

UNIVERSITI TEKNOLOGI MARA

**STABILITY ANALYSIS OF TWO-
SPECIES MUTUALISM MODEL
WITH TIME DELAY AND
HARVESTING**

RUSLIZA BINTI AHMAD

Thesis submitted in fulfillment
of the requirements for the degree of
Master of Science

Faculty of Computer and Mathematical Sciences

December 2013

AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

Name of Student : Rusliza Binti Ahmad

Student I.D. No. : 2010631316

Program : Master of Science (CS780)

Faculty : Computer and Mathematical Sciences

Thesis Title : Stability Analysis of Two-Species Mutualism Model
with Time Delay and Harvesting

Signature of Student :

Date : December 2013

ABSTRACT

In this thesis, we analyze a two-species mutualism model with and without time delay and harvesting functions. The time delay is considered in the model to improve accuracy because the growth rate of the population depends not only on the present size of population but also on past information while the harvesting function is introduced to describe the rate of removal of the species. The stability of the model involving time delay is determined by defining a time delay margin. The global stability analysis of the unique positive equilibrium point of mutualism model with two different harvesting functions is proven by constructing a suitable Lyapunov function. The two harvesting functions are harvesting at a constant rate and harvesting at a rate proportional to the population size. The effect of time delay and two different harvesting functions on the dynamics of the population has been examined. We consider time delay into the basic model without harvesting and discuss the effect of delay on the stability of the unique positive equilibrium point. From the analysis, it can be concluded that time delay can induce instability of a stable equilibrium point. The time delay model is then modified to include a harvesting function. Then we discuss the effect of harvesting and time delay on the dynamics of the model. The analysis shows that when there is no delay, varying the harvesting rate does not affect the stability of the model if the value of harvesting rate is under control while time delay can cause a stable equilibrium point to become unstable.

ACKNOWLEDGEMENTS

In the name of Allah, the Most Gracious and the Most Merciful

Alhamdulillah, all praise be to Allah SWT for giving me the strength and His blessing in completing this thesis. Special appreciation goes to my supervisor Associate Professor Dr. Harun Budin, for his valuable supervision and wise guidance. I am grateful for having the opportunity to work under his supervision. My appreciation also goes to my co-supervisor, Associate Professor Dr. Salemah Ismail for her support and knowledge regarding this topic.

I would like to express my appreciation to the Dean, Faculty of Computer and Mathematical Sciences, Prof. Dr. Azlinah Mohamed for her support and help towards my postgraduate affairs. My acknowledgement also goes to all staff of Faculty of Computer and Mathematical Sciences for their cooperation.

My deepest appreciation goes to Universiti Teknologi MARA for granting the Young Lecturer Scheme (TPM) scholarship for my master's degree.

Sincere thanks to all my friends especially Ainon, Nazifah, Samsiah, Aslina, Nora Baizura and others for their kindness and moral support during my study. Thanks for the friendship and memories.

Last but not least, my deepest gratitude goes to my beloved parents, Ahmad Bin Saleh and and also to my brothers, Rominor and Syafirul for their endless love, prayers and encouragement. I would like to express my heartfelt appreciation to my grandmother, Siti Mariam Binti Mahmud, uncle, Hamidon Bin Ramli and late grandfather, Ramli Bin Hassan. I hope this work makes them proud. To all those who have directly or indirectly contributed to this research your kindness means a lot to me. Thank you very much.

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