UNIVERSITI TEKNOLOGI MARA

SOLVING ORDINARY DIFFERENTIAL EQUATIONS USING MULTISTEP LEAPFROG METHOD

NUR SALMI BINTI JAMIL

BSc

July 2026

TABLE OF CONTENTS

		Page
DECLARATION BY THE CANDIDATE		**iii**
ABSTRACT		**iv**
ACKNOWLEDGEMENT		**v**
TABLE OF CONTENTS		**vi**
LIST OF TABLES		**x**
LIST OF FIGURES		**xvi**
LIST OF SYMBOLS		**xvii**
LIST OF ABBREVIATIONS		**xix**
CHAPTER 1 INTRODUCTION OF RESEARCH		1
1.1	Research Introduction	1
1.2	Research Background	1
1.3	Research Problem	3
1.4	Research Objectives	4
1.5	Research Questions	5
1.6	Research Significance	5
1.7	Research Scope	5
1.8	Research Benefit	6
1.9	Definition of Terms and Concepts	7
1.10	Research Outline	8
CHAPTER 2 LITERATURE REVIEW		**10**
2.1	Introduction	10
2.2	Fundamental Concept of Ordinary Differential Equations	10
2.3	Theoretical Methods for Solving ODEs	11
2.3.1	Separable ODE	12
2.3.2	Linear ODE	13
2.3.3	Bernoulli ODE	14
2.4	Numerical Methods for Solving ODEs	15

4.5 Error Calculation 66
4.6 Conclusion 67

CHAPTER 5 RESULTS AND DISCUSSION 68
5.1 Introduction 68
5.2 Numerical Results 68
5.3 Approximate Solutions 68
5.3.1 Approximate Solutions for $x_n = 3.0$ 69
5.3.2 Approximate Solutions for $x_n = 6.0$ 73
5.3.3 Approximate Solutions for $x_n = 9.0$ 77
5.4 CPU Time 81
5.4.1 CPU Time for $x_n = 3.0$ 81
5.4.2 CPU Time for $x_n = 6.0$ 85
5.4.3 CPU Time for $x_n = 9.0$ 89
5.5 Error Analysis 93
5.5.1 Error Analysis for $x_n = 3.0$ 94
5.5.2 Error Analysis for $x_n = 6.0$ 98
5.5.3 Error Analysis for $x_n = 9.0$ 102
5.6 Performance Profile Analysis 106
5.6.1 Performance Profile based on CPU Time 107
5.6.2 Performance Profile based on Relative Error 112
5.7 Conclusion 116

CHAPTER 6 CONCLUSIONS AND RECOMMENDATIONS 117
6.1 Introduction 117
6.2 Conclusions 117
6.3 Recommendations 118

REFERENCES 120

APPENDICES 123
APPENDIX 1 Maple Code for Euler's Method 124
APPENDIX 2 Maple Code for Second-Order Taylor Series Method 126

ACKNOWLEDGEMENT

Alhamdulillah, all praise and gratitude be to Allah SWT for His countless blessings, guidance and mercy, which have given me the strength, patience and perseverance to complete this Final Year Project successfully. Without His will, this achievement would not have been possible.

I would like to extend my sincere appreciation and heartfelt gratitude to my supervisor, Dr. Mohd Rivaie bin Mohd Ali, for his invaluable guidance, continuous encouragement, constructive feedback and unwavering support throughout the completion of this research. His patience, understanding and dedication have greatly contributed to the successful completion of this study. I have truly benefited from his knowledge, guidance and willingness to share his expertise throughout this journey.

My sincere thanks are also extended to Madam Noor Khairiah Binti Razali, the lecturer in charge of the Final Year Project, for her dedication and commitment in coordinating the course throughout the semester. Her guidance, encouragement and continuous support ensured that I remained on track and successfully fulfilled all the requirements of this research.

A very special note of appreciation goes to my Final Year Project partner, Nur Shamilah Binti Shah Kirit, for walking this journey alongside me from beginning to end. Thank you for her constant support, cooperation and encouragement throughout this research. The countless hours spent discussing ideas, solving problems and motivating one another have made this challenging experience much more meaningful. I am truly grateful for her dedication, patience and friendship, and I could not have asked for a better partner to share this journey with.

My deepest appreciation goes to my beloved parents and family for their endless prayers, unconditional love, unwavering encouragement and continuous support. Their sacrifices and belief in me have been my greatest source of strength and motivation throughout this journey. I am also sincerely grateful to my friends and classmates for their constant encouragement, assistance and memorable moments that have made this academic journey more enjoyable and meaningful.

Last but not least, I would like to express my gratitude to myself for remaining resilient, believing in my abilities and never giving up despite the challenges encountered along the way. This journey has taught me the importance of perseverance, determination and self-belief, lessons that I will carry with me beyond this academic milestone.

ABSTRACT

First-order ordinary differential equations (ODEs) are widely used to model various real-world phenomena in science, engineering and applied mathematics. While theoretical methods provide exact solutions for certain classes of ODEs, many practical problems require numerical methods due to the complexity of the equations involved. This study aims to solve ten different first-order ODE problems theoretically and numerically using Euler's method, the second-order Taylor series method, the fourth-order Runge–Kutta (RK4) method and the Leapfrog multistep method. The efficiency of the Leapfrog method as compared to other numerical methods is further analysed based on accuracy and CPU time using *Maple* code with different step sizes. Performance profiles are constructed using *SigmaPlot 15.0* to determine the best numerical method. The results indicate that Euler's method consistently demonstrates the best computational efficiency by producing the lowest CPU time, whereas RK4 provides the best accuracy with the lowest percentage of relative errors. The result also shows that the Taylor-Leapfrog and RK4-Leapfrog methods offer a balanced performance between CPU time and accuracy.