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PORTFOLIO OPTIMIZATION OF RISKY ASSETS USING MEAN-VARIANCE AND MEAN-CVAR

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Abstract

The aim of this research is to apply the variance and conditional value-at-risk (CVaR) as risk measures in portfolio selection problem. Consequently, we are motivated to compare the behavior of two different type of risk measures (variance and CVaR) when the expected returns of a portfolio vary from a low return to a higher return. To obtain an optimum portfolio of the assets, we minimize the risks using mean-variance and mean-CVaR models. Dataset with stocks for FBMKLCI is used to generate our scenario returns. Both models and dataset are coded and implemented in AMPL software. We compared the performance of both optimized portfolios constructed from the models in term of risk measure and realized returns. The optimal portfolios are evaluated across three different target returns that represent the low risk-low returns, medium risk-medium returns and high risk-high returns portfolios. Numerical results show that the composition of portfolios for mean-variance are generally more diversified compared to mean-CVaR portfolios. The in-sample results show that the seven optimal mean-$CVaR_{0:05}$ portfolios have lower $CVaR_{0:05}$ values as compared to their optimal mean-variance counterparts. Consequently, the standard deviation for mean-variance optimal portfolios are lower than the standard deviation of its mean-$CVaR_{0:05}$ counterparts. For the out-of sample analysis, we can conclude that mean-variance portfolio only minimizes standard deviation at low target return. While, mean-CVaR portfolios are favorable in minimizing risks at high target return.

Keywords: mean-risk, optimization, risk minimization, CVaR

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Introduction

Portfolio selection model focuses on dividing one's wealth among a set of securities. It is one of the leading problems in finance. Generally, if we have a set of n available assets that we may invest, we are interested in finding a solution on how to divide our wealth among this set of assets. It is common to consider the future returns of each assets to act as random variables as they are unpredictable; we denote this by $R_1, \dots, R_n$. A portfolio is denoted by $X = x_1, \dots, x_n$ where x_j is the fraction of the capital invested in asset $j, j = 1, \dots, n$. The fraction allocated among the assets is called portfolio weights which are required in the investment decisions. In order to construct a portfolio, there are a few constraints such as the portfolio weights must be non-negative and sum up to 1, which means no short selling is allowed. Then, the set of decision vectors can be expressed as:

$$X = (x_1, \dots, x_n) | \sum_{j=1}^{n} x_j = 1, x_j \geq 0, \forall\, j \in (1, \dots, n) \tag{1}$$

Consider that the return of the portfolio, R_x is random variables, it can be denoted as:

$$R_x = x_1 R_1 + \cdots + x_n R_n$$

To optimize a portfolio in which the risk is minimized for the expected target return, many risk measures have been introduced. Risk measures consist of various approaches to solve the portfolio selection problem. Variance is the first risk measure introduced by Markowitz (1952) to solve a portfolio selection problem. Markowitz (1952) has minimized the risk using the mean-variance model. Variance is known widely by researchers because it is easy to be implemented and interpreted.

Other than variance, another examples of risk measures are mean absolute deviation (MAD), lower partial moments (LPM), value-at-risk (VaR) and conditional value-at-risk (CVaR). In an article written by Markowitz, mean-variance model does not require the whole set of scenario returns as parameters, but only the expected returns and the covariance between the component assets (Markowitz, 1952). Steinbach (2001) stated that mean-variance approach has received comparatively little attention in the context of long-term investment planning.

In highlighting the importance of measuring risk for regulatory purposes, value-at-risk (VaR) was introduced in 1993 (Roman & Mitra, 2009). VaR has been used widely in the financial field. To estimate VaR of a market risk, we have to determine how much the value of a portfolio could decline over a given period of time with a given probability (Hendricks, 1997). Corresponding to the expected confidence level, VaR measures are most often expressed as percentiles. Despite its benefits, VaR imposes several shortcomings as it fails to solve the portfolio optimization problem. Rockafellar et. al. was the first to develop conditional value-at-risk (CVaR) as an alternative optimizable quantile-based risk measure (Rockafellar, Uryasev, et al., 2000). They demonstrated portfolio optimization through several cases. CVaR is quite similar to VaR for general distributions, but it has better specialties than VaR. Studies have shown that risk minimization can be done for large portfolios and scenarios with the CVaR performance function and constraints.

