Mathematics Letters, 2 (2): 263-273, 2023
eISSN: 2948-3735

INSIGHTS INTO THE KNAPSACK PROBLEM: A SYSTEMATIC LITERATURE REVIEW

Raja Ilham Raja Seman[1], Nur Hamizah Abdul Ghani[2*] and Sumarni Abu Bakar[3]

College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, 40450 Shah Alam, Selangor, Malaysia

[1]2023599419@student.uitm.edu.my, [2*]hamizahghani@uitm.edu.my, [3]sumarni164@uitm.edu.my
(indicates the corresponding author)*

ABSTRACT

This systematic literature review analyses 15 articles published in reputable international journals between 1992 and 2022 to gain comprehensive insights into the knapsack problem. The study focuses on three main themes: the purpose of the studies, data types used, and explored variants. The research reveals that the articles share a common goal of solving the knapsack problem, with a strong emphasis on proposing innovative algorithms. The problem demonstrates adaptability to various data types, and exploring different variants adds to the diversity of research. However, a limitation is the small number of articles analysed. Future reviews should encompass a larger quantity of articles for a more comprehensive understanding. This SLR contributes to a clearer understanding of the knapsack problem and its potential for practical applications in diverse domains.

Keywords*: Knapsack problem, Optimisation, Systematic Literature Review*

Received for review: 15-09-2023; Accepted: 22-10-2023; Published: 06-11-2023

1. Introduction

The knapsack problem is a classic optimization problem that has attracted significant attention from researchers across various disciplines, including computer science, operations research, and mathematics. The problem is then to choose a subset of the given items such that the corresponding profit sum is maximized without exceeding the capacity of the knapsack (Martello & Toth, 1990). Over the years, extensive research has been conducted to explore different aspects of the knapsack problem, including algorithmic techniques, problem variants, and real-world applications. Moreover, the extension of the knapsack problem to higher dimensions both in the number of constraints and in the number of knapsacks, as well as the modification of the problem structure concerning the available item set and the objective function, leads to a number of interesting variations of practical relevance which were the subject of intensive research (Kellerer et al., 2004). However, the abundance of research can make it challenging for researchers and practitioners to grasp the overall landscape, understand the state-of-the-art, and identify emerging trends in the field.

This study aims to provide comprehensive insights into the existing body of knowledge on the knapsack problem, synthesizing the findings of relevant studies and identifying key research trends, challenges, and opportunities according to themes of this study. The themes of this study are shown in Table 1.

Table 1. Themes of this Study

No.	Themes of this Study
1	Purpose of the Study
2	Data Type
3	Variant Knapsack Problem

2. Literature Review

The knapsack problem is a classic optimisation problem in which a limited capacity container must be filled with items of different value and weight. The goal is to maximise the total worth of the products in the container while maintaining the capacity constraint. The knapsack problem has been extensively researched in the literature, and a variety of solution strategies have been developed. However, the knapsack problem is NP-hard (Martello & Toth, 1990), which means that there is no known polynomial-time solution for solving it in the worst case. As a result, most knapsack problem solutions are heuristic in nature and may not necessarily offer optimal answers.

Different types of Knapsack Problems occur depending on the distribution of items and knapsacks: In the 0-1 Knapsack Problem each item may be chosen at most once, while in the Bounded Knapsack Problem we have a bounded amount of each item type. The Multiple-choice Knapsack Problem occurs when the items should be chosen from disjoint classes. The most general form is the Multi-constrained Knapsack Problem, which is basically a general Integer Programming (IP) Problem with non-negative coefficients (Pisinger & Toth, 1998). The solution for the knapsack optimization problems and its types described above can be found with several mathematic methods or algorithms. Martello and Toth (1990) and Kellerer et al. (2004) summarise studies on KP and its many variants.

In contemporary research, Systematic Literature Review (SLR) has become a pivotal methodology embraced across diverse disciplines. This structured approach offers a rigorous way to consolidate existing knowledge, standing as a distinct type of literature review. The primary objectives of conducting systematic reviews include summarizing existing evidence, pinpointing research gaps, and establishing a comprehensive framework for contextualizing new research endeavours. As emphasized by Kitchenham (2004), one of their major advantages lies in their ability to offer insights into the effects of various phenomena across a broad spectrum of settings and empirical methods.

