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Solving One-Predator Two-Prey System by using Adomian Decomposition Method

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Abstract: In this paper, a mathematical model of one-predator two-prey system is discussed. This model is derived from predator-prey Lotka-Volterra model by adding another population of prey into the system. The model derived is a nonlinear system of ODEs. So the approach to this model is different from the linear system of ODEs. With reference to that, Adomian Decomposition Method (ADM) is one of the semi-analytical approaches being applied in this paper to solve the system. The approximate solution is made until four terms. The solution obtained is analyzed graphically.

Keywords: Adomian Decomposition Method, Adomian Polynomials, Lotka-Volterra, Predator Prey

1 Introduction

Population is a group of organisms of the same species which live in the same region at the same time. A population consists of all the individuals of a given species in a specific region or area at a certain time. It contains genetic variation between themselves. From one individual to another, there might be a slight difference among them such as in terms of size, hair color, and so forth. Most importantly, not all individuals in the population have the same ability to survive and reproduce.
All living things within an ecosystem are dependent on each other. A change in the size of one population affects all other organisms within the ecosystem. There are many types of interaction that happen among the population and one of them is predation. According to Obaid and Hamoud [1], predation is the species interaction when one species, the predator eats another species, the prey as a source of food.
In the 1920's, a mathematical model was created independently by Alfred Lotka and Vito Volterra, and it is characterized by oscillations in the population size of both predator and prey [2]. This model was proposed to explain the dynamic, natural populations and describe the relationship of predators and prey. It is an application of a nonlinear system of differential equations in mathematics as it involves more than 1 population. This model is known as Lotka-Volterra predator-prey model.
From the model, a new system is derived by adding another prey population into the system. In formulating the system, there are a few assumptions made. The assumptions are :

1. Consider there are only one predator and two prey species in the system.
2. Both preys do not compete with each other.
3. Food sources for both preys are different and unlimited.
4. The system is free from any disease that can increase the death rate.
5. The same species are competing in nature.

In [3], many topics under the system of Lotka-Volterra were discussed, one of them was two species Lotka-Volterra systems. The author has reviewed about the evolution of predator and prey fish populations in Adriatic Sea. He stated that there are several points of criticism worth noting for the Lotka-Volterra model. The changes of birth and death rates do nothing but the changes in the period of oscillation - i.e. no population - can dominate, and it is impossible for either population being driven to extinction.

Another study was conducted by Schrum [4] where the author discussed the two-dimensional Lotka-Volterra systems where the interaction of two species was assumed as a closed system. The two given species are only affected to each other and no other external factors can influence the system. Schrum stated that the main weakness of this model is the exponential growth assumption. It is biologically unrealistic for the unbounded growth of the prey species without predators. He extended the discussion by stating that the system exhibits stable periodic behavior for all non-zero initial conditions.
Obaid and Hamoud [1] discussed two types of interaction in their article, which are one-prey one-predator system and one-predator two-prey system. The authors showed how both interaction was being modeled. Numerical simulation is also used in the verification of the results. After both mathematical model were examined, predator and prey populations seemed to be cyclical endless without settling down quickly. The same result is also stated in [5].
Kar and Batabyal [6] emphasized on the persistence and stability of the one-predator two-prey system. The authors used Holling type I and Holling type II response function to represent the interaction between the predator and prey. Kar and Chaudhuri [7] considered a two-prey one-predator harvesting model with interference. The model was based on Lotka-Volterra dynamics with two competing species which were affected not only by harvesting but also by the presence of a predator, the third species.
Biazar et al. [8] discussed solving the system of ordinary differential equations. It is known that among the many, Lotka-Volterra is a type of first order nonlinear system of differential equations. The author showed that Adomian Decomposition Method (ADM) was able to solve both linear and nonlinear system of differential equations. According to Babolian et al. [9], ADM was initially proposed by George Adomian in the early of 1980s. The authors stated that a solution of functional equation was assumed as the summation of infinite series that usually converges to the solution. There were two topics discussed which are system of linear equation and system of Volterra integral equation. The authors presented numerical examples to show a comparison between ADM and Jacobi iterative method for solving system of linear equations. The results indicate both methods are the same which means if Jacobi Iterative method diverges, so will ADM.
The main part in solving a system of differential equation using ADM is computing the Adomian polynomial. Many authors suggested different algorithms for computing Adomian polynomials [10-12]. This paper uses an alternate algorithm for computing Adomian polynomials [11].

