FUZZY TIME SERIES FORECASTING MODEL BASED ON VARIOUS TYPES OF SIMILARITY MEASURE APPROACH

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Abstract

Fuzzy time series (FTS) is a well-known method for forecasting the time series data in linguistic values. Recently, a few studies have used the similarity measure approach in determining the performance of the FTS forecasting model. In this paper, an FTS forecasting model based on seven intervals of equal length and trapezoidal fuzzy numbers is presented. Then, the performance of FTS forecasting model using various types of similarity measure is compared. The FTS model is implemented in the case of students' enrollment in the University of Alabama and the unemployment rate in Malaysia. The hybrid similarity measure of geometric distance, center of gravity, area, perimeter and height gives the best performance.

Keywords: Forecasting Model; Fuzzy Time Series; Similarity Measure; Trapezoidal Fuzzy Number

Introduction

Fuzzy time series (FTS) forecasting model has been widely used to make prediction for historical data in linguistic values. The concept was first introduced by Song and Chissom, (1993a, 1993b) with application on the forecasting of students' enrollments in the University of Alabama. After that, a lot of researchers have carried out their studies and developed various types of fuzzy forecasting methods such as Chen, (2002), Chen and Hsu, (2004), and Bas et al., (2018). They have been changing the number of partitions of the universe of discourse, the length of the intervals, the order of fuzzy logical relationships and the rules of calculating the forecasted values. All the methods have been mentioned used the fuzzy set to define the linguistic values, however, the forecasted range cannot be obtained for various degree of confidence.

Then, Liu, (2007) introduced a new method to forecast the enrollments in the University of Alabama, using the trapezoidal fuzzy numbers. A number of forecasting methods was developed using the trapezoidal fuzzy numbers such as Liu, (2009), Yadav et al., (2012) and Basyigit et al., (2014). The validity of the forecasting performance was proved by analyzing the forecasting results using the mean absolute percent error (MAPE), the mean square error (MSE) and the root mean square error (RMSE). However, to obtain the MAPE, MSE or RMSE, the forecasted values in trapezoidal fuzzy numbers form were defuzzified to crisp values. The defuzzification process has dissipated some of the information that has been kept on the data. Only a limited number of studies (Ramli et al., 2018; Mutalib et al., 2018; Mutalib et al., 2019) have used the similarity measure concept to evaluate the performance of the forecasting models. The similarity between the fuzzy forecasted values and fuzzy historical data were compared. Ramli et al., (2018) and Mutalib et al., (2019) used the center of gravity similarity measure to evaluate the performance in the Malaysian unemployment rate and Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) respectively. Meanwhile, Mutalib et al., (2018) used the area and height similarity measure to investigate the performance of

Malaysian unemployment rate.

An FTS forecasting model based on seven intervals of equal length and trapezoidal fuzzy numbers was presented. Then, the performance of FTS forecasting model using various types of similarity measure was compared. The similarity measure with best performance is determined. This paper is organized as follows; in section 2, the definition of FTS and trapezoidal fuzzy number are introduced; section 3 presents the FTS forecasting model; section 4 illustrates two numerical examples, which are the enrollments of the students in the University of Alabama and the unemployment rate in Malaysia; discussion is presented in section 5 and conclusion is shown in section 6.

Preliminaries

In this section, the definitions of FTS and trapezoidal fuzzy number are reviewed.

Definition 1: (Song & Chissom, 1993a): *Let Y(t) (t is integer) be a subset of* □ *and* $X(t)$ *is the universe of discourse defined by fuzzy set* $U_i(t)$ *(t is integer), then we call* $F(t)$ *fuzzy time series on* $X(t)$ *(t is integer).*

Definition 2: (Song & Chissom, 1993a): *If there exists a fuzzy relationship* $R(t-1,t)$ *such that* $F(t)=F(t-1)*R(t-1,t)$ *where * represents the fuzzy operator, then* $F(t)$ *is said to be caused by* $F(t-1)$. *The relationship can be noted as* $F(t-1)\rightarrow F(t)$.