3. Methodology

This study used the Systematic Literature Review (SLR) method, which aims to collect, identify, analyse, and synthesize various studies on the knapsack problem. A systematic literature review is a means of identifying, evaluating and interpreting all available research relevant to a particular research question, or topic area, or phenomenon of interest (Kitchenham, 2004). The PRISMA Statement consists a four-phase flow diagram which are identification, screening, eligibility, and inclusion (Moher, 2009). This study will apply this stage in order to improve the reporting of systematic reviews and meta-analyses. The following Figure 1 shows the methodical approach to answering this research question. Furthermore, based on the results of the data search as illustrated in the chart above, 15 articles were further analysed. The identity of the articles is summarized in Table 2.

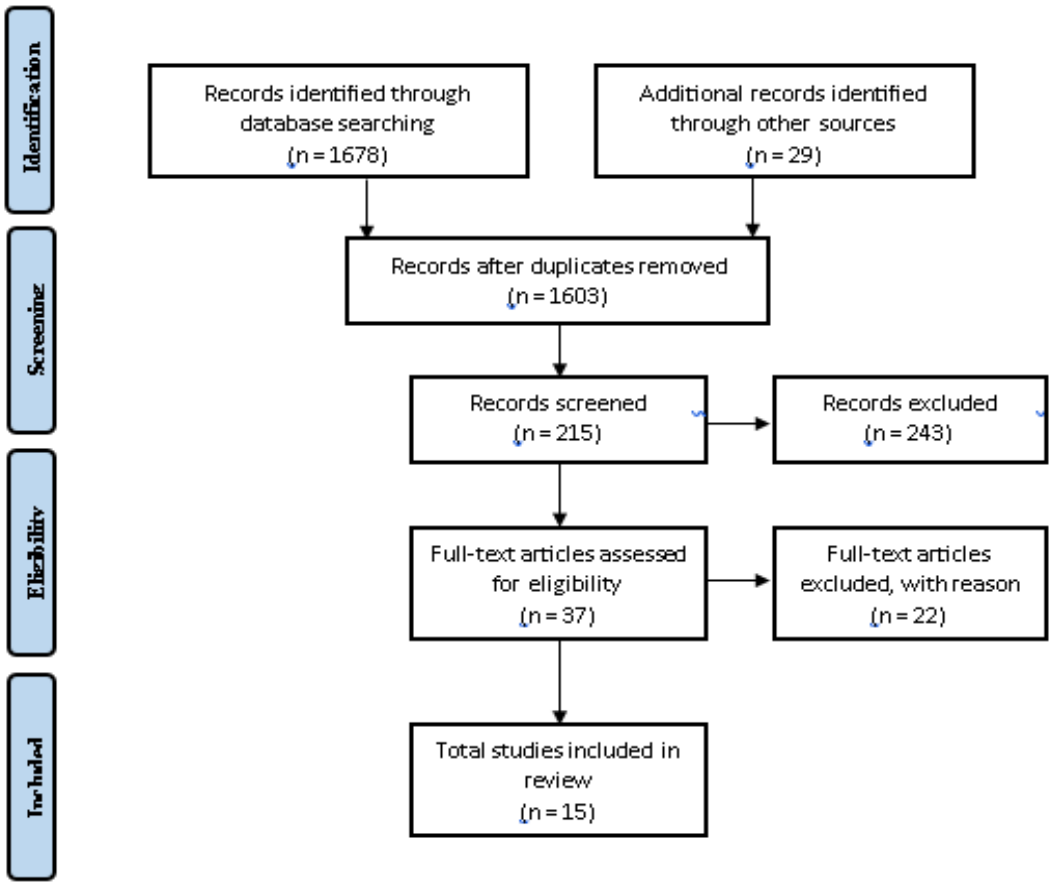

Figure 1. Research Data Search & Selection Flow

Table 2: Identity of the articles to be analysed further.

