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COMPREHENSIVE OVERVIEW OF THE USE OF MATHEMATICAL METHODS IN MODELING OF COVID-19

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ABSTRACT

The mathematical modeling of COVID-19 has been widely used in analyzing the dynamic model of an epidemic of COVID-19. It is beneficial to develop the epidemic model using mathematical language since it is the best way to describe the real-life phenomenon and make an accurate prediction compared to the regular theoretical method. The COVID-19 explanation is also considered in the analysis including the mathematical methods of Fractional Order Differential Equation (FODE) and Ordinary Differential Equation (ODE) to undergo further analysis of the model with different mathematical methods. A qualitative research approach is being used in this study to analyze the articles related to this research topic in the form of a systematic literature review (SLR). The PRISMA model is applied. The retrieval of 16 papers to construct the SLR is considered. Hence, the analysis of the article is conducted based on the main themes of the SLR: purpose of studies, type of article, research area, and findings.

Keywords*: COVID-19, FODE, Mathematical modeling, ODE, PRISMA, Systematic literature review.*

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1. Introduction

A comprehensive overview is a broad knowledge or explanation of a topic that covers most of the relevant facts and this study uses a Systematic Literature Review (SLR) as the comprehensive overview of the articles. Examining data and findings of other authors relative to specific research questions is the main purpose of SLR. Therefore, this study is conducted to accomplish two objectives which are to analyze the systematic literature review table of the proposed study and to provide a comprehensive summary and synthesis of the existing knowledge from the articles. This study involves the development of an SLR table that consists of the COVID-19, Fractional Order Differential Equation (FODE) model, and Ordinary Differential Equation (ODE) model.

COVID-19 has become a threat to human life causing thousands of deaths related to SARS-CoV-2 (Ciotti et al., 2020). The transmission of COVID-19 as referred by Rajagopal et al. (2020) would be very helpful with the application of mathematical models in real-world situations. Therefore, the mathematical model using Fractional Order Differential Equation (FODE) and Ordinary Differential Equation (ODE) are adopted in this study.

These two mathematical methods of the model are considered since many researchers have investigated the dynamic of epidemic models using ODE compared to FODE which is new in the field however it is more accurate. According to the articles used in SLR, the themes of studies are defined in Table 1. Furthermore, this study has retrieved 16 articles for conducting the systematic literature review.

Table 1. Themes of the studies

No.	Themes of Studies
1	Purpose of studies
2	Type of article
3	Research area
4	Findings

2. Literature Review

The COVID-19 outbreak is caused by the SARS-CoV-2 virus. COVID-19 replication cycles are longer than those seen with seasonal flu where it has an extra replication time of 10 days following maturation (Chatterjee & Ahmad, 2021). Mathematical models are applied in the study which seeks quantitative explanations of data that aim to define patterns of behavior of the underlying process of real-life situations. Precise predictions of the model from the underlying assumptions are enabled with the assistance of mathematical modeling which is required in enhancing the capacity to distinguish between models and hypotheses (Myung & Pitt, 2002). It is beneficial to solve the events related to the topic problems using mathematics (Bonin et al., 2017). Thus, the mathematical model in this study has considered the method of fractional order differential equation (FODE) and ordinary differential equation (ODE). According to Chatterjee and Ahmad (2021), a set of ordinary differential equations usually governs most models. However, fractional calculus also plays a crucial role in modeling various real-life situations that illustrate the memory impact of several models. It aids in reducing mistakes caused by overlooked factors in the traditional modeling of real-world issues.

Systematic Literature Review (SLR) differs from standard reviews in that it uses a repeatable scientific procedure to reduce the possibility of mistakes through a comprehensive search of previously published materials. The Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) criteria were utilized to select a sample of articles (Awan et al., 2022). Systematic reviews and meta-analyses are crucial methods for accurately and reliably summarizing information. However, meta-analyses are not included in all systematic reviews, although they are a sort of systematic review. Furthermore, it is helpful to use the PRISMA model since it contains further information about its origins and evolution. Besides, the development of the PRISMA model describes the systematic reviews and meta-analyses in a clear and comprehensive manner (Liberati et al., 2009).