Definition 3: (Liu, 2007): *A trapezoidal fuzzy number* $\tilde{A}$, *denoted by* $\tilde{A}=(n_1,n_2,n_3,n_4)$ *is defined as:*

$$\mu_{\tilde{A}}(x)=\begin{cases} 0 & , \quad x<n_1 \\ \dfrac{x-n_1}{n_2-n_1} & , \quad n_1\le x\le n_2 \\ 1 & , \quad n_2\le x\le n_3 \\ \dfrac{n_4-x}{n_4-n_3} & , \quad n_3\le x\le n_4 \\ 0 & , \quad x>n_4 \end{cases} \tag{1}$$

Methodology

In this section, the FTS forecasting model based on seven intervals of equal length and trapezoidal fuzzy numbers is presented as follows:

Step 1. Define the universe of discourse U as $[T_{\min}-T_1,T_{\max}+T_2]$ whereby $T_{\min}$ and $T_{\max}$ are the minimum and maximum historical data respectively, while T_1 and T_2 are two positive numbers.

Step 2. Partition the universe of discourse U into seven intervals of equal length (Song & Chissom, 1993b). These intervals are labelled as U_i where $i=1,2,3,...,7$.

Step 3. Establish the trapezoidal fuzzy numbers to represent the linguistic values of the intervals of U. Suppose $U_1=[u_1,u_2]$, $U_2=[u_2,u_3],\ldots,U_6=[u_6,u_7]$ and $U_7=[u_7,u_8]$, then the trapezoidal fuzzy numbers are defined as $\tilde{A}_1=(u_0,u_1,u_2,u_3)$, $\tilde{A}_2=(u_1,u_2,u_3,u_4),\ldots,\tilde{A}_6=(u_5,u_6,u_7,u_8)$ and $\tilde{A}_7=(u_6,u_7,u_8,u_9)$ (Liu, 2007).

Step 4. Fuzzify the historical data. If the value of the historical data is located in the range of U_i where $i=1,2,3,...,7$, then it belongs to $\tilde{A}_i$ where $i=1,2,3,...,7$ (Liu, 2007).

Step 5. Develop the first order fuzzy logical relation (FLR). If the value of time $t-1$ is $\tilde{A}_m$, then that of time t is $\tilde{A}_n$" as $\tilde{A}_m \rightarrow \tilde{A}_n$ (Liu, 2007). Note that if there are repeated relationships, they are not accounted (Song & Chissom, 1993b).
Step 6. Generate the first order FLR group.
Step 7. Calculate the forecasted value, $\tilde{F}_t$ of time t by using the rules proposed by Liu (2007).

The FTS model is summarized as in **Figure 1**.