2 Problem Formulation

The derivation of the system was discussed in Obaid and Hamoud [1]. The system was given by Eq. (1).

$$\begin{aligned}
\frac{dy_1}{dt} &= \left(-a + by_2 + b_2 y_3\right) y_1 \quad , \; y_1(0) = y_{10} \\
\frac{dy_2}{dt} &= \left(c - dy_2 - ey_1\right) y_2 \qquad , \; y_2(0) = y_{20} \\
\frac{dy_3}{dt} &= \left(c_2 - d_2 y_3 - e_2 y_1\right) y_3 \quad , \; y_3(0) = y_{30}
\end{aligned} \tag{1}$$

where $y_1(t)$ represents the predator's populations at time t, $y_2(t)$ represents the first prey's populations at time t and $y_3(t)$ represents the second prey's populations at time t. The parameter a represents per capita death rate of predator in the absence of prey, parameter c represents per capita birth rate of first prey in the absence of predator and parameter c_2 represents per capita birth rate of second prey in the absence of predator . Parameter b and b_2 represent interaction parameter of predator with the first and second prey respectively. Parameter d and d_2 represent the interaction parameter of first and second prey populations respectively. Lastly, parameter e and e_2 respresent the first and second prey death rate killed by predator respectively. Eq. (1) is called one-predator two-prey system. This nonlinear system will be solved by using Adomian decomposition method.

3 Adomian Decomposition Method

The Adomian Decomposition Method is a semi analytical method for solving Ordinary Differential Equations (ODE) and non-linear Partial Differential Equations (PDE). The purpose of this method is towards a unified theory for the solution of PDE, an aim which has been superseded by a more general theory of the homotopy analysis method.
The important aspect of ADM is employment of the Adomian Polynomials which allows for solutions to convergence of the nonlinear portion of the equation, without help to linearizing the system. These polynomials are mathematically generalized to Maclaurin series about an arbitrary external parameter that give more flexibility compared to the Taylor series.

Consider the one-predator two-prey system with the initial condition given by Eq. (1). The system can be represented in the form of L operator :

$$\begin{aligned} Ly_1 &= (-a + by_2 + b_2 y_3) y_1 \\ Ly_2 &= (c - dy_2 - ey_1) y_2 \\ Ly_3 &= (c_2 - d_2 y_3 - e_2 y_1) y_3 \end{aligned} \tag{2}$$

where L is linear operator $\frac{d}{dt}$ with inverse, $L^{-1} = \int_0^t (\cdot) dt$. By applying the L^{-1} on both sides of each Eq. (2), the equation becomes :

$$\begin{aligned} y_1(t) &= y_1(0) + \int_0^t (-a + by_2 + b_2 y_3) y_1 \; dt \\ y_2(t) &= y_2(0) + \int_0^t (c - dy_2 - ey_1) y_2 \; dt \\ y_3(t) &= y_2(0) + \int_0^t (c_2 - d_2 y_3 - e_2 y_1) y_3 \; dt \end{aligned} \tag{3}$$

Then, according to Babolian et. al [9], the solution is considered as the sum of infinite series with parameter λ.

$$\begin{aligned} y_1 &= \sum_{n=0}^{\infty} y_{1,n} \, \lambda^n \\ y_2 &= \sum_{n=0}^{\infty} y_{2,n} \, \lambda^n \\ y_3 &= \sum_{n=0}^{\infty} y_{3,n} \, \lambda^n \end{aligned} \tag{4}$$

Let each integrand in Eq. (3) as the sum of the following series :

$$
\begin{aligned}
f_1(y_1, y_2, y_3) &= \sum_{n=0}^{\infty} A_{1,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right)\lambda^n \\
f_2(y_1, y_2, y_3) &= \sum_{n=0}^{\infty} A_{2,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right)\lambda^n \\
f_3(y_1, y_2, y_3) &= \sum_{n=0}^{\infty} A_{3,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right)\lambda^n
\end{aligned}
\tag{5}
$$

where $A_{1,n}, A_{2,n}, A_{3,n}$ are called Adomian Polynomials. Then, substitute Eq. (4) and Eq. (5) into Eq. (3):

$$
\begin{aligned}
\sum_{n=0}^{\infty} \lambda^n y_{1,n} &= y_1(0) + \sum_{n=0}^{\infty} \lambda^n \int_0^t A_{1,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right) dt \\
\sum_{n=0}^{\infty} \lambda^n y_{2,n} &= y_2(0) + \sum_{n=0}^{\infty} \lambda^n \int_0^t A_{2,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right) dt \\
\sum_{n=0}^{\infty} \lambda^n y_{3,n} &= y_3(0) + \sum_{n=0}^{\infty} \lambda^n \int_0^t A_{3,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right) dt
\end{aligned}
\tag{6}
$$