3. Methodology

Developing a systematic literature review of several articles must undergo several processes as mentioned by Snyder (2019) where it is shown in Figure 1 below:

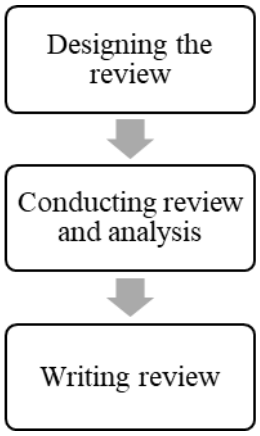

Figure 1. Flowchart of performing a systematic literature review

Phase 1: Designing the review

Designing the review is the first method to identify the best approach to conduct the literature review in a study. Since the goal of the review is to analyze and synthesize evidence, so systematic literature review is used, and a technique for locating relevant literature is created. The research topics considered are related to this study which are COVID-19, Fractional Order Differential Equation (FODE) model, and Ordinary Differential Equation (ODE) model.

Phase 2: Conducting the review and making analysis

Starting the process of conducting an actual review by selecting several samples of articles related to the research topics. Seeking articles through Google Scholar, Scopus, Research Gate, and others to obtain the articles for systematic literature review. Ensuring the quality and dependability of articles must regularly monitor data abstraction during the review process. Using the Preferred Reporting Items for Systematic Review and Meta-analysis (PRISMA) to make articles analysis and the flow is shown in Figure 2 (Hameed Ul Haq et al., 2019). Each step of identification, screening, eligibility, and included are exist in the analysis. Numerous articles have been found but only some of the reliable articles are taken. A total of 16 articles are used in this study. Thus, Table 2 shows the list of articles to be analyzed further in the systematic literature review.

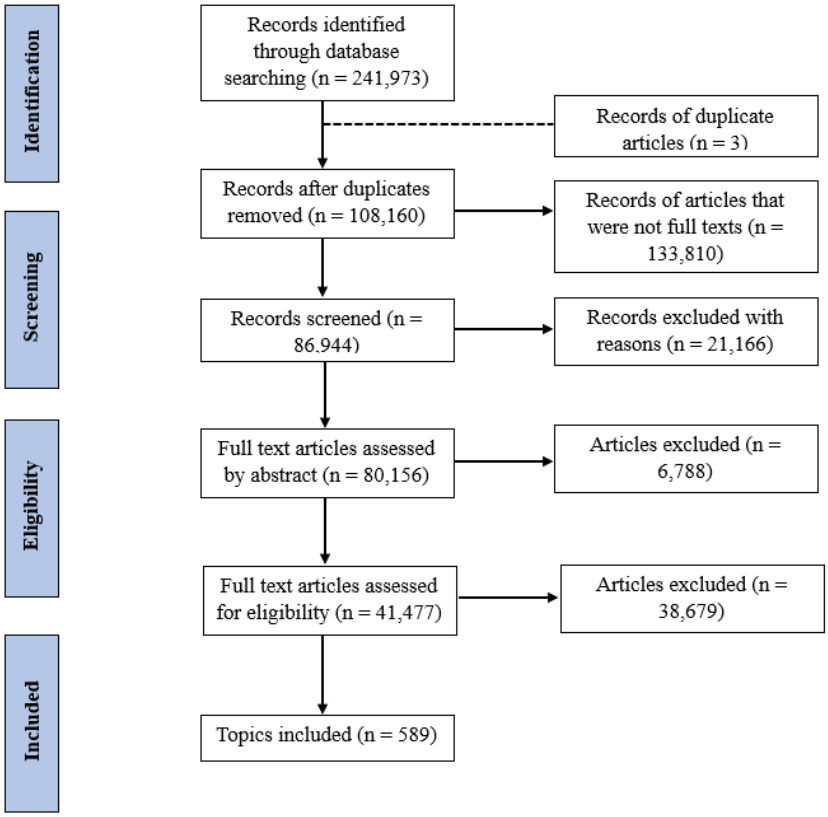

Figure 2. PRISMA model of literature analysis

Table 2. Identity of articles to be analyzed further.

