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ON A SUBCLASS OF TILTED STARLIKE FUNCTIONS WITH RESPECT TO CONJUGATE POINTS

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AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any other degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for the Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.

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ABSTRACT

This thesis is concerned with the function f defined on an open unit disk $E = \{z : |z| < 1\}$ of the complex plane. Let A_S be the class of analytic functions defined on E which is normalized and has the Taylor series representation of the form $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots + a_n z^n + \cdots = z + \sum_{n=2}^{\infty} a_n z^n$. Let H be the class of functions ω which are analytic and univalent in E of the form $\omega(z) = t_1 z + t_2 z^2 + \cdots + t_n z^n + \cdots = \sum_{n=1}^{\infty} t_n z^n$. We define the class $S_c^*(\alpha, \delta, A, B)$ for which functions in the class $S_c^*(\alpha, \delta, A, B)$ satisfy the condition $\left(e^{i\alpha} \frac{zf'(z)}{g(z)} - \delta - i\sin\alpha\right)\frac{1}{t_{\alpha\delta}} = \frac{1 + A\omega(z)}{1 + B\omega(z)}$, $\omega \in H$ where $g(z) = \frac{f(z) + \overline{f(\overline{z})}}{2}$ and $t_{\alpha\delta} = \cos\alpha - \delta$ with $|\alpha| < \frac{\pi}{2}$, $\cos\alpha > \delta$, $0 \leq \delta < 1$ and $-1 \leq B < A \leq 1$. Some of the basic properties are obtained for the class $S_c^*(\alpha, \delta, A, B)$ such as distortion theorem, growth theorem, argument of $\frac{zf'(z)}{g(z)}$ and coefficient bounds. The upper and lower bounds of $\mathrm{Re}\frac{zf'(z)}{g(z)}$ and $\mathrm{Im}\frac{zf'(z)}{g(z)}$ for functions in the class $S_c^*(\alpha, \delta, A, B)$ are also given. This thesis also discusses on the radius problems which are the radius of convexity and the radius of starlikeness for the defined class. Lastly, the coefficient inequalities problems which are the upper bounds for the Second Hankel determinant $|a_2 a_4 - a_3^2|$ and Fekete-Szegö functional $|a_3 - \mu a_2^2|$ are determined for functions in the class $S_c^*(\alpha, \delta, A, B)$. Also included the coefficient determinant with Fekete-Szegö parameter which is $|a_2 a_3 - \beta a_4|$.

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CHAPTER ONE
PRELIMINARIES

This chapter consists of seven sections in which are presented some of the basic definitions and results on the theory of univalent functions. Also included are the objectives of this study and the thesis outline.

1.1 INTRODUCTION

This study is concerned with the geometric properties of the class of analytic and univalent functions in the complex plane. Some of the definitions will be given in this section which is the basic theory of univalent functions.

Definition 1.1.1 (Duren, 1983)
Let f be the complex-valued function defined in a domain D. We say that f is differentiable at z_0 if $\lim_{z \to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ exists, it is denoted by f'. Then f is said to be analytic at z_0 if the function f is differentiable at z_0 and throughout a neighbourhood of z_0. A function f is analytic at z_0 if there exists a Taylor series expansion of the form $f(z)=\sum_{n=0}^{\infty} a_n (z-z_0)^n, a_n = \frac{f^n(z_0)}{n!}$ such that the series converges in some open unit disc centered at z_0.

According to Goodman (1983), the theory of univalent functions is so vast and complicated. Thus, certain assumptions are necessary and due to the Riemann Mapping Theorem, the arbitrary domain D is restricted to the unit disc $E=\{z:|z|<1\}$. By focusing on the scope of the domain D in which $E=\{z:|z|<1\}$ as a domain throughout the study, this assumption leads to create short formulas so that the computations involved later can be simplified. In what follows is