No	Article Title	Year	Author's Name	Journal Name
1	An improved Fruit Fly Optimization Algorithm for Solving the Multidimensional Knapsack Problem	2016	Tao Meng, Quan-Ke Pan	Applied Soft Computing
2	Integer Knapsack Problems with Set-up Weights	2007	Laura A. McLay · Sheldon H. Jacobson	Computational Optimization and Applications
3	Application of the Greedy Algorithm in Multiple Constrain Knapsack Optimization Problems in Goods Transportation	2015	Gradina Nur Fauziah	Syntax Admiration
4	Large-Scale Multi-Period Precedence Constrained Knapsack Problem: A Mining Application	2010	Eduardo Moreno a,2 Daniel Espinoza b,3 Marcos Goycoolea c,4	Proceedings of the Seminar on Applicable Mathematics
5	Multidimensional Knapsack Problem Optimization using a Binary Particle Swarm	2017	Luis Fernando Mingo López Nuria Gómez Blas • Alberto Arteta	Soft computing

	Model with Genetic Operations		Albert	
6	Basic Concepts on Integer Optimization	2005	GYAN BAHADUR THAPA	Seminar on Applicable Mathematics
7	Heuristics for Two-Dimensional Knapsack and Cutting Stock Problems with Items of Irregular Shape	2012	Aline M. Del Valle , Thiago Alves de Queiroz, , Fl´avio K. Miyazawa, , Eduardo C. Xavier	Expert Systems with Applications
8	An Approximation Algorithm for Solving Unconstrained Two-Dimensional Knapsack Problems	1992	Didier Fayard, Vassilis Zissimopoulos	European Journal of Operational Research
9	Combinatorial Approaches for the Trunk Packing Problem	2006	Joachim Reichel	PHD Thesis, SaarlandUniversity
10	Using Knapsack Problem Model to Design a Resource Aware Test Architecture for Adaptable and Distributed Systems	2017	Mariam, Lahami, Moez Krichen, Mariam Bouchakwa, Mohamed Jmaiel	International Conference
11	Hybrid Cat-Particle Swarm Optimization Algorithm on Bounded Knapsack Problem with Multiple Constraints	2021	Kiswara Agung Santoso, Muhammad Bagus Kurniawan, Ahmad Kamsyakawuni, Abduh Riski	International Conference on Mathematics, Geometry, Statistics, and Computation
12	Solving Knapsack Problems in a Sticker Based Model	2002	P´erez–Jim´enez, M.J. and Sancho–Caparrini, F.	DNA Computing: 7th International Workshop
13	Algorithms for the Bounded Set-up Knapsack Problem	2007	LA McLay, SH Jacobson	Discrete Optimization
14	Linear Time Algorithms for Knapsack Problems with Bounded Weights	1999	D PISINGER	Journal of Algorithms
15	Optimization Method of Knapsack Problem Based on BPSO-SA in Logistics Distribution	2022	Zhang Y, Wu T, Ding X	Journal of Information Processing Systems

Table 2 provides an inclusive overview of the time span covered by the 15 selected articles in our study. These articles were published between 1992 and 2022, representing a broad range of years that captures the historical context and progression of research in this field. This extended timeline emphasizes not only the longevity but also the continued relevance of the topics under examination. Moreover, the articles featured in Table 2 include traditional journal articles, as well as materials from conferences, workshops, theses, and proceedings. This diversity in publication types highlights the interdisciplinary and versatile nature of

research within our field, with each type of source offering its unique insights and depth to the collective body of knowledge.

Search for data sources in this research was accessed through various databases such as Google Scholar, Scopus, Research gate, and Science Direct. There are several keywords used in data search such “knapsack problem”, “maximize”, “optimization”, “packing problem”. In addition, this study also used several themes in analysing data. These themes were chosen using the SLR model equation, which stands for Preferred Reporting Items for Systematic Reviews and Meta-analysis (PRISMA). The articles chosen are articles published by reputable scientific journals. Table 3 shows the data collected from the articles included in this study.

Table 3: Analysis of selected articles using themes

No	Article Title	Purpose of the Studies	Data Type	Variant Knapsack Problem
1	An Improved Fruit Fly Optimization Algorithm for Solving the Multidimensional Knapsack Problem	To propose another variant of Fruit fly optimization, namely improved fruit fly optimization algorithm, to solve multidimensional knapsack problem	Fruit fly	Multi dimensional knapsack problem
2	Integer Knapsack Problems with Set-Up Weights	This paper provides dynamic programming algorithms for each problem that produce optimal solutions in pseudo-polynomial time. Moreover, four heuristics are presented that provide approximate solutions	Items	Integer Knapsack problem
3	Application of the Greedy Algorithm in Multiple Constrain Knapsack Optimization Problems in Goods Transportation	To provide insights into the effectiveness and limitations of the greedy algorithm in solving practical optimization problems in the context of goods transportation	Items	Multiple Constrain Knapsack problem
4	Large-scale multi-period precedence constrained knapsack problem: A mining application	To compare the performance of the Critical Multiplier algorithm with CPLEX LP algorithms	Mining Instances	Precedence Constrained Knapsack Problem
5	Multidimensional Knapsack Problem Optimization Using a Binary Particle Swarm Model with Genetic Operations	This paper proposes a hybrid, genetic inspired algorithm that uses random mutation/ crossover operations and adds penalty functions to solve a particular case: the multidimensional knapsack problem	Binary Particle Swarm	Multi dimensional Knapsack Problem

