AN OBSERVER DESIGN OF NONLINEAR QUARTER CAR MODEL FOR ACTIVE SUSPENSION SYSTEM BY USING BACKSTEPPING CONTROLLER

Raden Sre Emikaningseh Bt. R.A.Othman
(2004347256)
Faculty of Electrical Engineering
University Teknologi MARA (UiTM)
40450 Shah Alam
Selangor Darul Ehsan

ABSTRACT

This paper presents the use of backstepping controller in order to construct an active suspension for a quarter car model with observer design. The implementation of backstepping controller is presented in non-linear quarter car model for active suspension system. The movement of the rotational motion of arm has been taking into consideration for this system which has been neglected by the linear system. Backstepping control is a recursive control procedure which breaks a design problem for the full system into a sequence of a design problem for lower order systems [1]. The performance of the proposed controller will be compared with the linear quadratic regulator controller to help validate the result.

Keywords—backstepping, observer design, rotational motion arm, and linear quadratic regulator

I. INTRODUCTION

The term of suspension is referring to the system of springs, shock absorbers and linkages that connects a vehicle to its wheels. Two purposes that serve by the suspension are contributing to the car's handling and braking for good active safety and driving pleasure, and keeping vehicle occupants comfortable and reasonably well isolated from disturbances such as road noise, bumps, and vibrations. The suspension also protects the vehicle itself and any cargo or luggage from damage and wear. Different characteristics can be considered in a suspension system design namely, ride comfort, body displacement, wheel displacement and suspension travel.

At the beginning, modeling of the active suspension systems has only considered the linear force input to the system. Recently, with the developments of new control theories, the force input to the active suspensions has been replaced by an input to control the actuator [5]. The active suspension system now consists of the dynamics of suspension system plus the dynamic of the actuator system.

The purpose of this paper is to utilize the concept of backstepping controller to the non-linear quarter car motion equation in the state space form.

II. METHODOLOGY

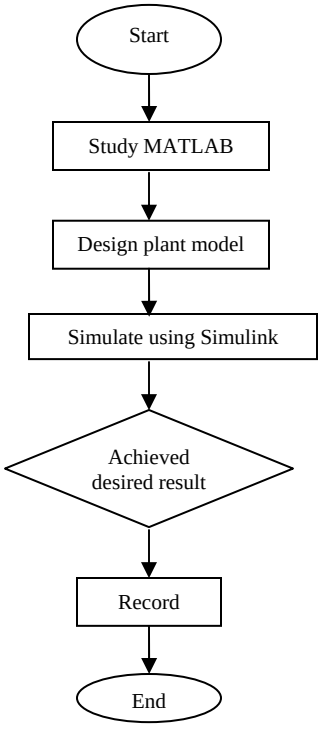

Figure1: Flow chart for the methodology of work

Figure 1 above shows the flow of methodology been use during completing this paper.

III. SYSTEM MODELING

The schematic in Figure 2[2] shows the model of a suspension system which the rotational motion of sprung mass and rotational motion of control arm is admitted. If the joint between the control arm and the car body is assumed to be bushing and the mass of the control arm is included, the degree of freedom of a quarter car system is four. However, if mass of the control arm is neglected and the bushing is assumed to be a pin joint, the degree of freedoms becomes two [1]. Therefore, the generalized coordinates are z_s and θ. The schematic diagram of the suspension system with the degree two is shown in Figure 3 [1, 2].