No.	Article Title	Year	Author's Name	Journal Name
1.	COVID-19: Transmission, prevention, and potential therapeutic opportunities	2020	Lotfi, M., Hamblin, M. R., & Rezaei, N.	Clinica Chimica Acta
2.	Beyond the Pandemic: COVID-19 Pandemic Changed the Face of Life	2021	Khalifa et al.	International Journal of Environmental Research and Public Health
3.	COVID-19 Epidemic in Malaysia: Epidemic Progression, Challenges, and Response	2021	Hashim, J. H., Adman, M. A., Hashim, Z., Mohd Radi, M. F., & Kwan, S. C.	Frontiers in Public Health
4.	Integer versus fractional order SEIR deterministic and stochastic models of measles	2020	Islam, M. R., Peace, A., Medina, D., & Oraby, T.	International Journal of Environmental Research and Public Health
5.	Mathematical models in infectious disease epidemiology	2010	Kretzschmar, M., & Wallinga, J.	Modern infectious disease

				epidemiology: Concepts, methods, mathematical models, and public health
6.	Modeling COVID-19: Forecasting and analyzing the dynamics of the outbreaks in Hubei and Turkey	2022	Aslan, I. H., Demir, M., Wise, M. M., & Lenhart, S.	Mathematical Methods in the Applied Sciences
7.	Relevant mathematical modelling efforts for understanding COVID-19 dynamics: An educational challenge	2023	Meyer, J., & Lima, M.	ZDM-Mathematics Education
8.	Why ODE models for COVID-19 fail: Heterogeneity shapes epidemic dynamics	2021	Großmann, G., Backenköhler, M., & Wolf, V.	medRxiv
9.	Modeling with ODE	2005	Howard, P.	Texas A&M University
10.	Studying of COVID-19 fractional model: Stability analysis	2023	Khalaf, S. L., Kadhim, M. S., & Khudair, A. R.	Partial Differential Equations in Applied Mathematics
11.	Fractional order mathematical modeling of COVID-19 transmission	2020	Ahmad, S., Ullah, A., Al-Mdallal, Q. M., Khan, H., Shah, K., & Khan, A.	Chaos, Solitons & Fractals
12.	Caputo fractional-order SEIRP model for COVID-19 Pandemic	2022	Akindeinde, S. O., Okyere, E., Adewumi, A. O., Lebelo, R. S., Fabelurin, O. O., & Moore, S. E.	Alexandria Engineering Journal
13.	Basic Epidemic Model of Dengue Transmission Using The Fractional Order Differential Equations	2019	Hamdan, Nur 'Izzati and Kilicman, Adem	Malaysian Journal of Science
14.	A method for solving differential equations of fractional order	2012	Demirci, E., & Ozalp, N.	Journal of Computational and Applied Mathematics
15.	A fractional-order mathematical model for COVID-19 outbreak with the effect of symptomatic and asymptomatic transmission	2022	Ali, Z., Rabiei, F., Rashidi, M. M., & Khodadadi, T.	The European Physical Journal Plus

16.	A fractional-order model for the novel coronavirus (COVID-19) outbreak	2020	Rajagopal, K., Hasanzadeh, N., Parastesh, F., Hamarash, I. I., Jafari, S., & Hussain, I.	Nonlinear Dynamics

Phase 3: Writing the review

The various sorts of information and levels of depth are required to write the review. The systematic literature review of each reliable article is analyzed in Table 3 and consists of four themes of studies which are purpose, type of articles whether it is a discussion or analysis-oriented, the research area of study, and findings obtained from the articles. The summarization and synthesis of the systematic literature review are created to complete the analysis of this study. Hence, a comprehensive overview of all articles in SLR is produced.

Table 3. Systematic literature review of the articles.

No.	Article Title	Purpose of Studies	Type of Article	Research Area	Findings
1.	COVID-19: Transmission, prevention, and potential therapeutic opportunities	To propose solutions for controlling the pandemic	Discussion	COVID-19	• Direct and indirect contact with the SARS-CoV-2 led to COVID-19 where precautions against COVID-19 are required to prevent the spread of the virus. • Antiviral drugs that target the SARS-CoV-2 virus, anti-inflammatory drugs that can help to reduce the severity of the disease, and immunomodulatory drugs to boost the immune system are potential therapeutic strategies.