Risk measures are categorized into two types; deviation measures and left-tail measures. The first type of risk measure consists of symmetric and asymmetric risk measures. Variance, MAD and LPM are examples of this kind of risk measures. While the other type of risk measure focuses on the possible losses which are measured on the left-tail of a distribution. VaR and CVaR are included in this type of risk measure. Mean-risk models and expected utility maximization are the well-established models for optimizing portfolios. Mean-risk models are used to minimize risk subject to a constraint of different expected return, where expected utility maximization used to maximize return with different level of expected risk.

In the paradigm of mean-risk optimization models, a good portfolio has the lowest risk for specified level of expected return. Varying the level of expected return, we will obtain different performance of portfolios. In an article by Roman, Darby-Dowman, and Mitra (2007), an efficient portfolio consistently has the lowest risk for a specified level of expected return in a multi-objective approach. They observed that the mean-variance efficient portfolios are not dominated by CVaR; as well as the mean-CVaR efficient portfolios are not dominated with respect to variance. Hoe, Hafizah, and Zaidi (2010) provided a comparison of different risk measures in portfolio optimization. Their findings show that the minimax model outperforms other mean-risk models that employ risk measures of variance, absolute deviation, and semi-variance. In 2013, they compare the composition and performance employing different risk measures for Malaysian share market data in three different economic scenarios. Results show variations in both composition and the performance of these portfolios for the three selected economic periods (see Jaaman, Lam and Isa (2003)). Maasar, Roman, et al. (2016) also observed that the mean-variance portfolios are the most diversified and mean-CVaR efficient portfolios are the least diversified portfolio when applying the mean risk models onto risky assets in London Stock Exchange.

The objective of this paper is to minimize the risk measure of portfolio of risky assets using mean-variance and mean-conditional value-at-risk (CVaR). We construct portfolios to obtain the minimum risk measures using mean-variance and mean-conditional value-at-risk (CVaR) at different level of specified return. The specified returns set for this study is under the low risk-low return, medium risk-medium return, and the high risk-high return cases. Consequently, we compare the performance of the portfolio obtained based on their profitability and risk by using in-sample and out-of sample analyses. From these analyses we will examine the behavior of variance (CVaR) when CVaR (variance) is minimized for the mean-CVaR (mean-variance) efficient portfolio.

Methods

The first risk measure is proposed by Markowitz (1952), which is known as variance. In an article written by Maasar et al. (2016), they reviewed that risk models are divided into two categories. The first category is deviation measures from a target, and outcomes of the whole distributions are concerned. Variance in one of the examples of the first kind of risk measure, as it measures risk based on a specified target, and it involves the results of whole distribution. Second category of risk measure only concerned of the left-tail in a distribution, rather than whole distribution. Conditional value-at-risk (CVaR) that was introduced by Rockafellar et al. (2000), is categorized under this type of risk measure. This section explains the risk measures (variance and CVaR) that will be used in our numerical work. The first model used in this project is the mean-variance. Mean-variance used variance as its risk measure in portfolio selection problem. The variance of random variable R_x, denoted as σ^2, of the expected return of the deviations of R_x is given by:

$$\sigma^2(R_x) = E\left[R_x - \left(E(R_x)\right)^2\right]$$

where $E(R_x)$ is the expected value of R_x. Equation 2 is used in portfolio optimization problem to express variance of the portfolio return $R_x = x_1R_1 + \ldots + x_nR_n$. Thus, the variance of the return, R_x is defined as:

$$\sigma^2(R_x) = \sum_{j=1}^{n}\sum_{k=1}^{n} x_j\, x_k \sigma_{jk} \tag{3}$$

Value-at-Risk

VaR measures the risk of loss investments and has been used widely in finance. To calculate VaR of a given portfolio X with a return R_x for a holding period, let $A\% = \alpha \in [0,1]$ be a percentage that represents a sample of worse cases for the outcomes of R_x, where α is usually close to 0 ($\alpha = 0.01(1\%)$ or $\alpha = 0.05(5\%)$). Therefore, VaR as returns is expressed as:

$$\mathrm{VaR}_\alpha(R_x) = -q^\alpha(R_x); \text{for } \alpha \in [0,1] \tag{4}$$