Then, define the following scheme :

$$
\begin{aligned}
y_{1,0} &= y_1(0), \quad y_{n+1} = \int_0^t A_{1,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right) dt \\
y_{2,0} &= y_2(0), \quad y_{n+1} = \int_0^t A_{2,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right) dt \\
y_{3,0} &= y_3(0), \quad y_{n+1} = \int_0^t A_{3,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right) dt
\end{aligned}
\tag{7}
$$

In computing all the Adomian Polynomials, an alternate algorithm method is used [11]. From the series in Eq. (4), if we derive $f(y_1, y_2, y_3)$ as a power series of λ, then the coefficient of λ^n will be $A_{i,n}\left(y_{1,0},\dots,y_{1,n}, y_{2,0},\dots,y_{2,n}, y_{3,0},\dots,y_{3,n}\right)$. Then, the expansion will be as follows :

Predator equation :

$$
\begin{aligned}
A_{1,0} &= -a y_{1,0} + b y_{1,0} y_{2,0} + b_2 y_{1,0} y_{3,0} \\
A_{1,1} &= -a y_{1,1} + b\left(\sum_{r=0}^{1} y_{1,r} y_{2,1-r}\right) + b_2\left(\sum_{r=0}^{1} y_{1,r} y_{3,1-r}\right) \\
A_{1,2} &= -a y_{1,2} + b\left(\sum_{r=0}^{2} y_{1,r} y_{2,2-r}\right) + b_2\left(\sum_{r=0}^{2} y_{1,r} y_{3,2-r}\right) \\
A_{1,3} &= -a y_{1,3} + b\left(\sum_{r=0}^{3} y_{1,r} y_{2,3-r}\right) + b_2\left(\sum_{r=0}^{3} y_{1,r} y_{3,3-r}\right)
\end{aligned}
\tag{8}
$$

1st Prey equation :

$$
\begin{aligned}
A_{2,0} &= cy_{2,0} - d\left(y_{2,0}\right)^2 - ey_{1,0}y_{2,0} \\
A_{2,1} &= cy_{2,1} - d\left(\sum_{r=0}^{1} y_{2,r}y_{2,1-r}\right) - e\left(\sum_{r=0}^{1} y_{1,r}y_{2,1-r}\right) \\
A_{2,2} &= cy_{2,2} - d\left(\sum_{r=0}^{2} y_{2,r}y_{2,2-r}\right) - e\left(\sum_{r=0}^{2} y_{1,r}y_{2,2-r}\right) \\
A_{2,3} &= cy_{2,3} - d\left(\sum_{r=0}^{3} y_{2,r}y_{2,3-r}\right) - e\left(\sum_{r=0}^{3} y_{1,r}y_{2,3-r}\right)
\end{aligned}
\tag{9}
$$

2nd Prey equation :

$$
\begin{aligned}
A_{3,0} &= c_2y_{3,0} - d_2\left(y_{3,0}\right)^2 - e_2y_{1,0}y_{3,0} \\
A_{3,1} &= c_2y_{3,1} - d_2\left(\sum_{r=0}^{1} y_{3,r}y_{3,1-r}\right) - e_2\left(\sum_{r=0}^{1} y_{1,r}y_{3,1-r}\right) \\
A_{3,2} &= c_2y_{3,2} - d_2\left(\sum_{r=0}^{2} y_{3,r}y_{3,2-r}\right) - e_2\left(\sum_{r=0}^{2} y_{1,r}y_{3,2-r}\right) \\
A_{3,3} &= c_2y_{3,3} - d_2\left(\sum_{r=0}^{3} y_{3,r}y_{3,3-r}\right) - e_2\left(\sum_{r=0}^{3} y_{1,r}y_{3,3-r}\right)
\end{aligned}
\tag{10}
$$

So, in general :

$$
\begin{aligned}
A_{1,n} &= -ay_{1,n} + b\left(\sum_{r=0}^{n} y_{1,r}y_{2,n-r}\right) + b_2\left(\sum_{r=0}^{n} y_{1,r}y_{3,n-r}\right) \\
A_{2,n} &= cy_{2,n} - d\left(\sum_{r=0}^{n} y_{2,r}y_{2,n-r}\right) - e\left(\sum_{r=0}^{n} y_{1,r}y_{2,n-r}\right) \\
A_{3,n} &= c_2y_{3,n} - d_2\left(\sum_{r=0}^{n} y_{3,r}y_{3,n-r}\right) - e_2\left(\sum_{r=0}^{n} y_{1,r}y_{3,n-r}\right)
\end{aligned}
\tag{11}
$$

In this paper, the solution is approximated until four terms. So, all the Adomian Polynomials will be generated for n = 0, 1, 2. The four terms approximation to the solution are considered as

$$
y_i(t) \approx y_{i,0} + y_{i,1} + y_{i,2} + y_{i,3} \tag{12}
$$

The three populations for y_1, y_2 and y_3 are fox, rabbit and turkey populations respectively where their initial population are 100, 90 and 80. The parameters a = 0.5, b = 0.01, $b_2 = 0.02$, c = 1, d = 0.01, e = 0.01, $c_2 = 3.2$, $d_2 = 0.04$ and $e_2 = 0.03$ are used for solving the model [1].