2.	Beyond the Pandemic: COVID-19 Pandemic Changed the Face of Life	To discuss the impact of the COVID-19 pandemic on different life aspects and re-evaluate the efforts in defeating the crisis.	Discussion	COVID-19	• COVID-19 leads to negative and positive impacts on society in the economic sector, lifestyle, education sector, health sector, and environment.
3.	COVID-19 Epidemic in Malaysia: Epidemic Progression, Challenges, and Response	To provide a comprehensive overview of the COVID-19 epidemic in Malaysia	Analysis-oriented	COVID-19	• Two waves of the COVID-19 epidemic in Malaysia occurred in March-May 2020 and November 2020-March 2021; low cases in the first wave but rise in the second wave. • Challenges in the prevention of COVID-19 such as shortage of protective equipment and the spread of misinformation lead to slow and inconsistent response to the epidemic. • The response implemented by the government including social distancing, mask-wearing, and quarantine slowed the spread of the virus but also affected the economy.
4.	Integer versus fractional order SEIR deterministic and stochastic models of measles	To study the comparison of the performance between systems of ordinary and (Caputo) fractional differential equations depicting the susceptible-exposed-infectious-recovered (SEIR) models of diseases.	Analysis-oriented	Measle infections	• A fractional stochastic of SEIR (susceptible-exposed-infected-recovered) measles model depicted using a continuous time Markov chain (CTMC). • Construction of Fractional Differential Equation (FDE), Ordinary Differential Equation (ODE) and α-Dependent ODE of measles model to conduct analysis of the models. • Comparison among models in numerical simulations shows that FDE performs slightly better than the ODE in Portsmouth data fittings where the FDE model is solved by using the subordination of the solutions of ODE with the help of Ordinary least squares (OLS) while ODE is solved using the Runge-Kutta method.
5.	Mathematical models in infectious disease epidemiolo	To describe the basic concepts of mathematical models in infectious disease epidemiology	Discussion	Infectious disease	• Introduce and explain in detail the basic concepts of mathematical modeling, reproduction number, final size, endemic steady state, and critical vaccination coverage.

	gy				• Information on the stochastic transmission models which are formulated in terms of integers with probabilities describing the transitions between states and using the Monte Carlo as simulations while network models are related to the duration and speed of spread of an epidemic through the population.
6.	Modeling COVID-19: Forecasting and analyzing the dynamics of the outbreaks in Hubei and Turkey	To analyze the dynamics of local outbreaks of COVID-19 via a coupled system of ordinary differential equations (ODEs) particularly highlighting the effect of testing and the effects of quarantine and social distancing in Hubei.	Analysis-oriented	COVID-19	• A deterministic ODE of susceptible S(t), susceptible in quarantine (isolated class S_q((t), exposed E(t), infected (asymptomatic or having mild symptoms) I(t), infected cases (hospitalized if get severe symptoms or quarantined if get mild symptoms) I_q(t) and recovered R(t) model is used. • Conduct the stability analysis using the next-generation matrix (NGM) to obtain basic reproduction number and Ordinary Least Squares (OLS) method to make predictions on the model. • The simulations show that any change in the quarantine rate significantly changes the total number of cases and deaths. • Partial rank correlation coefficient (PRCC) and p-value with the range of parameters used have proven that quarantine greatly reduced the number of cases and deaths seen in Hubei's COVID-19 outbreak.
7.	Relevant mathematical modelling efforts for understanding COVID-19 dynamics: An educational challenge	To present modeling techniques that students can use to learn to adapt models to slight changes and prepare for the creation of new models. To evaluate modelling choices of state variables, parameters and mathematical tools. To recognize modelling	Analysis-oriented	COVID-19	• Comparison of ODE models; SIR model (susceptible, infected, and removed), SIRV model includes vaccinated individuals(V), SCIR model where C is a confined person and SCIRV includes both vaccinated and confined persons. • The comparison provides the critical evaluation of the models by simulating the social scenarios and the importance of pedagogical values in mathematical modeling whereby a third pedagogical value justified the use of modeling hence, giving a better data interpretation.

