decimals, and money. The multiple-choice format was chosen for the Estimation Test to safeguard against students doing precise calculations (Bana and Dolma, 2006).

385 selected respondents from four colleges in the North Zone of Malaysia participated in the study by answering both sets of tests. Random students were selected to sit for the Probing Interview. The responses to the tests were analyzed using the Rasch Measurement Model.

Reliability of a measure indicates the "stability and consistency with which the instrument measures the concept and helps to assess the "goodness" of a measure" (Sekaran, 2003). Reliability indicates "the degree to which measures are free from error and therefore yield consistent results" (Zikmund, 2003). In particular, person reliability index indicates the replicability of person ordering one could expect if the sample of persons were given another parallel set of items measuring the same construct (Bond and Fox, 2007). Likewise, the item reliability index indicates the replicability of item placements along the pathway if the same items were given to another sample of the same size that behaved the same way (Bond and Fox, 2007).

The following Table 1 summarizes the statistics of all responses to the Computation Test.

TABLE 1
SUMMARY STATISTICS OF ALL RESPONSES

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   TABLE 3.1 ALL - COMPUTATION - RASCH 2010   ZOU804WS.TXT Jul 15 15:31 2010
INPUT: 385 Persons  15 Items  MEASURED: 385 Persons  15 Items  2 CATS       1.0.0
--------------------------------------------------------------------------------

          SUMMARY OF 379 MEASURED (NON-EXTREME) Persons
+-----------------------------------------------------------------------------+
|          RAW                          MODEL         INFIT        OUTFIT     |
|         SCORE     COUNT     MEASURE   ERROR      MNSQ   ZSTD   MNSQ   ZSTD |
|-----------------------------------------------------------------------------|
| MEAN      11.3      15.0        1.63     .78       .99     .1   1.00     .2 |
| S.D.       2.0        .0        1.03     .17       .37     .8    .82     .8 |
| MAX.      14.0      15.0        3.48    1.16      2.05    3.2   6.67    3.7 |
| MIN.       3.0      15.0       -1.81     .59       .41   -1.4    .12   -1.0 |
|-----------------------------------------------------------------------------|
| REAL RMSE    .86  ADJ.SD     .57  SEPARATION   .66  Person RELIABILITY  .30 |
|MODEL RMSE    .79  ADJ.SD     .65  SEPARATION   .82  Person RELIABILITY  .40 |
| S.E. OF Person MEAN = .05                                                   |
+-----------------------------------------------------------------------------+
          MAXIMUM EXTREME SCORE:      6 Persons

      SUMMARY OF 385 MEASURED (EXTREME AND NON-EXTREME) Persons
+-----------------------------------------------------------------------------+
|          RAW                          MODEL         INFIT        OUTFIT     |
|         SCORE     COUNT     MEASURE   ERROR      MNSQ   ZSTD   MNSQ   ZSTD |
|-----------------------------------------------------------------------------|
| MEAN      11.4      15.0        1.68     .79                                |
| S.D.       2.0        .0        1.10     .22                                |
| MAX.      15.0      15.0        4.93    1.92                                |
| MIN.       3.0      15.0       -1.81     .59                                |
|-----------------------------------------------------------------------------|
| REAL RMSE    .88  ADJ.SD     .65  SEPARATION   .74  Person RELIABILITY  .35 |
|MODEL RMSE    .82  ADJ.SD     .73  SEPARATION   .88  Person RELIABILITY  .44 |
| S.E. OF Person MEAN = .06                                                   |
+-----------------------------------------------------------------------------+
    Person RAW SCORE-TO-MEASURE CORRELATION = .97
    CRONBACH ALPHA (KR-20) Person RAW SCORE RELIABILITY = .49

             SUMMARY OF 15 MEASURED (NON-EXTREME) Items
+-----------------------------------------------------------------------------+
|          RAW                          MODEL         INFIT        OUTFIT     |
|         SCORE     COUNT     MEASURE   ERROR      MNSQ   ZSTD   MNSQ   ZSTD |
|-----------------------------------------------------------------------------|
| MEAN     286.4     379.0         .00     .16      1.00    -.1   1.00     .1 |
| S.D.      75.6        .0        1.36     .06       .05     .7    .17    1.1 |
| MAX.     370.0     379.0        3.26     .34      1.09    1.3   1.27    1.9 |
| MIN.      78.0     379.0       -2.56     .12       .92   -1.5    .74   -1.7 |
|-----------------------------------------------------------------------------|
| REAL RMSE    .17  ADJ.SD    1.35  SEPARATION  7.85  Item   RELIABILITY  .98 |
|MODEL RMSE    .17  ADJ.SD    1.35  SEPARATION  7.91  Item   RELIABILITY  .98 |
| S.E. OF Item MEAN = .36                                                     |
+-----------------------------------------------------------------------------+
    UMEAN=.000 USCALE=1.000
    Item RAW SCORE-TO-MEASURE CORRELATION = -.96
    5685 DATA POINTS. APPROXIMATE LOG-LIKELIHOOD CHI-SQUARE: 4477.11
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Table 1 gives an item reliability index of 0.98. This implies that a line of inquiry has been developed in which some items are more difficult and some items are easier and consistency can be expected of these inferences (Bond and Fox, 2007). This simply means that the item ordering has a very high probability of being replicated if these same items are given to a different group of students.