4 Results and Discussion

The approximations to the solutions with four terms for $y_1(t), y_2(t)$ and $y_3(t)$ are as follow:

$$y_1(t) \approx 100 + 200t - 80.5t^2 - 97.36667t^3$$

$$y_2(t) \approx 90 - 81t - 17.1t^2 + 66.54t^3$$

$$y_3(t) \approx 80 - 240t + 504t^2 - 1265.2t^3$$

From the solutions obtained, some of the approximated values are shown below :

Table 1 : Numerical results obtained from the solutions using ADM

t	$y_1(t)$	$y_2(t)$	$y_3(t)$
0.1	119.0976333	81.79554	59.7748
0.2	136.0010666	73.64832	42.0384
0.3	150.1260999	65.95758	19.1996
0.4	160.8885331	59.12256	−16.3328
0.5	167.7041662	53.54250	−72.1500
0.6	169.9887993	49.61664	−155.8432
0.7	167.1582322	47.74422	−275.0036
0.8	158.6282650	48.32448	−437.2224
0.9	143.8146976	51.75666	−650.0908
1.0	122.1333300	58.44000	−921.2000
1.1	92.9999622	68.77374	−1258.1412
1.2	55.8303942	83.15712	−1668.5056
1.3	10.0404260	101.98938	−2159.8844
1.4	−44.9541425	125.66976	−2739.8688

Table 1 shows some numerical results after using Adomian Decomposition Method in approximating the predator and both preys population for some value of *t*. The negative value means that the population extincts. This result is analyzed further by a graph that has been plotted using MAPLE software.

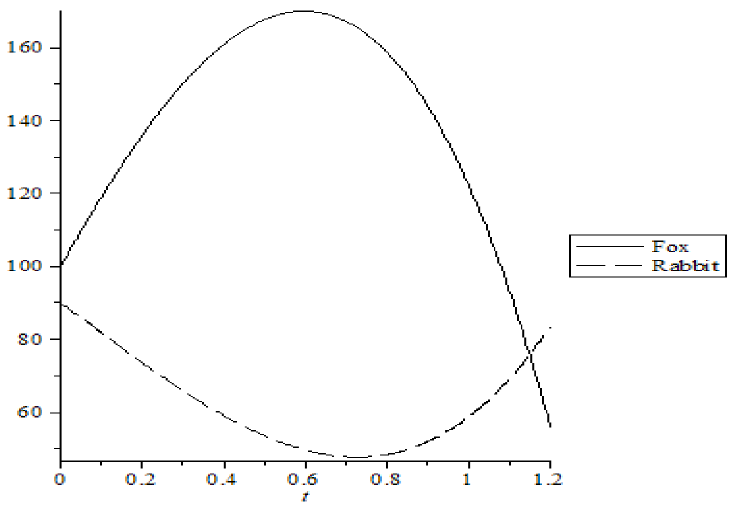

Figure 1: Population of foxes and rabbits versus time

Figure 1 shows the population of foxes and rabbit. This graph agrees with the graph plotted in Biazar and Montazeri [13].

