

**UNIVERSITI TEKNOLOGI MARA**

**PROPERTIES OF TILTED  
UNIVALENT ANALYTIC  
FUNCTIONS OF ORDER 8**

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## ABSTRACT

This thesis deals with the functions  $f$ , analytic and univalent in the open unit disk denoted as  $U = \{z \in \mathbb{C} : |z| < 1\}$ . Let  $A$  be the class of analytic functions/defined in  $U$  and  $S$  be the subclass of  $A$  normalized by  $f(0) = 0, f'(0) = 1$  and has the Taylor series expansion of the form

$$f(z) = z + a_2 z^2 + a_3 z^3 + \dots = z + \sum_{n=2}^{\infty} a_n z^n.$$

Also, let  $P$  be the subclass of  $A$  consisting functions  $h$ , such that  $\operatorname{Re}\{f'(z)\} > 0$ ,  $h(0) = 1$  and has the form of

$$h(z) = 1 + h_1 z + h_2 z^2 + \dots = \sum_{n=0}^{\infty} h_n z^n.$$

In this thesis, we investigate on the class  $P(A, S)$  of  $A$ -tilted Caratheodory functions of order  $\delta$  and the subclasses of  $S$  denoted by  $C_g(A, S)$  of  $A$ -close-to-convex functions of order  $\delta$ . Such functions in  $P(X, d)$  and  $C_g(A, S)$  satisfies

$$\operatorname{Re}\{e^{i\theta} f'(z)\} > \delta \quad \text{and} \quad \operatorname{Re}\{e^{i\theta} h(z)\} > \delta \quad (zeU),$$

respectively with  $|\theta| < \frac{\pi}{2}$ ,  $\cos(\theta) > \delta$ ,  $0 < \delta < 1$  and  $g_a(z) = \frac{z}{(1-cuz)^a}$  for  $0 < a < 1$ .

Some basic properties such as representation function, coefficient bounds, distortion theorem and growth theorem for the class  $P(X, d)$  and  $C_g(X, S) = C_{g_X}(I, S)$  are obtained. The bounds for real and imaginary part of  $h \in P(A, S)$  and  $f \in C_g(A, S)$  are also determined. We also discuss on the coefficient inequalities which consist of the upper bounds for the second Hankel determinant,  $a_2 a_4 - a_3^2$ , and the Fekete-szegő functional,  $a_3 - \lambda a_2^2$ . We determined the upper bound for  $a_2 a_4 - a_3^2$  for function in  $C_g(A, S)$  and the upper bound for  $a_3 - \lambda a_2^2$  for function in  $C_g(X, S)$ . Lastly, we discuss on the radius problems which focuses on finding the radius of convexity and the radius of starlikeness for the class  $C_g(A, S)$ .

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