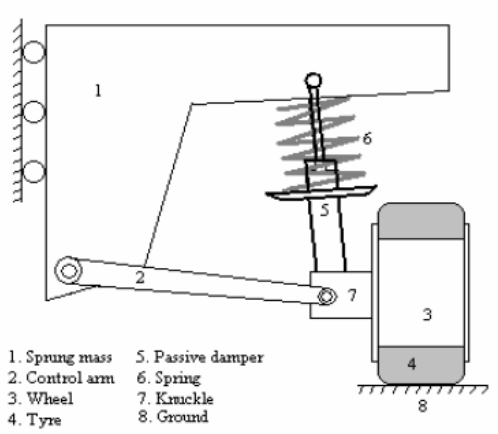

Figure 2: A schematic diagram for a Machperson suspension system for a quarter car model

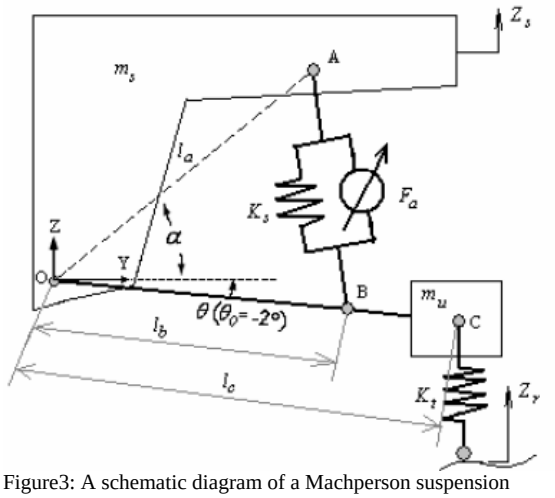

Figure3: A schematic diagram of a Machperson suspension system with 2 DOF

The state variable is stated as [1, 2, 5]

$$\left[x_1\ x_2 x_3 x_4\right]^T = \left[z_s\ \dot{z}_s \theta\ \dot{\theta}\right]^T \quad (1)$$

Shown in Figure 3, the m_s is the body mass, k_s is the spring constant coil spring k_t is the spring constant of the tire, m_u is the mass of the wheel, C_s is the damper damping constant, f_a is the control input, α is the angle between the OA and axis-x, z_s is the vertical displacement of the sprung mass, z_r is the irregular excitation from the road and θ_0 is the angular displacement of the control arm at static equilibrium point [1,2].

By following the state representation of the equations of motion for the sprung and unsprung masses of the passive and active quarter car model [1, 2, 5]

$$\dot{x}(t) = Ax(t) + B_1 f_a(t) + B_2 z_r(t) \quad (2)$$

the system matrix A, control input matrix B_1 and disturbance matrix B_2 are represented as [1, 2, 5],

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} & \frac{\partial f_1}{\partial x_4} \\ 0 & 0 & 0 & 1 \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} & \frac{\partial f_2}{\partial x_4} \end{bmatrix} \quad (3)$$

$$B_1 = \begin{bmatrix} 0 \\ \frac{\partial f_1}{\partial f_a} \\ 0 \\ \frac{\partial f_2}{\partial f_a} \end{bmatrix} \quad (4)$$

$$B_2 = \begin{bmatrix} 0 \\ \frac{\partial f_1}{\partial z_r} \\ 0 \\ \frac{\partial f_2}{\partial z_r} \end{bmatrix} \quad (5)$$

Referring to the Hong et al (1999; 2002) [2], the elements of A, B1 and B2 matrices are defined as,

$$\frac{\partial f_1}{\partial x_1} = \frac{-K_t l_c \sin^2(-\theta_0)}{m_s l_c + m_u l_c \sin^2(-\theta_0)} \quad (6)$$

$$\frac{\partial f_1}{\partial x_3} = \frac{1}{(m_s l_c + m_u l_c \sin^2(-\theta_0))^2} \left\{ \begin{array}{l} \left\{ \begin{array}{l} \left(\frac{1}{2} K_s \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) \cos(\alpha' + \theta_0) + \right. \\ \left. \left\{ \frac{1}{2} K_s \sin\alpha' \cos(-\theta_0) \left(\frac{d_l^2 \sin\alpha'}{2(c_l - d_l \cos(\alpha'))^{3/2}} \right) \right\} \right) \times \\ K_t l_c^2 \sin^2(-\theta_0) \cos(-\theta_0) \end{array} \right. \\ \left(m_s l_c + m_u l_c \sin^2(-\theta_0) \right) - \\ m_u K_s l_c \sin\alpha' \sin(-\theta_0) \cos^2(-\theta_0) \times \\ \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) \end{array} \right\} \quad (7)$$