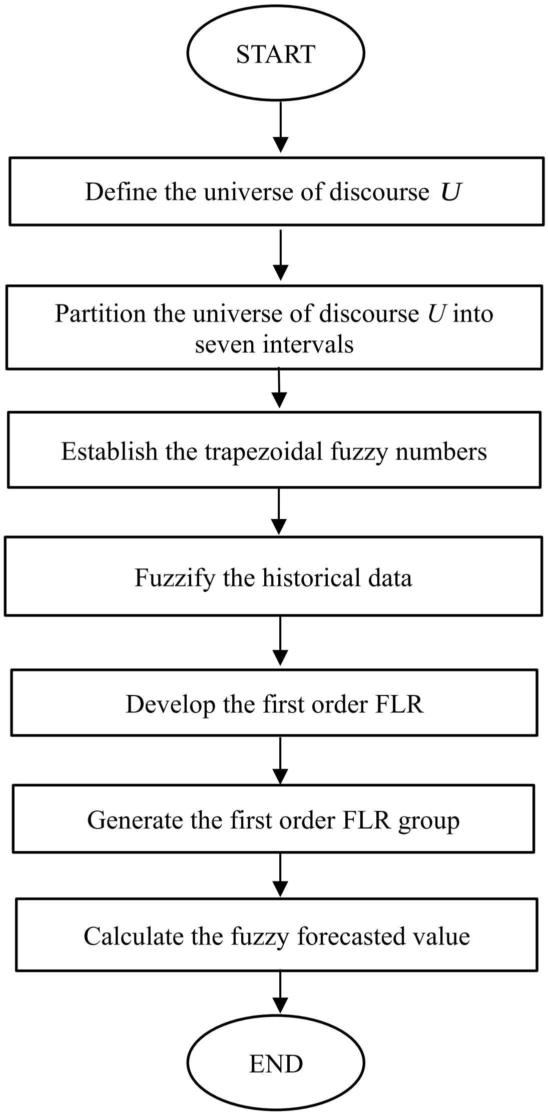

Figure 1 The Proposed FTS Forecasting Model

Numerical Examples
In this study, two numerical examples to illustrate the proposed FTS forecasting model was considered. The numerical examples used are the historical data of students' enrollment of the University of Alabama and the unemployment rate in Malaysia.
Case 1: Students' Enrollment of the University of Alabama
The students' enrollment data, adopted from Song and Chissom (1993b) takes into account the number of students' enrollment at the University of Alabama from year 1971 until 1992, as shown in **Figure 2**.

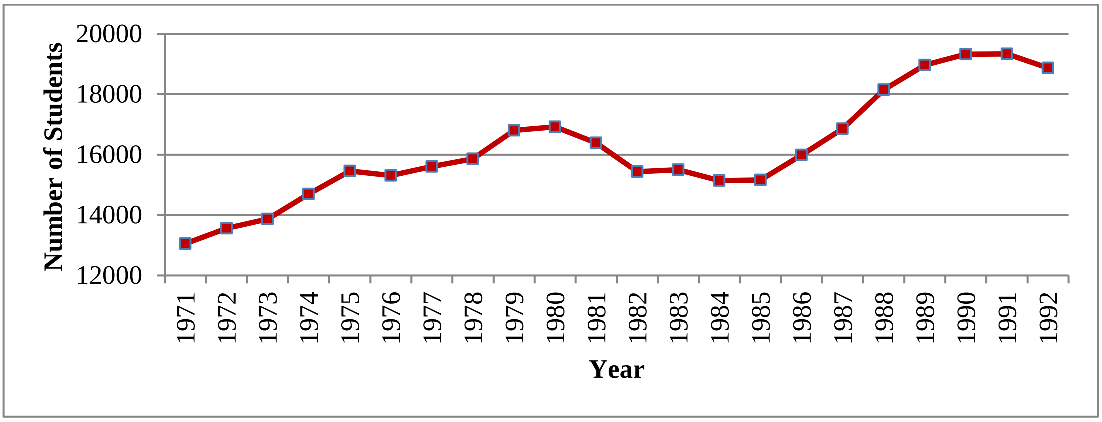

Figure 2 Students' Enrollment of the University of Alabama

Step 1. Note that $T_{\min} = 13055$ and $T_{\max} = 19337$. $T_1 = 55$ and $T_2 = 663$ were chosen such that $U = [13055 - 55, 19337 + 663] = [13000, 20000]$.

Step 2. By using seven intervals, we get partitions $U_1 = [13000,14000]$, $U_2 = [14000,15000]$, $U_3 = [15000,16000]$, $U_4 = [16000,17000]$, $U_5 = [17000,18000]$, $U_6 = [18000,19000]$ and $U_7 = [19000, 20000]$.

Step 3. Next, the trapezoidal fuzzy numbers $\tilde{A}_1 = (12000,13000,14000,15000)$, $\tilde{A}_2 = (13000,14000,15000,16000),\ldots,\tilde{A}_6 = (17000,18000,19000,20000)$ and $\tilde{A}_7 = (18000, 19000,20000,21000)$ for U_1, $U_2,\ldots,U_6$ and U_7 were established respectively.

Step 4. Since the trapezoidal fuzzy numbers have been established, the data of historical enrollments at the University of Alabama were fuzzified. The fuzzified data is shown in **Table 1**.

Step 5. From **Table 1**, the first order FLR was developed.

Table 1 The Fuzzified Data of Students'Enrollment at the University of Alabama (1975-1983)

Year	Enrollment	Fuzzy Number	Trapezoidal Fuzzy Numbers
1975	15460	$\tilde{A}_3$	(14000,15000,16000,17000)
1976	15311	$\tilde{A}_3$	(14000,15000,16000,17000)
1977	15603	$\tilde{A}_3$	(14000,15000,16000,17000)
1978	15861	$\tilde{A}_3$	(14000,15000,16000,17000)
1979	16807	$\tilde{A}_4$	(15000,16000,17000,18000)
1980	16919	$\tilde{A}_4$	(15000,16000,17000,18000)
1981	16388	$\tilde{A}_4$	(15000,16000,17000,18000)
1982	15433	$\tilde{A}_3$	(14000,15000,16000,17000)
1983	15497	$\tilde{A}_3$	(14000,15000,16000,17000)