		limitations and critically understand how to use mathematical models.			
8.	Why ODE models for COVID-19 fail: Heterogeneity shapes epidemic dynamics	To identify the consequences of a population's heterogeneity regarding connectivity and individual viral load levels by using stochastic simulations.	Analysis-oriented	COVID-19	• ODE model of susceptible (S), exposed (E) (infected but not yet infectious), removed (R) (immune or dead), as well as mild, severe, and critical infection stages (I_1, I_2, I_3) is converted into a stochastic multi-agent system. • The methods of a complete graph, Watts–Strogatz (WS) random networks, power-law Configuration Model networks, and synthetically generated Household networks are applied. • Based on point estimators of population averages, models are not adequate for analyzing or predicting the dynamics of an epidemic and the widely used class of ODE models cannot accurately capture population heterogeneity. • Cautious must always be taken in models' interpretation since models are prone to hidden assumptions.
9.	Modeling with ODE	To identify the three critical aspects in the theory of ordinary differential equations: Developing models of physical phenomena Determining whether the models are mathematically "well-posed" Solving ODE numerically with MATLAB.	Discussion and analysis-oriented	Physical phenomena	• Modeling with ODE required some outside conditions to solve ODE uniquely using Taylor approximation and it also includes the compartment analysis in a variety of areas such as chemical reactions, rates of reaction, the population dynamics of ODE, Newtonian mechanics, Hamiltonian mechanics, and variational methods. • The theory involves in modeling is the well-posedness theory which considers the existence, uniqueness, and stability of the model. • Solving ODE in MATLAB: solve ODE with the number of tools (the built-in functions ode23 and ode45) which implement versions of Runge–Kutta 2nd/3rd-order and

					Runge–Kutta 4th/5th-order, solve ODE system with MATLAB M-file containing the Lorenz equations and solve boundary value problem by using MATLAB's built-in solver bvp4c. In addition, the Laplace transform is the most useful mathematics tool. • Euler's method is used as a numerical method to solve the ODE process. Initiatives have been taken by advance the ODE solvers using the multistep solvers (ode113, ode15s, ode23s, ode23t, ode23tb) if encountered error in solving the ODE. • Fundamental theorems available; Implicit Function Theorem, Mean Value Theorem, Extreme Value Theorem, The Ratio Test, Cauchy's Convergence Condition, Cauchy's Convergence Condition for required functions, and Taylor expansion with remainder. However, the Implicit Function Theorem is the most useful theorem.
10.	Studying of COVID-19 fractional model: Stability analysis	To investigate the five-dimensional fractional order Covid-19 mathematical model.	Analysis-oriented	COVID-19	• Propose the Caputo fractional mathematical model of SEIQR: susceptible S, exposed E, hospitalized infected I, quarantined Q, and recovered or removed R. • The existence, uniqueness, and positively invariant solutions of the model are proven. • The next-generation matrix is used to calculate R_0. Thus, the model is locally asymptotically stable for both free-disease and endemic equilibrium points. • Analyze the global stability using Ulam-Hyers stability where the Covid-19 fractional model is Hyers–Ulam stability on [0, T]. • Fractional Euler's method (FEM) is applied to identify the numerical solutions of the model which then shows numerical simulations are consistent with the stability analysis.

11.	Fractional order mathematical modeling of COVID-19 transmission	To observe the fractional-order mathematical modeling of Covid-19 transmission	Analysis-oriented	COVID-19	• Using the Caputo fractional derivatives model of order γ where the total population N is divided into eight epidemiological classes: S susceptible class, E exposed class, I symptomatic and infectious class, P super-spreaders class, A infectious but asymptomatic class, H hospitalized, R recovery class, and F fatality class. • The existence and uniqueness of the model are proven through fixed point theory using nonlinear analysis. • The equilibrium points are locally asymptotically stable with the assistance of the Next Generation Matrix (NGM). • Applied Fractional Euler's method to obtain the general numerical solutions for each model compartment. • The fractional differential operator provides the global dynamics of the model. Furthermore, stability occurs rapidly on smaller fractional orders by comparing the simulated data in different fractional orders of the model with real data
12.	Caputo fractional-order SEIRP model for COVID-19 Pandemic	To propose a Caputo-based fractional compartmental model for the dynamics of the novel COVID-19 pandemic	Analysis-oriented	COVID-19	• Integer order model of the interaction between human populations N(t) and pathogens P(t) which is SEIR-P model; susceptible population S(t), the exposed E(t), the asymptomatic infected $I_A(t)$, the symptomatic infected $I_S(t)$ and the recovered population R(t) interacts with pathogens P(t). • A unique solution exists for the model and the state variables remain bounded and non-negative for all times $t > 0$. • The multistage technique transformed the model into the equivalent polynomial system (SEIRPUW). • Caputo fractional derivative is proposed to the SEIRP integer-order model since it models disease epidemics more realistically and captures the memory effect often associated with the human body's response to diseases.