$$\frac{\partial f_2}{\partial x_1} = \frac{-m_s K_t l_c \cos(-\theta_0)}{m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)} \quad (8)$$

$$\frac{\partial f_1}{\partial x_3} = \frac{1}{(m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(\theta_0))^2} \left\{ \begin{array}{l} \left\{ \begin{array}{l} \left(\frac{1}{2}(m_s + m_u) K_s \sin\alpha' \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) - \right. \\ \left. \left\{ \frac{1}{2}(m_s + m_u) K_s \sin\alpha' \left(\frac{d_l^2 \sin\alpha'}{2(c_l - d_l \cos(\alpha'))^{3/2}} \right) \right\} \right) + \\ m_s K_t l_c^2 \sin^2 \cos(-\theta_0) \end{array} \right. \\ \times \left(m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0) \right) + \\ \frac{1}{2}(m_s + m_u) m_u^2 K_s l_c^2 \sin\alpha' \sin(-\theta_0) \\ \left(b_l + \frac{d_l}{(c_l - d_l \cos(\alpha'))^{1/2}} \right) \end{array} \right\}$$

(10)

$$\frac{\partial f_1}{\partial f_s} = \frac{l_b \cos(-\theta_0)}{m_s l_c + m_u l_c \sin^2(-\theta_0)} \quad (11)$$

$$\frac{\partial f_2}{\partial f_s} = \frac{-(m_s + m_u) l_b}{m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)} \quad (12)$$

$$\frac{\partial f_1}{\partial z_r} = \frac{K_t l_c \sin^2(-\theta_0)}{m_s l_c + m_u l_c \sin^2(-\theta_0)} \quad (13)$$

$$\frac{\partial f_2}{\partial z_r} = \frac{m_s K_t l_c \cos(-\theta_0)}{m_s m_u l_c^2 + m_u^2 l_c^2 \sin^2(-\theta_0)} \quad (14)$$

With regards from the car parameter in Table 1[1, 2, 5],

Table 1
CAR PARAMETER

Mass of the car body, m_s	453kg
Wheel mass, m_u	36kg
Spring constant, k_s	17,658 N/m
Tire constant, k_t	183,887 N/m
Damper damping constant, C_s	1,500N/m
AO length, l_A	0.66m
BO length, l_B	0.34m
CO length, l_C	0.37m
Angle between y-axis and OA, α	74 degree
Angular displacement of the control arm at a static equilibrium point, θ_o	-2 degree

The calculation with regards to equation [6] to [14] can be done to define matrix A, B_1 and B_2 and yields,

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.49437 & 0 & 21.177 & 0 \\ 0 & 0 & 0 & 1 \\ -13796 & 0 & -5105.4 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0 \\ 0.002 \\ 0 \\ -0.074 \end{bmatrix} \text{ and } B_2 = \begin{bmatrix} 0 \\ 0.494 \\ 0 \\ 13796 \end{bmatrix}$$

For passive suspension, f_a is represented by the multiplication of the conventional damping of damper with the relative velocity between the car body and wheel. The relative velocity of the car body and the wheel is represented by [1],

$$\dot{\Delta l} = \frac{2 l_a l_b \sin(\alpha' - \theta)\dot{\theta}}{2(l_a^2 + l_b^2 - 2 l_a l_b \cos(\alpha' - \theta))^{1/2}} \quad (15)$$

Therefore the input force f_a is,

$$f_{q,passivre} = c_s \times \dot{\Delta l}$$

$$= c_s \times \frac{2 l_a l_b \sin(\alpha - \theta)\dot{\theta}}{2\left(l_a^2 + l_b^2 - 2 l_a l_b \cos(\alpha' - \theta)\right)^{1/2}} \quad (16)$$

IV. BACKSTEPPING CONTROLLER

Nonlinear backstepping control is a design approach for a quarter-car model. In Some researches, backstepping control is used to investigate nonlinear behavior of actuator in different road condition. The backstepping control approach developed by Kanellakopoulos and Kokotovic, has shown itself very effective in dealing with systems with multiple dynamics, such as mechanical systems driven by electrical systems, or multiple coupled mechanical systems[1].