Table 2 FLR Group of Enrollments at the University of Alabama

Group	FLR
Group 1	$\tilde{A}_1 \rightarrow \tilde{A}_1$, $\tilde{A}_1 \rightarrow \tilde{A}_2$
Group 2	$\tilde{A}_2 \rightarrow \tilde{A}_3$
Group 3	$\tilde{A}_3 \rightarrow \tilde{A}_3$, $\tilde{A}_3 \rightarrow \tilde{A}_4$
Group 4	$\tilde{A}_4 \rightarrow \tilde{A}_4$, $\tilde{A}_4 \rightarrow \tilde{A}_3$, $\tilde{A}_4 \rightarrow \tilde{A}_6$
Group 5	$\tilde{A}_6 \rightarrow \tilde{A}_6$, $\tilde{A}_6 \rightarrow \tilde{A}_7$
Group 6	$\tilde{A}_7 \rightarrow \tilde{A}_7$, $\tilde{A}_7 \rightarrow \tilde{A}_6$

Table 3 The Forecasted Enrollments (1975-1983)

Year	Actual Enrollment	Trapezoidal Fuzzy Numbers	Forecasted Enrollments
1975	15460	(14000,15000,16000,17000)	(14000,15000,16000,17000)
1976	15311	(14000,15000,16000,17000)	(14500,15500,16500,17500)
1977	15603	(14000,15000,16000,17000)	(14500,15500,16500,17500)
1978	15861	(14000,15000,16000,17000)	(14500,15500,16500,17500)
1979	16807	(15000,16000,17000,18000)	(14500,15500,16500,17500)
1980	16919	(15000,16000,17000,18000)	(15000,16000,17000,18000)
1981	16388	(15000,16000,17000,18000)	(15000,16000,17000,18000)
1982	15433	(14000,15000,16000,17000)	(15000,16000,17000,18000)
1983	15497	(14000,15000,16000,17000)	(14500,15500,16500,17500)

Case 2: Unemployment Rate in Malaysia

The data of unemployment rate in Malaysia was taken from the Department of Statistics Malaysia. The data concerns from year 1982 to 2017. However, the data for years 1991 and 1994 have gone missing. This missing data was replaced by using the linear interpolation method, as used by Adenan and Noorani, (2015). The data of unemployment rate in Malaysia is shown in **Figure 3**.

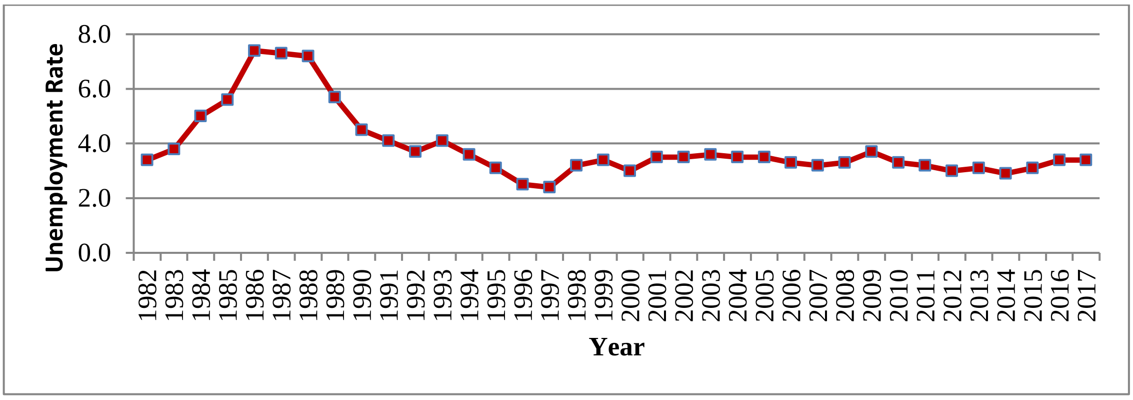

Figure 3 Unemployment Rate in Malaysia

Step 1. Note that $T_{\min} = 2.4$ and $T_{\max} = 7.4$. $T_1 = 0.4$ and $T_2 = 0.6$ were chosen such that $U = [2.4 - 0.4, 7.4 + 0.6] = [2.0, 8.0]$.