where $-q^\alpha(R_x)$ represents the greatest lower bound from the probability of a distribution that is more than α. Example: Let $\alpha = 0.05$ with the confident level of 0.95, and $\mathrm{VaR}_{0.05}$ of a random variable, $R_x = 100$. From this example, there is a probability of 5% that the losses will be greater than 100, which also means that a probability of 95% confident that the losses will be less than 100. However, VaR is difficult to optimize for discrete distributions, when it is calculated using scenarios (Krokhmal, Palmquist, & Uryasev, 2002). Moreover, the losses beyond the value-at-risk cannot be estimated. Due to these problems, researchers use conditional value-at-risk (CVaR) to minimize risks for a targeted return in portfolio.

Conditional Value-at-Risk

Due to the shortcomings of VaR, researchers introduced a new risk measure which is called conditional value-at-risk (CVaR). CVaR has superior properties in many respects as a tool in optimization modelling. This model is an alternative measure to calculate risk to overcome the shortcomings in VaR. CVaR is defined as the conditional expected loss under the conditions that exceeds VaR. For general distributions, CVaR, which is quite similar to VaR measure of risk has more attractive properties than VaR (Krokhmal et al., 2002). An important result is proven by Rockafellar and Uryasev, that CVaR of a portfolio X can be calculated by solving a convex optimization problem (Rockafellar et al., 2000). So, in this project, we are going to minimize CVaR, and calculate VaR at the same time. Consider the decision vector, x represents a portfolio, such that $x = x_1, \ldots, x_n$ with x_j be the position of asset j;

$$x_j \geq 0 \text{ for } j = 1, \ldots, n, \text{with } \sum_{j=1}^{n} x_j = 1 \tag{5}$$

As defined in Equation 4, we let $\alpha = 0.01$ and $\alpha = 0.05$, which is in the interval of $[0,1]$. CVaR is considered to be approximately equal to the average losses greater than or equal to VaR at the same α. The CVaR at level α of R_x is defined as:

$$\mathrm{CVaR}_\alpha(R_x) = -\frac{1}{\alpha}\{E(R_x 1_{\{Rx \le q^\alpha(R_x)\}})\} - q^\alpha(R_x)[P(R_x \le q^\alpha(R_x)) - \alpha]$$

where

$$1_{\text{Relation}} = \begin{cases} 1, & \text{if Relation is true;} \\ 0, & \text{if Relation is false.} \end{cases}$$

Rockafellar et al. (2000) have proven the following results that is used in CVaR optimization. Let R_x be a random variable depends on a decision vector x that belongs to a feasible set X, let $\alpha \in [0,1]$. CVaR of the random variable R_x for the confidence level α is denoted by the $\mathrm{CVaR}_\alpha(x)$. The function is as follows:

$$F_\alpha(x, v) = \frac{1}{\alpha}E[-R_x + v]^+ - v,$$

$$[u]^+ = \begin{cases} u, & \text{if } u \ge 0; \\ 0, & \text{if } u \le 0. \end{cases}$$

Mean-Risk Models

In this section, we present the formulation of the mean-risk models that will be used in this research, that named mean-variance and mean-CVaR optimization models. The following is the form of the mean-risk models used in our numerical work.

- The input data:
  s = the number of scenarios;
  n = the number of assets;
  r_{ij} = the return of asset j under scenario $i; i = 1, \dots, s$;
  μ_j = the expected rate of return asset $j; j = 1, \dots, n$;
  σ_{kj} = covariance of scenario returns between assets;

- The decision variables:
  x_j = the fraction of the portfolio value invested in asset j.