6	Basic Concepts on Integer Optimization	To provide an introduction to the theoretical foundations and practical applications of integer optimization	Plant, Person, Milk, Raw Material	0-1 Knapsack Problem
7	Heuristics for Two-Dimensional Knapsack and Cutting Stock Problems with Items of Irregular Shape	To presented heuristics to2D cutting/packing problems where items have irregular shape and heuristics for the 0/1Knapsack, Unconstrained Knapsack and Cutting Stock problems	Irregular items	Two-Dimensional Knapsack Problem
8	An Approximation Algorithm for Solving Unconstrained Two-Dimensional Knapsack Problems	To propose a new efficient heuristic for solving the Two-dimensional Knapsack Problem	Instances	Two-Dimensional Knapsack Problem
9	Combinatorial Approaches for the Trunk Packing Problem	To present combinatorial approach to a three-dimensional real-world packing problem which to pack a maximum number of rigid boxes with side length ratios 4: 2:1 into the trunk car	Boxes	Knapsack-like Problem
10	Using Knapsack Problem Model to Design a Resource Aware Test Architecture for Adaptable and Distributed Systems	To propose a resource aware test architecture design phase before executing runtime tests while the system evolves dynamically	Resource (CPU load, Memory, Battery level)	Multiple Multi-dimensional Knapsack Problem
11	Hybrid Cat-Particle Swarm Optimization Algorithm on Bounded Knapsack Problem with Multiple Constraints	To propose a hybrid Cat-Particle Swarm Optimization algorithm for solving the bounded knapsack problem with multiple constraints	Cat-Particle Swarm	Bounded Knapsack Problem with Multiple Constraints
12	Solving Knapsack Problems in a Sticker Based Model	To give molecular solutions for two NP–complete problems, namely Subset-sum and Knapsack in a Sticker based model for DNA computations	Tubes (Memory Complexes),	Bounded Knapsack Problem
13	Algorithms for the Bounded Set-Up Knapsack Problem	To provides three dynamic programming algorithms that solve Bounded Set-up	Instance	Bounded Knapsack Problems

		Knapsack Problem in pseudo-polynomial time and a fully polynomial-time approximation scheme		
14	Linear Time Algorithms for Knapsack Problems with Bounded Weights	To presented a new technique for deriving tight pseudo polynomial time bounds for Knapsack Problems	Instance	0-1 Knapsack Problem
15	Optimization Method of Knapsack Problem Based on BPSO-SA in Logistics Distribution	To proposes an algorithm to overcome the drawback of basic Binary Particle Swarm Optimization and Simulated Annealing algorithms to solve the knapsack problem in logistics distribution	Binary Particle Swarm	0-1 Knapsack Problem

4. Result and Discussion

In connection with this SLR, the data analysis used in this study is content analysis. There are three themes are chosen in analysing data with this technique which are purpose analysis, data type population analysis, and variant knapsack problem analysis.

4.1. Purpose of the Studies

Table 4, which is presented below, serves as a valuable reference point, systematically outlining the purpose of studies found within the 15 articles under consideration. Some of these studies share common objectives, shedding light on recurring themes and areas of focus within the broader research landscape.

Table 4: Purpose of the studies

Improve performance algorithm	Propose new algorithm/ approach	To provide insights effectiveness and limitations	To compare performance algorithm	To provide theoretical and applications	To evaluate a new approach
n=1	n=10	n=1	n=1	n=1	n=1

It shows that all the selected papers share a common goal which solving the knapsack problem. As a result, we can conclude that the purpose of these articles is similar, which is to address the knapsack problem. While some papers may have specific focuses related to the knapsack problem, the majority of them concentrate on proposing new algorithms. This suggests that developing a new algorithm is a promising approach for solving the knapsack problem effectively. The fact that multiple articles are centred around introducing novel algorithms indicates that there is ongoing research and interest in finding more efficient solutions for this problem. Therefore, it can be inferred that a new algorithm has the potential to effectively solve the knapsack problem.