Step 2. $U_1 = [2.000, 2.857]$, $U_2 = [2.857, 3.714]$, $U_3 = [3.714, 4.571]$, $U_4 = [4.571, 5.429]$, $U_5 = [5.429, 6.286]$, $U_6 = [6.286, 7.143]$ and $U_7 = [7.143, 8.000]$ have been obtained using the

seven intervals.
Step 3. Then, the trapezoidal fuzzy numbers were established.
Step 4. After the trapezoidal fuzzy numbers was established, then, data on historical unemployment rate in Malaysia were fuzzified. The fuzzified data is shown in **Table 4** below.

Table 4 The Fuzzified Data of Unemployment Rate in Malaysia (1991-2003)

Year	Unemployment Rate	Fuzzy Number	Trapezoidal Fuzzy Numbers
1991	4.1	$\tilde{A}_3$	(2.857,3.714,4.571,5.429)
1992	3.7	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
1993	4.1	$\tilde{A}_3$	(2.857,3.714,4.571,5.429)
1994	3.6	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
1995	3.1	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
1996	2.5	$\tilde{A}_1$	(1.143,2.000,2.857,3.714)
1997	2.4	$\tilde{A}_1$	(1.143,2.000,2.857,3.714)
1998	3.2	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
1999	3.4	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
2000	3.0	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
2001	3.5	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
2002	3.5	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)
2003	3.6	$\tilde{A}_2$	(2.000,2.857,3.714,4.571)

Step 5. From **Table 4**, the first order of FLR was developed.
Step 6. From the FLR obtained in the previous step, the data then were classified into some groups which resulted the data in **Table 5**.

Table 5 FLR Group of Unemployment Rate in Malaysia

Group	FLR
Group 1	$\tilde{A}_1 \to \tilde{A}_1$, $\tilde{A}_1 \to \tilde{A}_2$
Group 2	$\tilde{A}_2 \to \tilde{A}_1$, $\tilde{A}_2 \to \tilde{A}_2$, $\tilde{A}_2 \to \tilde{A}_3$
Group 3	$\tilde{A}_3 \to \tilde{A}_2$, $\tilde{A}_3 \to \tilde{A}_3$, $\tilde{A}_3 \to \tilde{A}_4$
Group 4	$\tilde{A}_4 \to \tilde{A}_5$
Group 5	$\tilde{A}_5 \to \tilde{A}_3$, $\tilde{A}_5 \to \tilde{A}_7$
Group 6	$\tilde{A}_7 \to \tilde{A}_5$, $\tilde{A}_7 \to \tilde{A}_7$

Step 7. By analysing the FLR groups, the forecasted values were calculated and the forecasted enrollments were listed in **Table 6**.

Table 6 The Forecasted Unemployment Rate (1991-2003)

Year	Actual Enrollment	Trapezoidal Fuzzy Numbers	Forecasted Enrollments
1991	4.1	(2.857,3.714,4.571,5.429)	(2.857,3.714,4.571,5.429)
1992	3.7	(2.000,2.857,3.714,4.571)	(2.857,3.714,4.571,5.429)
1993	4.1	(2.857,3.714,4.571,5.429)	(2.000,2.857,3.714,4.571)

1994	3.6	(2.000,2.857,3.714,4.571)	(2.857,3.714,4.571,5.429)
1995	3.1	(2.000,2.857,3.714,4.571)	(2.000,2.857,3.714,4.571)
1996	2.5	(1.143,2.000,2.857,3.714)	(2.000,2.857,3.714,4.571)
1997	2.4	(1.143,2.000,2.857,3.714)	(1.571,2.429,3.286,4.143)
1998	3.2	(2.000,2.857,3.714,4.571)	(1.571,2.429,3.286,4.143)
1999	3.4	(2.000,2.857,3.714,4.571)	(2.000,2.857,3.714,4.571)
2000	3.0	(2.000,2.857,3.714,4.571)	(2.000,2.857,3.714,4.571)
2001	3.5	(2.000,2.857,3.714,4.571)	(2.000,2.857,3.714,4.571)
2002	3.5	(2.000,2.857,3.714,4.571)	(2.000,2.857,3.714,4.571)
2003	3.6	(2.000,2.857,3.714,4.571)	(2.000,2.857,3.714,4.571)

