

**UNIVERSITI TEKNOLOGI MARA**

**EXTREMAL PROPERTIES, COEFFICIENT  
INEQUALITIES AND RADIUS PROPERTIES OF A  
CERTAIN CLASS OF CLOSE-TO-CONVEX  
FUNCTIONS**

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## AUTHOR'S DECLARATION

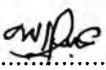
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## ABSTRACT

This thesis is concerned with the function  $f$  defined on the open unit disk  $E = \{z : |z| < 1\}$  of the complex plane. Let  $A$  be the class of analytic functions defined on  $E$  which is normalised and has the Taylor series representation of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \cdots + a_n z^n + \cdots = z + \sum_{n=2}^{\infty} a_n z^n.$$

This thesis concentrates on the class  $\mathcal{G}(\alpha, \delta)$  where functions in this class satisfy the condition

$$\operatorname{Re} \left\{ e^{i\alpha} \frac{zf'(z)}{g(z)} \right\} > \delta, \quad (z \in E)$$

with  $|\alpha| < \pi$ ,  $\cos \alpha > \delta$  and  $g(z) = \frac{z}{1-z^2}$ . Some of the basic properties are attained for  $\mathcal{G}(\alpha, \delta)$  including the representation theorem and extremal properties such as coefficient bounds, distortion theorems, covering theorems and rotation theorems. The arc length and the area for functions in this class are also obtained.

This thesis also discusses on the coefficient inequalities for  $\mathcal{G}(\alpha, \delta)$ . Lastly, we deal with the bounds for  $\frac{f(z)}{z}$  and  $f'(z)$  and obtain results for the radius of univalence, the radius of starlikeness, and the radius of convexity for this class.

# TABLE OF CONTENTS

	Page
<b>AUTHOR'S DECLARATION</b>	ii
<b>ABSTRACT</b>	iii
<b>ACKNOWLEDGEMENTS</b>	iv
<b>TABLE OF CONTENTS</b>	v
<b>LIST OF SYMBOLS</b>	vii
<b>CHAPTER ONE : PRELIMINARIES</b>	
1.1 The Classes of Analytic and Univalent Functions	1
1.2 Class of Functions with Positive Real Part, $P$	2
1.3 Some Subclasses of Univalent Functions	5
1.4 Multivalent Functions	9
1.5 Subordination	9
1.6 Function with Negative Coefficient	10
1.7 Radius Problem	10
1.8 Some Properties of Functions in a Certain Class	11
1.9 Objective of the Study	15
1.10 Thesis Outline	15
<b>CHAPTER TWO : EXTREMAL PROPERTIES</b>	
2.1 Introduction	16
2.2 Representation Theorem	19
2.3 Some Extremal Properties	25
2.4 Arc Length and Area	48
2.5 Summary	56
<b>CHAPTER THREE : COEFFICIENT INEQUALITIES FOR THE CLASS <math>\mathcal{G}(\alpha, \delta)</math></b>	
3.1 Introduction	57
3.2 Second Hankel Determinant	58
3.3 Summary	69

<b>CHAPTER FOUR : RADIUS PROPERTIES FOR <math>\mathcal{G}(\alpha, \delta)</math></b>	
4.1 Introduction	70
4.2 Bounds for $\frac{f(z)}{z}$ and $f'(z)$	71
4.3 Radius of Univalence, $R_U$	77
4.4 Radius of Starlikeness, $R_{St}$	79
4.5 Radius of Convexity, $R_K$	83
4.6 Summary	86
<b>CHAPTER FIVE : CONCLUSION AND RECOMMENDATION</b>	87
<b>REFERENCES</b>	88