4.2. Data Type

Data types are fundamental components in the world of problem-solving and data analysis, serving as the foundational building blocks that define how information is structured and processed within a given context. These data types are not uniform; they vary widely based on the unique requirements and objectives of each study or problem at hand.

Table 5: Data Type in The Studies

Fruit fly	Mining Instances	Particle Swarm	Plant, Person, Milk, Raw Material	Tubes (Memory Complexes)	Resource (CPU load, Memory, Battery level)	Boxes	No Specific data
n=1	n=1	n=3	n=1	n=1	n=1	n=1	n=6

Table 5 above displays the various data types used in the selected studies, each chosen according to the study's purpose. Among the 15 articles, one study employed a diverse set of data types, including fruit fly, mining instances, raw material, memory complexes, resource, and boxes. Additionally, three articles utilized particle swarm data types, specifically binary and cat. However, for the remaining studies, no specific data types were mentioned. This data diversity underscores the versatility of applying the Knapsack Problem to different situations and contexts. Based on the information presented, we can deduce that the Knapsack Problem is amenable to a wide variety of data types.

To further enhance our understanding and practical applications, future research should focus on exploring specific data types and their impact on the problem's solution. By delving into particular data scenarios, researchers can gain deeper insights into the Knapsack Problem's behaviour and performance, ultimately contributing to more tailored and effective solutions for real-world applications. This targeted investigation of specific data types holds the potential to unlock new perspectives and advance the problem-solving capabilities in diverse domains.

4.3. Variant Knapsack Problem

The Knapsack Problem has evolved to encompass a diverse range of variants tailored to specific real-world scenarios. These variants represent nuanced adaptations of the core problem, each addressing unique constraints and objectives.

Table 6: Variant Knapsack Problem in The Studies

Multi-dimensional / Multiple Constrain / Two-Dimensional Knapsack problem	0-1 Knapsack problem	Precedence Constrained Knapsack Problem	Bounded Knapsack problem	Integer Knapsack problem	Knapsack-like Problem	Hybrid Problem
n=5	n=3	n=1	n=2	n=1	n=1	n=2

Table 6 presents an overview of the different variants used in the knapsack problem. Among the selected research, various variants were explored. One study focused on the precedence constrained knapsack problem, integer knapsack problem, and knapsack-like problem.

Additionally, two studies examined the bounded knapsack problem and the hybrid problem, while another study delved into the multidimensional knapsack problem. It is worth noting that the hybrid problem appears to be an area that requires further investigation to develop new variants.

In conclusion, the analysis of Table 6 reveals that studying different variants of the knapsack problem can lead to the discovery of new and interesting variations. The inclusion of various variants, such as the precedence constrained knapsack problem and the hybrid problem, showcases the diverse nature of research in this domain. Focusing on the hybrid problem, in particular, offers an opportunity to explore novel combinations and potential solutions. By investing more research effort in studying the different variants, scholars can broaden their understanding of the knapsack problem and potentially identify new avenues for advancements and practical applications.

5. Conclusion

This study is a systematic literature review (SLR) focusing on the Knapsack problem, analysing 15 articles published in reputable international journals between 1992 and 2022. The articles were selected based on predetermined criteria, allowing for a clear understanding of the current trends and insights related to the knapsack problem. Through the analysis, three main themes emerged: the purpose of the studies, the data types used, and the variants of the knapsack problem explored. The research on the knapsack problem has proven to be versatile, generating various variations of the problem and showcasing its applicability across a wide range of data scenarios.

However, one limitation of this review is the relatively small number of articles analysed. The strict criteria used to capture specific variations of the knapsack problem may have limited the scope of the review. To achieve a more comprehensive understanding, future literature reviews should consider a broader range of articles to provide a more encompassing analysis of the topic. Expanding the scope both in terms of quantity and inclusiveness of literature review articles would allow for a more robust and comprehensive exploration of the knapsack problem and its evolving research landscape.

6. Acknowledgement

The authors would like to thank the College of Computing, Informatics and Mathematics, Universiti Teknologi MARA for supporting this project.

7. Funding

The author(s) received no specific funding for this work.

8. Conflict of Interest

The authors have no conflicts of interest to declare.