					• The solution of the fractional model is bounded and remains positive as long as a positive initial condition is given. Hence, the existence of the solution is proven on [0,b] which is uniformly Lyapunov stable. • Banach fixed point theorem is used in established the well-posedness of the fractional model where P has a unique fixed point on [0,b]. • Obtaining R_0 through next-generation matrix which then constructs the Jacobian matrix that leads to locally asymptotically stable disease-free equilibrium point for all E [0,1). • Ulam-Hyers stability obtained the global stability of model while Adams-type predictor-corrector iterative scheme can solve fractional order differential equations numerically.
13.	Basic Epidemic Model Of Dengue Transmission Using The Fractional Order Differential Equations	To study a basic fractional order epidemic model of dengue transmission is conducted using the SIR-SI model, including the aquatic phase of the vector	Analysis-oriented	Dengue transmission	• Basic fractional order model formulation of human population (susceptible H_s, infectious H_i, and recovered H_r individuals) and mosquito one serotype of dengue viruses; the dynamics of female Aedes mosquito includes aquatic phase, A_m and the adult stage which are susceptible M_s and infectious M_i. • Obtained R_0 using the next generation matrix in the stability analysis where the disease-free equilibrium of the system is locally asymptotically stable if the corresponding $R_0<1$. • The fractional order model solution approaches the fixed point for a longer period even though both integer order and fractional order system equilibrium points are equal. • The period of the disease elimination is significantly affected by the memory of the mosquito and human population in the form of the ordinary differential equation. • Fractional-order derivative gives a more realistic way to model vector-borne disease dynamics as it possesses memory. Therefore, the smaller value of α, gives a better

					approximation in reducing the intensity of the dengue transmission
14.	A method for solving differential equations of fractional order	To determine the solution of the nonlinear Caputo fractional differential equations of order $0 < \alpha < 1$ with the initial condition of $x(0) = x_0$ by using a specialized technique.	Analysis-oriented	Real-life phenomena	• Nonlinear Caputo fractional differential equations of SEIR epidemic model of a population (N); susceptible (S), exposed (E), infectious (I), and recovered (R). • Analytic and numerical methods are used to evaluate the exact and numerical solutions respectively using the integer order differential equation technique. • The numerical solution of the system is evaluated based on the fractional order SEIR epidemic model using the generalized Adams–Bashford–Moulton algorithm and 4th-order Runge–Kutta method. Hence, 4th-order Runge–Kutta method is more effective than the Adams–Bashford–Moulton algorithm. • The comparison shows that the method of integer order differential equations of FDEs are more accurate and higher in the rate of convergence
15.	A fractional-order mathematical model for COVID-19 outbreak with the effect of symptomatic and asymptomatic transmission	To investigate the effect of symptomatic and asymptomatic transmissions on the coronavirus (COVID-19) outbreak using a fractional-order mathematical model	Analysis-oriented	COVID-19	• COVID-19 Fractional order model of SEI_1I_2R model; susceptible S, exposed E, asymptomatic infected I_1, symptomatic infected I_2, and recovered or removed R. • The Schaefer-and Banach-type fixed point theorems has proven the existence theory and uniqueness of the model. • Established local stability analysis through the Routh–Hurwitz criteria; the disease-free equilibrium is stable for $R_0 < 1$, whereas the endemic equilibrium becomes stable for $R_0 > 1$ and unstable otherwise. • The numerical analysis (recruitment rate and symptomatic transmission rate) impacts the parameters of reproduction number R_0 using the