Backstepping control design is started with the process of choosing the regulated variable [1]. The choice of this regulated variable is essential to the performance of the closed loop system. For the non linear suspension system, one possible choices of the regulated variable is [1]

$$z_1 = x_1 - \bar{x}_3 \tag{17}$$

where,

x_1*:* Wheel displacement

$\bar{x}_3$*:* Filtered version of the angular displacement

$$\bar{x}_3 = \frac{\varepsilon_0}{s+\varepsilon_0} x_3 \tag{18}$$

After choosing the regulated variable on equation (below), the design of the backstepping controller and proceed with two steps [1]:

Step 1: Compute the derivative of z_1

$$\dot{z} = \dot{\bar{x}}_1 - \dot{\bar{x}}_3 \tag{19}$$

$$= x_2 + \varepsilon_0(\bar{x}_3 - x_3)$$

By substituting equation (17) into (19)

$$= x_2 + \varepsilon_0(x_1 - z_1 - x_3)$$

$$= x_2 + \varepsilon_0(x_1 - x_3) - \varepsilon_0 z_1 \tag{20}$$

And by defining x_2 as the virtual control variable and which the stabilizing function is chosen as

$$\alpha_1 = -c_1 z_1 - \varepsilon_0(x_1 - x_3) \tag{21}$$

where $c_1 > 0$ and defined as the controller designed constant.

The resultant error variable would be

$$z_2 = x_2 - \alpha_1) \tag{22}$$

The resulting error equation is defined as,

$$\dot{z}_2 = -(c_1 + \varepsilon_0)z_1 + z_2 \tag{23}$$

Step 2: Compute derivative z_2

$$\dot{z}_2 = \dot{x}_2 - \dot{\alpha}$$

$$= \left[\frac{\partial f_1}{\partial x_1} z_s + \frac{\partial f_1}{\partial x_3}\theta + \frac{\partial f_1}{\partial x_4}\dot{\theta} + \frac{\partial f_1}{\partial f_a} f_a + \frac{\partial f_1}{\partial z_r} z_r\right] - \tag{24}$$

$$\left[-c_1(-c_1 z_1 - \varepsilon_0 z_1 + z_2) - \varepsilon_0(x_2 - x_4)\right]$$

Let the Lyapunov function be,

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \tag{25}$$

Therefore, the control input f_a of a backstepping controller is

$$f_a = \frac{1}{\frac{\partial f_1}{\partial f_a}}\left[(c_1^2 + \varepsilon_0 c_1 - 1)z_1 - (c_1 + c_2)z_2 - (x_2 - x_4)\varepsilon_0\right] \tag{26}$$

$$-\frac{1}{\frac{\partial f_1}{\partial f_a}}\left[\frac{\partial f_1}{\partial x_1} x_1 + \frac{\partial f_1}{\partial x_3} x_3 + \frac{\partial f_1}{\partial x_4} x_4 + \frac{\partial f_1}{\partial z_r} z_r\right]$$

where $c_2 > 0$ is controller design constant to render the derivative of Lyapunov function

$$\dot{V} = -(c_1 + \varepsilon_0)z_1^2 - c_2 z_2^2 \tag{27}$$

This implies the error of the system

$$\dot{z}_1 = -(c_1 + \varepsilon_0)z_1 + z_2$$
$$\dot{z}_2 = -c_2 z_2 - z_1 \tag{28}$$

has a generally exponentially stable equilibrium at $(z_1, z_2) = (0,0)$.