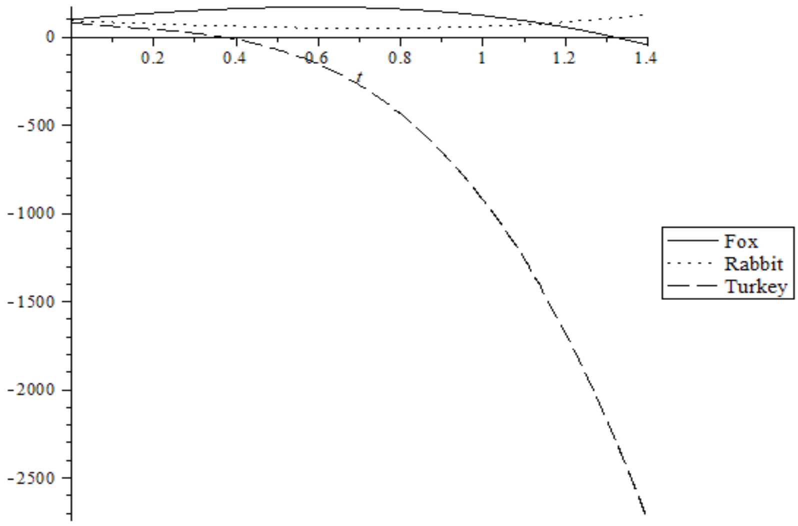

Figure 2: Population of foxes, rabbits and turkey versus time

In Figure 2, all the three populations are plotted. We can summarize that there are patterns we can see. First, populations go up and down, this refers to a graph where shape of the graph is oscillated up and down. Second, the number of predator population depends upon the number of prey population. As can be seen in the table and graph, the number of foxes increases gradually over the time, *t* at the beginning. On the other hand, the number of rabbits and turkeys decreases as we can say the population is in the process of adapting to the environment.

Foxes are known as the predator that readily adapt to new environments and so are very successful invasive species. Foxes grow quickly and occupy new area and generally prey on native species, which are easily caught. So, it is relevant to say that number of foxes increases at the beginning.

Yet, the number of rabbits decreases at first and then going up back when foxes turn to decrease. It is logically realistic for the rabbits population to show an increase when there are less foxes. For turkeys population, we can see the values continuously go down until they reach 0 and drop to a negative. It means that the turkeys population is experiencing an extinction. The graph agrees with Vlastmil and Eisner [14] who said, when resources grow exponentially, handling times are zero and apparent competition always leads to extinction of the weaker resource.

5 Conclusion

This paper discusses solving one-predator two-prey system by using Adomian Decomposition Method. This method is claimed to be a powerful device for solving nonlinear problems. From the two preys' population, one prey goes to extinction while the other prey survives. The predator population is also heading towards extinction. All predators and preys can neither leave these populations nor enter them from the outside system. This means that there is no possible immigration into the system. This model also assumes that the predator has only two types of prey as food sources. This is rarely the case in nature and the real world. As for the recommendation, this method should be applied to an improved model which considers realistic assumptions and an open system.

References

[1] Obaid, T. A. S., and Hamoud, A. K. Predator-Prey Relationships System. International Journal of Computer Applocations. 2016. 140(5): 42-44

[2] Bacaër N. Lotka, Volterra and the predator–prey system (1920–1926). In: A Short History of Mathematical Population Dynamics. Springer, London. 2011

[3] Baigent, S. Lotka –Volterra Dynamical Systems. Dynamical and Complex Systems: 2017

[4] Schrum, J. Modelling Population Dynamics with Volterra-Lotka Equations, Retrieved from http://people.southwestern.edu/~schrum2/MathCapstone.pdf

[5] Green, E. The Effect of a `Smart' Predator in a One Predator, Two Prey System, Rose-Hulman Undergraduate Mathematics Journal. 2004. 5(2)

[6] Kar, T. K. and Batabyal, A. Persistence and stabilitiy of a two prey one predator system. International Journal of Engineering, Science and Technology. 2010. 2(2): 174-190.

[7] Kar, T. and Chaudhuri, K. Harvesting in a Two-Prey One-Predator Fishery: A Bioeconomic Model. Anziam Journal. 2004. 45(3)3: 443-456.

[8] Biazar, J., Babolian, E. and Islam, R. Solution of the system of ordinary differential equations by Adomian decomposition method. Applied Mathematics and Computation. 2004. 147(1): 713-719.

[9] Babolian, E., Biazar, J. and Vahidi, A. On the decomposition method for System of Linear Equations and System of Linear Volterra Integral Equations. Applied Mathematics and Computation. 2004. 147(1): 19-27.

[10] Wazwaz, A. M. A new algorithm for calculating Adomian polynomials for non-linear operator. Applied Mathematics and Computation. 2000. 111: 53-69.

[11] Biazar, J., Babolian, E., Kember, G., Nouri, A. and Islam, R. An alternate algorithm for computing Adomian polynomials in special cases. Applied Mathematics and Computation. 2003. 138: 523-529.

[12] Biazar, J. and Shafiof, S. M. A Simple Algorithm for calculating Adomian Polynomials. Int. J. Contemp. Math Sciences, 2007. 2(2): 975-982.

[13] Biazar, J. and Montazeri, R. A computational method for solution of the prey and predator problem. Applied Mathematics and Computation. 2005. 163: 841-847.

[14] Vlastmil, K. and Eisner, J. The effect of Holling Type II functional response on apparent competition. Theoretical Population Biology. 2006. 70: 421-430.