Mean-variance optimization model is used to minimize variance. The equation in 3 is used as the measure of risk and Markowitz (1952) formulated the portfolio optimization as a parametric quadratic programming problem:

$$\text{minimize} \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{kj},$$
$$\text{subject to} \sum_{j=1}^{n} \mu_j x_j \ge d$$

where μ_j is the expected return of assets j, $\sigma_{kj} = E[(R_k - E(R_k)) - (R_j - E(R_j))]$ be the covariance of scenario returns between assets k and j, and d is a target expected return for the portfolio. Mean-variance model will optimize the portfolio of 22 constituent assets, by evaluating all the scenarios and the returns. In an article written by Roman, mean-variance model does not require the whole set of scenario returns as parameters, but only the expected returns and the co variances between the component assets (Roman & Mitra, 2009). For CVaR model, the decision variables xj, there are $p + 1$ decision variables. The variable v represents the negative of an α -quantile of the portfolio return distribution. Therefore, to solve this model, the maximum value of variable v may be used as an approximation for VaR_α. The remaining decision variables of p portray the magnitude of negative deviations of the portfolio return from α-quantile, for every scenario $i = 1, \dots, s$:

$$y_i = \begin{cases} -v - \sum_{j=1}^{n} x_j r_{ij} & \text{, if } \sum_{j=1}^{n} x_j r_{ij} \leq -v; \\ 0 & \text{, otherwise.} \end{cases}$$

$$\min v + \frac{1}{\alpha s} \sum_{j=1}^{s} y_i$$

subject to:

$$\sum_{j=1}^{n} -x_j r_{ij} - v \leq y_i \, ; \forall i \in \{1, \dots, s\}$$

$$y_i \geq 0; \forall i \in \{1, \dots, s\}$$

$$\sum_{j=1}^{n} \mu_j x_j \geq d \, ; \forall x \in X$$

Same as mean-variance, mean-CVaR also will optimize the portfolio after the CVaR risk is evaluated. Therefore, through this project, we compare the minimum CVaR with the minimum variance approach to determine an optimum portfolio selection.

Result and Discussion

This section presents the results on the performance of the mean-risk models used for this research. We consider two risk measures, variance and conditional value-at-risk, with risk minimized by mean-variance and mean-CVaR respectively. We construct the portfolios in the mean-risk models above for different target return. We analyze their performances of in-sample and out-of-sample in terms of their risk measures. We consider the data set of 22 constituent assets from FBMKLCI index. We compare sets of two constructed portfolios each having the expected values d of low, medium and high. We analyzed these portfolios using in-sample parameters of standard deviation and CVaR. For a portfolio construction, it is desirable to have smaller CVaR and standard deviation.

The composition of portfolios is the number of selected assets based on the different level of target returns. We analyzed the composition of the in-sample portfolios based on their di-versification. From the total of 22 constituent assets, only 13 of them are selected in optimizing in-sample portfolios. The selected assets in the portfolio are Hong Leong, Hapseng, TNB, KLCC, Airport, Digi, Nestle, Telekom, Genting Malaysia, Press Metal, Public, MISC and PPB. Based on the results obtained in AMPL, we observed the selected assets of each portfolios from both models used. From our analysis, we found that mean-variance portfolios are more diversified than mean-CVaR for different target returns. In Table 1, it is shown that for low target return, mean-variance optimal portfolios are the most diversified, while for medium target return, most of the mean-variance portfolios are more diversified than mean-CVaR portfolios. But, for the high target return, only three efficient mean-variance portfolios are more diversified while the others are not diversified.

Table 1. Number of assets selected in in-sample portfolios for each target returns

Model	**M-V**	**M-CVAR**	**M-V**	**M-CVAR**	**M-V**	**M-CVAR**
d	**Low (0.5%)**		**Medium (1.5%)**		**High (2.2%)**	
1	9	7	7	6	4	4
2	9	8	8	5	4	3
3	9	8	8	6	3	3
4	9	8	6	6	3	2
5	9	8	5	6	2	2
6	9	8	5	5	2	2
8	9	8	4	6	2	2

The risk measures of mean-variance and mean-CVaR are calculated from the portfolio construction. For mean-variance portfolios, the standard deviation is calculated in AMPL, and the CVaR is calculated from the distribution constructed in Microsoft Excel. As well as for mean-CVaR portfolios, the tail CVaR measure are obtained from AMPL, while the standard deviation is calculated in Microsoft Excel. From all the risk measures obtained, we did comparison between two different portfolios of mean-variance and mean-CVaR.