However, the person reliability index of 0.44 is considered low. Since person reliability is not dependent on sample ability variance, this low index may imply that there is a small ability range between the respondents or there is not much difference between their abilities, thus making it impossible for the samples to be discriminated into different levels. There is just not a large enough spread of ability across the sample for the measures to demonstrate a hierarchy of ability (Bond and Fox, 2007).

According to Fisher Jr., Elbaum and Coulter (2010), reliability and Rasch separation statistics are practical in the sense that they can indicate number of ranges exist in the measurement continuum that are repeatedly reproducible, and a reliability lower than about 0.60 implies that one cannot confidently distinguish the top measure from the bottom one. As can be seen from Table 1, the person raw score-to-measure correlation is reported as 0.97. For this value to hold true, the proportion of very high and very low scores is low (Winsteps, 2011).

In general, Fisher Jr. et al (2010) said, when reliability increases, the number of ranges in the scale that can be distinguished with confidence across samples also increases, and specifically, measures with reliabilities of 0.67 will tend to vary within two groups that can be separated with 95% confidence, while those with reliabilities of 0.80 will vary within three groups; of 0.90, four groups; 0.94, five groups; 0.96, six groups; 0.97, seven groups, and so on. On the other hand, if person reliability is not dependent on sample size, low person reliability may also mean that the test is not long enough, or there are not many categories per item (Winsteps, 2011).

I. RESULTS AND DISCUSSIONS

In order to evaluate how students compare values of fractions to another number, we will take a look at how students respond to three items, namely questions 2, 3 and 6 on the Computation test. Table 2 lists the objectives of these items according to the curriculum.

TABLE 2
OBJECTIVES FOR ITEMS 2, 3 AND 6

Item	Objectives
Item 2: Which of the following has a value which is nearest to 10? 19/2 29/3 39/4 49/5	• Understand improper fraction • Compare the value of two improper fractions
Item 3: Which of the following is larger than 4 150/1000? 4.145 4.053 4.154 4.115	• Convert fraction to decimals of tenths, hundredths, tenths and hundredths, and thousandths and vice versa
Item 6: Which of the following sum has a total nearest to 5? 3 4/5 + 3/5 4 7/8 + 3/8 3 9/10 + 4/5 4 7/10 + 4/5	• Add two mixed numbers with the same denominators up to 10.

Figure 1 provides the person map of items for the responses to the Computation test. Items 2, 3 and 6 are displayed as S2, S3 and S6 on the map. As can be seen from the map, in terms of hierarchy of difficulty, the least difficult is Item 3 (S3) followed by Item 2 (S2) and the most difficult is Item 6 (S6).

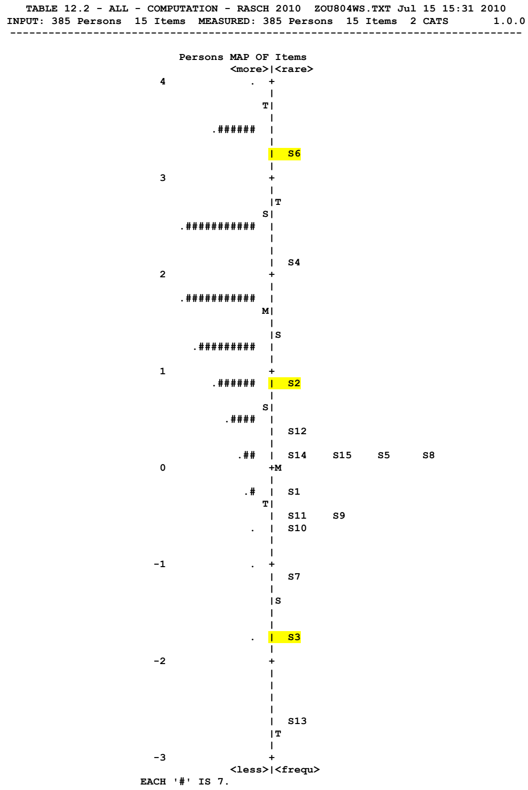

Figure 1. Person map of items

On the first item, out of selected 385 students from these colleges, 365 students (94.81%) were able to decide that 4.154 was larger than 4 150/1000. Most of the students converted the fraction 4 150/1000 to the decimal number 4.150 and compared it to the given decimal numbers or likewise, converted the decimal numbers 4.145 as 4 145/1000, 4.053 to 4 53/1000, 4.154 to 4 154/1000 and 4.115 to 4 115/1000 and then compared the values to 4 150/1000 before making the decision that 4.154 was larger. Scanning through the incorrect responses, the other 20 students (4.19%) reported responses as either 4.145 or 4.053 or no response.

