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IMPROVED HYBRID METHODS IN SOLVING SINGLE VARIABLE NONLINEAR ALGEBRAIC EQUATIONS

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CONFIRMATION BY PANEL OF EXAMINERS

I certify that a panel of examiners has met on 30th September, 2016 to conduct the final examination of Nor Hanim Binti Abd Rahman on his Doctor of Philosophy thesis entitled "Improved Hybrid Methods in Solving Single Variable Nonlinear Algebraic Equations" in accordance with Universiti Teknologi MARA Act 1976 (Akta 173). The Panel of Examiners recommends that the student be awarded the relevant degree. The panel of Examiners was as follows:

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ABSTRACT

Nonlinear problem is one of the most frequently occurring problems in scientific works especially in science and engineering applications. Amongst the most popular schemes are the Newton's method and homotopy perturbation method. However, the duration to converge are heavily depends on how close the guess value is to the real root/s and the rate of convergence for Newton's method is only order-2 and its efficiency index is only $\sqrt[2]{2} \approx 1.41421$. Secondly, some of the methods utilized successive approximation procedure to ensure every step of computing will converge to the desired root and one of the most common problems is the improper initial values for the iterative methods. Thus, this particular research aims to develop an improved numerical solution for solving nonlinear equations by using hybrid concept and higher order correctional terms. Higher order successive approximations are applied and evaluated to ensure it converges to the desired root/s more effectively. Two sets of schemes of hybrid algorithms, the Higher Order Taylor-Perturbation method (HTP) and Higher Order Homotopy Taylor Perturbation method (HHTP) with higher order correctional terms up to 6^{th} order are derived and evaluated. The theoretical and numerical results used to verify the stability, consistency and convergence of the schemes. Numerical examples and comparison studies are used to illustrate and to support the efficiency of the suggested method. Furthermore, a new definition of computational order of convergence are defined and analyzed. Next, in order to jumpstart the process of iteration, an improved way to choose the initial-value is also discussed and evaluated numerically. As a result of hybriding several methods, both improved algorithms of HTP and HHTP established faster, more reliable and better outputs, in comparison to other classical methods. The computational tools such as Maple 14 and Mathematica 7.0 are used for this research.

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CHAPTER ONE
INTRODUCTION

1.1 OVERVIEW

Root finding has been one of the oldest nonlinear problems in mathematics, yet it is far from being solved. Every time a solution is suggested by a researcher, algebraically or numerically, each has its own limitations and boundaries of applications and level of convergences. Researchers are long to create an overall formula that applicable unlimited field, not only in science and engineering. Furthermore, over the decades we have observed an increasing trend towards replacing costly real-life tests and experiments with computer simulations, from automobile crash tests to space-craft auto-pilot. This emerging trend prompts the need for reliable and efficient self-verified computing methods that can guarantee prediction of results one hundred percent. This also includes in cases of root-finding where calculation of finding roots are more toward computer-oriented and no longer manually numerical calculated. An approximation in decimals can be always obtained to any degree of accuracy which may be required. The ordinary tables give the values continue to four decimal places; but tables recently have been computed extending to twelve or even higher decimal places. These are the factors that motivate us to find a more reliable, faster and stable way to solve nonlinear equation problems.

1.1.1 A Brief History of Root Finding

Even though root finding of nonlinear equations of $f(x)=0$ equal zero might be an ancient problem, but it is still relevant today. This is because in most nonlinear cases, it is very difficult to obtain an analytical solution of the equation, $f(x)=0$. It is one of the significant elements in mathematics and is the determination of solutions to single-variable equations or to systems of n equations in n unknowns which is used enormously in science and engineering fields. The basics of the method revolve around the determination of roots, an analytical solution to an equation (or a system) which able to produce with an exact answer. Some equations, such as