Then, we analyze the portfolio distribution obtained in terms of its risk measure, standard deviation and CVaR 5%. We have constructed 7 in-sample efficient portfolios for both mean-variance and mean-CVaR models from their returns. Table 2 shows the standard deviation results of all 7 in-sample portfolios. We can deduce that mean-variance portfolios have the lowest risk for all levels of target returns, d because mean-variance only minimizing variance of portfolios. For an example, the first two columns and the first rows in Table 2 show the standard deviation of mean-variance and mean-CVaR for the In-sample 1 portfolios respectively. The previous explanations mean that 0.0244 is obtained from AMPL and 0.0269 is calculated in Microsoft Excel. Therefore, the risk measures for the in-sample portfolios are calculated for 7 times at different levels of target return. Meanwhile, mean-CVaR only minimizing CVaR of portfolios as we can observe that mean-CVaR portfolios has the lowest risk measured from low and medium target returns. Whereas for high target return, only the first 4 portfolios of mean-CVaR has the lowest risk, while the other portfolios have equal risk measured for mean-variance and mean-CVaR. These results can be seen from Table 3 presents the CVaR for the in-sample portfolios. As an example, from the table given, at high target return of In-sample 1, mean-CVaR has a lower CVaR value which is 0.0828, compared to mean-variance portfolios with 0.0914 as its CVaR value.

Table 2. Standard deviation of in-sample portfolios for each target returns *d*

	Standard Deviation					
Models	M-V	M-CVaR	M-V	M-CVaR	M-V	M-CVaR
d	Low (0:5%)		Medium (1:5%)		High (2:2%)	
1	0.024	0.027	0.031	0.037	0.059	0.064
2	0.025	0.028	0.033	0.038	0.067	0.069
3	0.025	0.029	0.035	0.041	0.073	0.074
4	0.025	0.028	0.035	0.040	0.070	0.071
5	0.025	0.028	0.039	0.044	0.097	0.097
6	0.024	0.028	0.037	0.041	0.091	0.091
7	0.0245	0.028	0.035	0.039	0.078	0.078

Table 3: CVaR 5% of in-sample portfolios for each target returns *d*

	CVaR 5%					
Models	M-V	M-CVaR	M-V	M-CVaR	M-V	M-CVaR
d	Low (0:5%)		Medium (1:5%)		High (2:2%)	
1	0.047	0.042	0.056	0.045	0.091	0.083
2	0.050	0.042	0.058	0.049	0.101	0.091
3	0.050	0.042	0.059	0.051	0.105	0.099
4	0.050	0.042	0.058	0.050	0.092	0.090
5	0.050	0.042	0.066	0.058	0.162	0.162
6	0.049	0.042	0.064	0.056	0.151	0.151
7	0.049	0.042	0.062	0.054	0.122	0.122

Next, we constructed out-of sample portfolios using the remaining 30 scenarios from the set of data used in this research. We expect to calculate the realized returns using the portfolio weights of in-sample portfolios. Our numerical works show that the results of the analysis in not consistent between portfolios. We analyzed the favorable results according to the levels of target return.

For low target return, 0.5%, the result shows that the average return is higher for Mean-Variance portfolios, while the standard deviation and CVaR are lower in mean-variance and mean-CVaR respectively. It is because mean-variance models only minimized variance, while mean-CVaR minimized CVaR. The maximum and minimum values are obtained from the portfolio returns which both are from mean-CVaR portfolios. As for medium target return, the results are bit different from the results of low target return. The higher average still is from mean-variance portfolios, but for the risk measure, it is shown that lower standard deviation on mean-CVaR portfolios and lower CVaR on Mean-Variance portfolios. For maximum and minimum returns are from mean-CVaR and mean-variance portfolios respectively. The out-of-sample portfolios for the highest target return, 2.2% also shows different results from the other target returns. The lower standard deviation and CVaR are both obtained from Mean-Variance portfolios. As the targeted return increases, CVaR can capture more risks as it is a tail-based risk measure. Therefore, the out-of-sample results differ as the targeted returns differ. Although mean-variance and mean-CVaR could minimize variance and CVaR respectively, it is only favorable for low target return. While for medium and high target return, the results contradict the previous results. The difference in the realized results show that mean-CVaR minimize risks better as the target returns increases. We provided the results of one of the portfolios in Table 4 for the reference.