Result and Discussion

In this section, the similarity of the results obtained were compared. The existing similarity measures such as Chen and Lin, (1995), Chen and Chen, (2003), Wei and Chen, (2009), Xu et al., (2010), Patra and Mondal, (2015), Li and Zeng, (2017), Khorshidi and Nikfalazar, (2017) and Chutia and Gogoi, (2018) were used in order to compare the closeness of the forecasted values to the actual values. **Tables 7** and **8** show the similarity measures for the enrollments in the University of Alabama and the unemployment rate in Malaysia respectively.

The similarity measures involving the years of 1975, 1980 and 1981 are 1 because the actual value is exactly the same as the forecasted value. From Table 7, the proposed similarity measure suggested by Khorshidi and Nikfalazar, (2017) gives the highest similarity between the actual and forecasted values, which is 99.79% on average, followed by Xu et al., (2010) and Chen and Lin, (1995) with 99.76% and 99.75% on average respectively.

Table 7 Similarity Measures for Enrollments in the University of Alabama (1975-1983)

Year	(Chen & Lin, 1995)	(Chen & Chen, 2003)	(Wei & Chen, 2009)	(Xu et al., 2010)	(Patra & Mondal, 2015)	(Li & Zeng, 2017)	(Khorshidi & Nikfala-zar, 2017)	(Chutia & Gogoi, 2018)
1975	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1976	0.997479	0.994964	0.995349	0.997612	0.995803	0.986092	0.997844	0.993782
1977	0.997479	0.994964	0.995349	0.997612	0.995803	0.986092	0.997844	0.993782
1978	0.997479	0.994964	0.995349	0.997612	0.995803	0.986092	0.997844	0.993782
1979	0.997619	0.995244	0.995604	0.997745	0.996036	0.986854	0.997962	0.994129
1980	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1981	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1982	0.995098	0.990220	0.990967	0.995357	0.991846	0.973129	0.995798	0.987938
1983	0.997479	0.994964	0.995349	0.997612	0.995803	0.986092	0.997844	0.993782
Average	0.997511	0.995032	0.995410	0.997643	0.995858	0.986310	0.997868	0.993870

Table 8 Similarity Measures for the Unemployment Rate in Malaysia (1991-2003)

Year	(Chen & Lin, 1995)	(Chen & Chen, 2003)	(Wei & Chen, 2009)	(Xu et al., 2010)	(Patra & Mondal, 2015)	(Li & Zeng, 2017)	(Khorshidi & Nikfala-zar, 2017)	(Chutia & Gogoi, 2018)
1991	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1992	0.914275	0.835847	0.914245	0.918798	0.914252	0.842319	0.993395	0.794236
1993	0.914275	0.835847	0.914245	0.918798	0.914252	0.842319	0.993395	0.794236
1994	0.914275	0.835847	0.914245	0.918798	0.914252	0.842319	0.993395	0.794236
1995	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
1996	0.914300	0.835944	0.914300	0.918824	0.914300	0.842485	0.993431	0.786784
1997	0.957125	0.916043	0.957094	0.959391	0.957101	0.917690	0.998324	0.886885

1998	0.957175	0.916128	0.957144	0.959433	0.957151	0.917781	0.998327	0.889269
1999	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2001	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2002	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2003	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
Average	0.965714	0.935086	0.965706	0.967523	0.965708	0.938178	0.996706	0.922206

From **Table 8**, the highest average of similarity between the actual and forecasted values is 99.67%, which is the similarity measure proposed by Khorshidi and Nikfalazar, (2017). This is followed by Xu et al., (2010) and Chen and Lin, (1995) with 96.75% and 96.57% on average respectively.

Conclusion

In this study, two cases to investigate the performance of the forecasted result using the proposed forecasting model were used. From both cases, it is found out that the hybrid similarity measure based on geometric distance, centre of gravity, area, perimeter and height which was proposed by Khorshidi and Nikfalazar, (2017) shows the best performance.

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Conflict of interests

Author hereby declares that there is no conflict of interests with any organization or financial body for supporting this research.

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