With the use of designed constant parameter c_1, c_2 and ε_0 as stated in Table 2, the performance of the suspension system with backstepping controller is able to determine. The value of c_1, c_2 and ε_0 had been tuned and the initial value is referred from [1],

Table 2
BACKSTEPPING PARAMETER

c_1	100
c_2	100
ε_0	20

V. OBSERVER DESIGN

The main issue of practical design of active suspension system is the sensor requirement which can be minimized using the state observer. In recent years, some researchers have applied an

optimal state observer based on Kalman filter to active suspension system [4].

In the design of the observer it can explained by [4, 7],

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B_1 u(t) + B_2 \hat{w}(t) + L(y - C\hat{x}) \quad (29)$$

and the uncertain system is described as,

$$\dot{x}(t) = Ax(t) + B_1 u(t) + B_2 w(t)$$
$$y(t) = Cx(t) \quad (30)$$

VI. SIMULATION AND RESULTS

In order to see the effectiveness, the technique of the proposed controller backstepping, will be compared with the LQR controller. The best possible linear feedback control law is explained as [1, 6, 3, 4, 5],

$$u = -Kx(t) \quad (31)$$

By assuming the quadratic performance index in the form of

$$J = \frac{1}{2}\int_0^t (x^T Qx + u^T Ru)dt \quad (32)$$

Table 3 state the parameter in obtaining LQR controller performance. The parameter of K, as the designed matrix is obtained with regards to Ackerman's formula [7] and which had been set in Simulink program while the value of matrix Q and R are fixed.

Table 3

LQR CONTROLLER PARAMETER

K=[77.21 -22.5 -98.42 261.7]
P=[-300 -450 -770 -80]
Q=[1 0 0 0;0 1 0 0;0 0 1 0;0 0 0 1]
R=[1]

As an addition, the performance of passive system also will be included as the system exists without any controller needed.

The typical road disturbance that generated for the system is as in Figure 4 an the road profile is [1, 2, 3, 4, 5, 6]

$$w(t) = \begin{cases} a(1-\cos 8\pi t), 2.25\text{sec} \le t \le 2.5\text{sec} \\ 0, otherwise \end{cases} \quad (33)$$