Table 4: Out-of-sample portfolios for each target returns *d*

	Realized Returns					
Models	M-V	M-CVaR	M-V	M-CVaR	M-V	M-CVaR
Scenarios	d_1 = 0:5%		d_2 = 1:5%		d_3 = 2:2%	
1	-0.044	-0.034	-0.025	-0.003	-0.002	-0.016
2	-0.013	-0.028	-0.018	-0.039	-0.016	-0.009
3	-0.006	-0.004	-0.006	-0.014	-0.004	-0.012
4	-0.029	-0.032	-0.021	-0.012	-0.065	-0.036
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
28	0.0023	0.002	0.014	0.001	0.060	0.036
29	0.002	0.015	0.010	0.015	0.016	0.011
30	0.009	0.009	0.019	-0.005	0.038	0.023
Average	**0.0023**	0.002	**0.0065**	0.004	**0.021**	0.018
Std. Dev.	**0.017**	0.019	0.017	**0.018**	0.033	**0.0251**
CVaR 5%	0.048	**0.044**	**0.032**	0.046	0.069	**0.056**
Max	0.030	**0.038**	0.029	**0.038**	**0.075**	0.061
Min	-0.044	**-0.034**	**-0.025**	-0.039	**-0.065**	-0.048

Conclusion

We consider two mean-risk models with variance and conditional value-at-risk as risk measures. Variance measures the deviations from the average on any side of distribution, while CVaR measures the worst outcomes that may occur in distribution on left-tail. Both approaches are evaluated at 3 different level of target returns; low, medium and high returns. To obtain an optimum portfolio, we implemented mean-risk models in AMPL and analyzed the results in Microsoft Excel. Mean-variance and mean-CVaR are the models used in our numerical work. The target returns are specified at 0.5%, 1.5% and 2.2%; corresponding to low risk-low return, medium risk-medium return and high risk-high return respectively. The models are implemented on a set of data drawn from FBMKLCI, containing 130 scenarios of 22 risky assets. A total of 131 closing prices of the risky assets taken from December 2016 until October 2017 are considered to generate 130 monthly scenario returns. The returns of each scenarios of risky assets are calculated in Microsoft Excel. Out of 130 scenario re-turns generated, 100 scenarios are used to construct efficient in-sample portfolios. While, the remaining 30 scenarios are used to back test and validate the in-sample results in out-of-sample analysis. Then, we coded mean-variance and mean-CVaR models in AMPL to optimize the portfolios, for different level of target returns. The numerical results from AMPL are then analyzed in Excel to evaluate the portfolio performance and the composition of portfolios. Based on the results found, the composition of portfolios show that mean-variance portfolios are more diversified than mean-CVaR portfolios, but with very slight difference. However, there are also portfolios that are not diversified given the selected assets for each portfolio are same. We found that the optimal portfolios become less diversified as the level of target return increases. In-sample portfolios are constructed using 100 scenario returns, in which the concept of rolling windows is applied. We constructed seven optimal in-sample portfolios using the first 100 scenarios until the seventh 100 scenarios. Mean-variance and mean-CVaR models are applied in this research. Their performance is compared in term of the risk measure, which are standard deviation and CVaR. For the in-sample results, mean-variance shows more favorable results in term of standard deviation as Mean-Variance applicable in minimizing variance as its risk measure. While mean-CVaR portfolios have favorable results of CVaR 5% because mean-CVaR minimized CVaR as the risk measure on the left-tail distribution. From the total of 130 scenario returns, the remaining 30 scenarios are used to back test the results of the in-sample portfolios. The consistency of the realized returns calculated in Microsoft Excel observed. Based on our analysis, mean-CVaR portfolios are favorable in capturing risk for high target return. This aligned with the mean-CVaR assumption where it minimized the worst cases in the scenarios. While for medium target return, the results obtained fluctuates and not consistent. At low target return, the results obtained in out-of-sample analysis are mostly consistent with the results from the performance of in-sample portfolios. The realized returns of the out-of-sample analysis are inconsistent from the in-sample portfolios. Based on the obtained results, we can conclude that mean-variance and mean-CVaR have their own specialties in minimizing risks. As for mean-variance, it is applicable and widely used as the method is easy to be calculated, but only favorable at low target return. Mean-CVaR is a tail measure, which focuses on the worst cases in the scenarios. It is favorable for mean-CVaR to minimize risks at high target returns.

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