9. References

Del Valle, A. M., De Queiroz, T. A., Miyazawa, F. K., & Xavier, E. G. (2012c). Heuristics for two-dimensional knapsack and cutting stock problems with items of irregular shape. Expert Systems with Applications, 39(16),12589–12598. https://doi.org/10.1016/j.eswa.2012.05.025

Fauziah, G. N. (2022). Application of the Greedy Algorithm in Multiple Constrain Knapsack Optimization Problems in Goods Transportation. *Journal Syntax Admiration*, 3(5), 725-732. https://doi.org/10.46799/jsa.v3i5.425

Fayard, D., & Zissimopoulos, V. (1995). An approximation algorithm for solving unconstrained two-dimensional knapsack problems. *European Journal of Operational Research*, 84(3), 618–632. https://doi.org/10.1016/0377-2217(93)e0221-i

Kellerer, H., Pferschy, U., & Pardalos, P. M. (2004). Knapsack Problems. In *Springer eBooks*. https://doi.org/10.1007/978-3-540-24777-7

Kitchenham, B. (2004). *Procedures for performing systematic reviews*. Keele, UK, Keele University, 33(2004), 1-26.

Khan, M., Blas, N. G., & Albert, A. A. (2017c). Multidimensional knapsack problem optimization using a binary particle swarm model with genetic operations. *Soft Computing*, 22(8), 2567–2582. https://doi.org/10.1007/s00500-017-2511-0

Lahami, M., Krichen, M., Bouchakwa, M., & Jmaiel, M. (2012). Using Knapsack Problem Model to design a resource aware test architecture for adaptable and distributed systems. *In Springer eBooks* (pp. 103–118). https://doi.org/10.1007/978-3-642-34691-0_9

McLay, L. A., & Jacobson, S. H. (2007). Algorithms for the bounded set-up knapsack problem. *Discrete Optimization*, 4(2), 206–212. https://doi.org/10.1016/j.disopt.2006.11.002

McLay, L. A., & Jacobson, S. H. (2007). Integer knapsack problems with set-up weights. *Computational Optimization and Applications*, 37(1), 35–47. https://doi.org/10.1007/s10589-007-9020-5

Meng, T., & Pan, Q. (2017c). An improved fruit fly optimization algorithm for solving the multidimensional knapsack problem. *Applied Soft Computing*, 50, 79–93. https://doi.org/10.1016/j.asoc.2016.11.023

Moher, D. (2009). Preferred reporting items for Systematic Reviews and Meta-Analyses: *the PRISMA statement. Annals of Internal Medicine*, 151(4), 264. https://doi.org/10.7326/0003-4819-151-4-200908180-00135

Moreno, E., Espinoza, D., & Goycoolea, M. V. (2010b). Large-scale multi-period precedence constrained knapsack problem: A mining application. *Electronic Notes in Discrete Mathematics*, 36, 407–414. https://doi.org/10.1016/j.endm.2010.05.052

Pardalos, P. M. (1999). Linear Time Algorithms for Knapsack Problems with Bounded Weights. *Journal of Algorithms*, 33(1), 1–14. https://doi.org/10.1006/jagm.1999.1034

Pérez-Jiménez, M. J., & Sancho-Caparrini, F. (2002). Solving knapsack problems in a sticker-based model. In Springer eBooks (pp. 161–171). https://doi.org/10.1007/3-540-48017-x_15

Pisinger, D., & Toth, P. (1998). Knapsack problems. *Handbook of Combinatorial Optimization*: Volume1–3, 299-428. doi:10.1007/978-1-4613-0303-9_5

Reichel, J. (2006). Combinatorial approaches for the trunk packing problem. https://doi.org/10.22028/d291-25883

Santoso, K. A., Kurniawan, M. R., Kamsyakawuni, A., & Riski, A. (2022). Hybrid Cat-Particle Swarm Optimization Algorithm on Bounded Knapsack Problem with Multiple Constraints. https://doi.org/10.2991/acsr.k.220202.045

S. Martello, P. Toth, Knapsack Problems: *Algorithms and Computer Implementations*, Wiley & Sons, New York, NY, 1990

Thapa, G. B. (2005, September). Basic concepts on integer optimization. In *Proceedings of the Seminar on Applicable Mathematics* (pp. 20-21).

Yan Zhang, Tengyu Wu, & Xiaoyue Ding (2022). Optimization Method of Knapsack Problem Based on BPSO-SA in Logistics Distribution. *Journal of Information Processing Systems*, 18(5), 665-676. DOI: 10.3745/JIPS.01.0090