The responses indicated that majority of students were able to convert fraction to decimals of tenths, hundredths, tenths and hundredths, and thousandths and vice versa. Using prior knowledge on doing counting on or counting back sequence of numbers to position this decimal numbers in a descending or increasing order as learnt in Mathematics Year 3, these students were able to make judgment on the results obtained to decide that 4.154 was larger than 4.150, thus demonstrating their ability to do computational estimation.

The second item required students to decide which among the improper fractions 19/2, 29/3, 39/4 and 49/5 was nearest to 10. Only 64.68% of the students were able to conclude that 49/5 is nearest to 10. Students were observed to have converted these improper fractions to either mixed numbers or decimal numbers before deciding on which of these fractions were closer to 10.

Table 3 displays the percentage of students from each college who responded to Item 3. Scanning through the responses, the most common incorrect response was 19/2, followed by 39/4 and 29/3. This might indicate the possibility that most students who gave 19/2 as their answer misunderstood the word “nearest to” to mean “the farthest from”. This might also indicate that the students do not fully understand that the number following another number in the counting on sequence is larger and likewise, the number following another number in the counting back sequence is smaller, causing them to be unable to do accuracy check of the position of the numbers, and this affected their judgment (Mathematics Year 3, 2003).

TABLE 3
PERCENTAGE OF STUDENTS FROM EACH COLLEGE RESPONDING TO ITEM 3

Response given	% students from each college			
	College I	College 2	College 3	College 4
49/5	62.50	73.33	54.84	66.30
19/2	21.88	12.38	24.73	17.39
39/4	6.25	6.67	10.75	4.35
29/3	3.13	3.81	6.45	4.35
NR	4.17	3.81	3.23	5.43
OTHERS	2.07	0	0	2.18

The third item was seen to have caused lots of confusion to the students. It required students to determine which among the sums 3 4/5 + 3/5, 3 9/10 + 4/5, 4 7/8 + 3/8 or 4 7/10 + 4/5 was nearest to 5. Only 21.82% of these students were able to correctly decide that 4 7/8 + 3/8 had the smallest increment from 5, hence concluding that this particular sum was nearest to 5. A big majority of the students (78.18%) were not able to decide which among the sums were nearest to 5.

Scrutinizing the responses of the students from one of these colleges, out of 105 responses, only 18.1% selected the correct answer 4 7/8 + 3/8. The answer 3 9/10 + 4/5 was favoured more by 38.10% of the students, followed by 28.57% choosing 4 7/10 + 4/5 and 3.81% choosing 3 4/5 + 3/5. 11.41% left the question unanswered. Majority of students from this college were observed able to add both fractions correctly, giving answers 4 7/8 + 3/8 = 5 1/4, 3 9/10 + 4/5 = 4 7/10, 4 7/10 + 4/5 = 5 1/2, and 3 4/5 + 3/5 = 4 2/5. Some went further and converted these results into decimal numbers, namely, 5 1/4 = 5.25, 4 7/10 = 4.7, 5 1/2 = 5.5 and 4 2/5 = 4.4. They were then required to decide on which of these results were nearest to 5. These students had to find the distance between these results and 5 before they can decide which value was nearest to 5. It is obvious that the students were having a great difficulty in deciding which of these results if measured in terms of distance or length away from 5 would produce the smallest increment. For this particular college, it came down to deciding between 4 7/8 + 3/8 and 3 9/10 + 4/5. They were just not able to decide with certainty that an increment of 0.25 was smaller than 0.3, thus, this impeded their progress in the estimation process.

Behr & Post (1986) stated that students needed to be able to understand the size of numbers in order to be able to estimate numbers, and likewise, knowing how to estimate can help develop this understanding of number size. This did not take place in the students' estimation process of Items 2 and 6. In Item 2, we observe that students had difficulties in deciding whether which among 1/2, 1/3, 1/4 or 1/5 was smallest in size. In Item 6, students had difficulty deciding whether an increment of 1/4, 1/2 or 3/10 (or their equivalent values 0.25, 0.5 or 0.3) was smallest in size. All these values are proper fractions. This lack of ability has interfered with the progress in the estimation process, thus causing them to make incorrect judgments.

ACKNOWLEDGMENT

The researchers would like to thank Universiti Teknologi Mara Malaysia for funding this research project. We would like to also extend our sincere appreciation to Majlis Amanah Rakyat, the principals, teachers and students of all the colleges who took part in this study for their support in achieving the objectives of the study. Last but not least, we thank all who participated directly and indirectly in making this study possible.

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