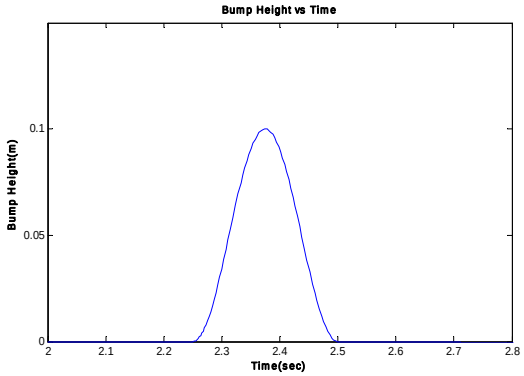

Figure 4: Road profile for a single bump; 10cm

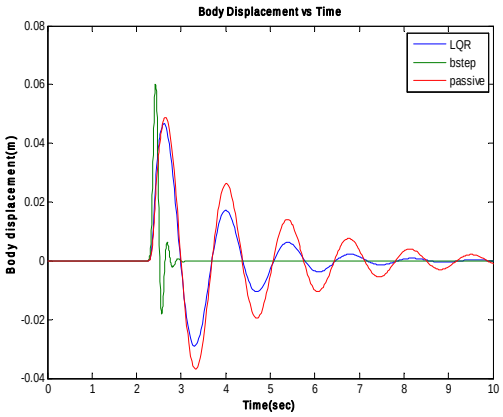

Figure 5: body displacement for LQR backstepping and passive performance

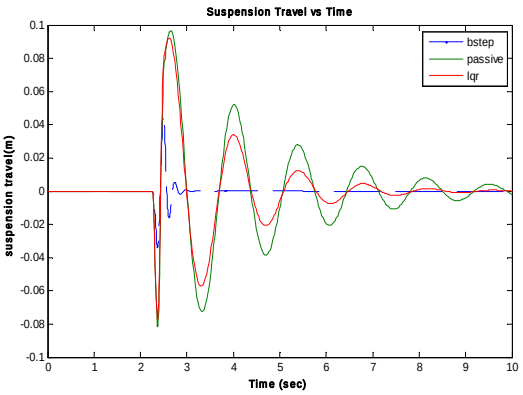

Figure 6: Suspension travel for LQR, backstepping and passive performance

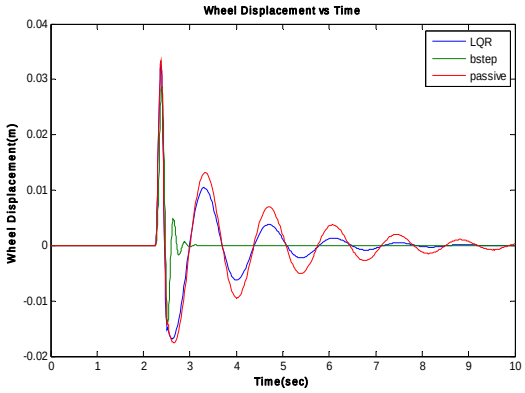

Figure 7: Wheel displacement for LQR, backstepping and passive performance

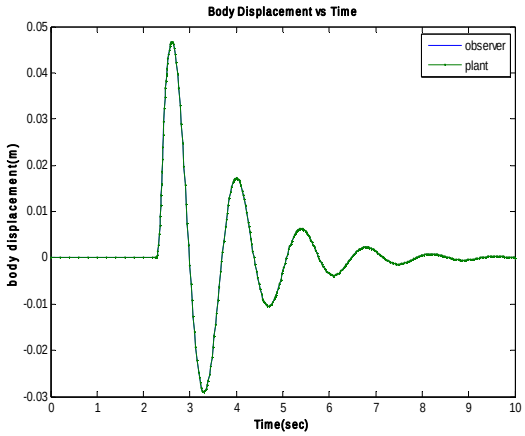

Figure 8: Body displacement between plant and observer for LQR

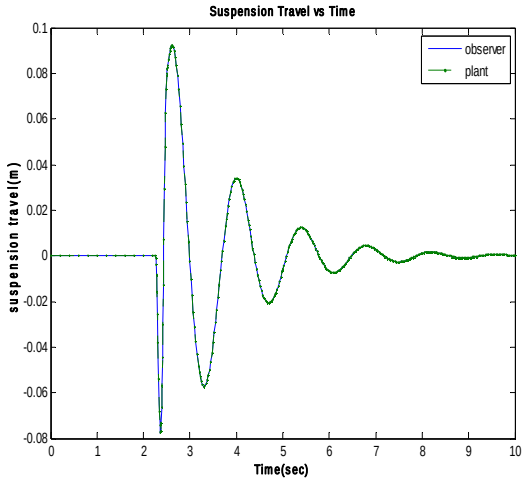

Figure 9: Suspension travel between plant and observer for LQR

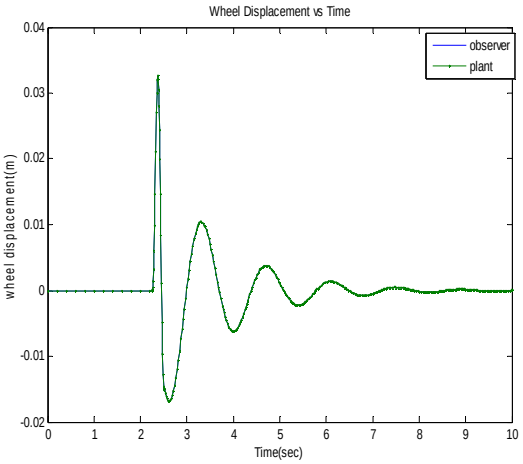

Figure 10: Wheel Displacement between Plant and observer for LQR

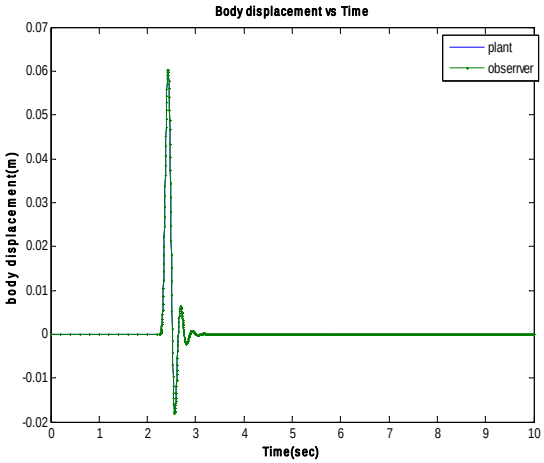

Figure 11: Body displacement between plant and observer for backstepping controller

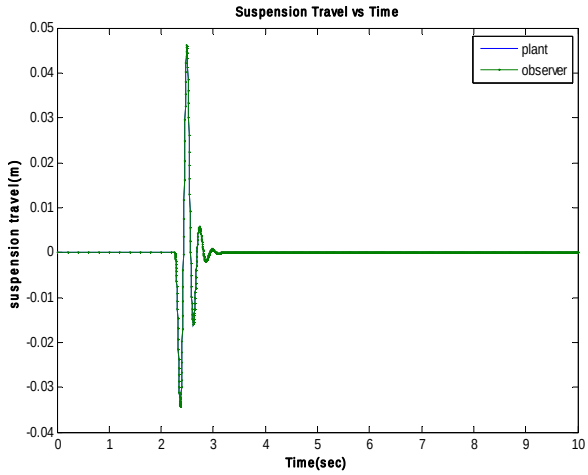

Figure 12: Suspension travel between plant and observer for backstepping controller

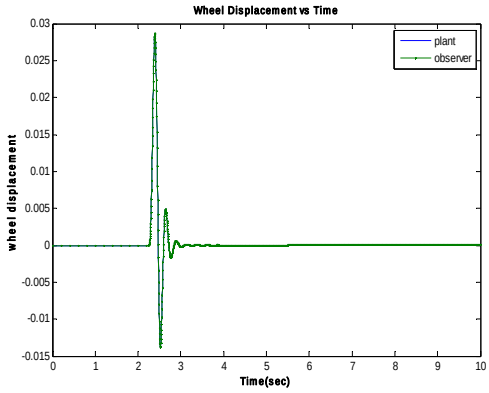

Figure 13: Wheel displacement between plant and observer for backstepping

For the purpose of performance comparison, the body displacement, suspension travel and wheel displacement were illustrated in the Figure 5, Figure 6 and Figure 7 respectively. Based on the three figures, it shows that the ride quality of active suspension system is better the passive suspension system. The waves based on the three figures also show that the ripple produced from the proposed controller with respect to time is less rather than using the LQR method. This explained backstepping gives greater ride performance.

Figure 8 to figure 9 help to compare between the result from the plant and observer for both, LQR and backstepping. Based on the results obtained, the performances from the plant and observer are exactly the same.

VII. CONCLUSIONS

The implementation of the backstepping controller has successfully achieved. Two conclusions can be made here after conducting the simulation with MATLAB simulink representation. The results had proved that the active suspension system which had been applied with the backstepping controller has a greater result compared to passive suspension system and also the backstepping controller can give better ride compared to LQR controller.

VIII. FUTURE DEVELOPMENTS

This paper is basically implemented with the a quarter car model of active suspension. As the objective had been successfully achieved it is possible to apply the backstepping method with the half car, full car model and also for steering system. The longer period of analysis and other parameter values may further help to improve the controller performance. Although the simulation is a virtual car performance design, it also can help in doing initial design for a practical car.

ACKNOWLEDGEMENT

Praise to Allah Almighty with His blessing I am able to complete this paper. My gratitude firstly goes to my parents and family because without them I would not have the strength to through my study years in UiTM. I also would like to thank Encik Adizul, my supervisor and Pn. Norlela as the co. supervisor for the help and guidance they been giving me during completing this project. Lastly, appreciation to my friends for their suggestions and support they been giving me.

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