					fractional Euler method. Besides, the COVID-19 epidemic can be significantly reduced by controlling people from migrating and strictly enforcing personal measures simultaneously.
16.	A fractional-order model for the novel coronavirus (COVID-19) outbreak	To predict and analyze the spread of the epidemic outbreak of the novel coronavirus in Italy	Analysis-oriented	COVID-19	• Caputo-type fractional derivative model of susceptible individuals, asymptomatic infected, symptomatic infected, recovered, and deceased (SEIRD). • The generalized mean value theorem is used in proving the existence and uniqueness of the solution. • The fractional-order model has less root-mean-square error than the classical one. • A test data set is used to evaluate the ability of integer- and fractional-order models in prediction. The fractional model provides a closer forecast to the real data which provides a better prediction.

4. Results and Discussion

4.1 Purpose of the studies

Every study conducted comes with a purpose or objectives to accomplish in the research area. The same goes for this study aiming to analyze the systematic literature review table of the proposed study and to provide a comprehensive summary and synthesis of the existing knowledge from the articles. Therefore, based on all 16 articles in the systematic literature review table, a summarization of the purpose of the studies is obtained which is the first one is to provide knowledge on infectious diseases, especially COVID-19, and precautions taken. Secondly is to recognize the different types of mathematical models used. Then, to conduct an analysis of the mathematical model of the epidemic and lastly, to make predictions based on the analyzed model. The first purpose is gained by referring to Articles 1, 2, and 3. The other purposes are based on the balance of 13 articles in the SLR.

4.2 Type of article

Table 4. Type of articles in the studies

Discussion	Analysis-oriented
n = 4	n = 13

n = Total number of articles

The discussion structure of the study only relates to the knowledge of the specific topic of the study which is not optimizing the analysis-oriented process that developing solutions critically to a specific problem. Table 4 represents two different categories of articles which are discussion-type and analysis-oriented type. The discussion consists of a total of 4 articles and analysis-oriented has a total of 13 articles has led to the subtotal of 17 articles although

the SLR contains 16 articles only. Therefore, it shows that one of the articles contains both discussion and analysis-oriented types which is Article 9. Furthermore, the analysis-oriented type of article is mostly used in the research area since most of the study conduct the thorough investigation for the research topic.

4.3 Research area

Table 5: Research area of the studies

COVID-19	Measles infections	General infectious disease	Physical and real-life phenomena	Dengue transmission
n = 11	n = 1	n = 1	n = 2	n = 1

n = Number of articles

The research area of the study is the body of information on which the investigation is focused. Referring to Table 5 there are five research areas adopted from all 16 articles. The research areas are COVID-19, measles infections of individuals, unmentioned general infectious diseases, physical and real-life phenomena, and dengue transmission. All these research areas are also related to the mathematical model. COVID-19 has the highest number of articles which is 11 articles since COVID-19 is the focus of this study compared to other research areas that only contain a small value number of articles. The value of measles infections, infectious disease, and dengue transmission is similar which is only 1. However, the physical and real-life phenomena have risen in 1 value from those three research areas of measles, infectious disease, and dengue transmission which has a total of 2 articles.

4.4 Findings

Findings are obtained from each article as in the systematic literature review of Table 3. Besides, a summarization has been made for all the articles. COVID-19 leads to a great impact on societies around the world including positive and negative impacts on the economic sector, education sector, and health sector. Thus, alternatives have been taken in the precautions against COVID-19 by social distancing, wearing medical masks, washing hands with soap, and sanitizing. According to the articles starting from Article 4 until Article 16, the mathematical modeling method used by the articles is analyzed to identify what kind of model is used by the articles. Table 6 shows the summarization of the mathematical modeling method used whether it is an Ordinary Differential Equation (ODE) or Fractional Order Differential Equation (FODE) or both. It shows that the fractional order differential equation (FODE) model is frequently used in research nowadays since it is a new approach method compared to ODE which has been numerously studied. Besides, the FODE model has better accuracy than the ODE model in making analysis and prediction.

Table 6: Mathematical modeling method

Ordinary Differential Equation (ODE)	Fractional Order Differential Equation (FODE)
n = 6	n = 8

n = Total number of articles

The process of analyzing the mathematical model of the epidemic is also measured in this study. The development of model formulation, stability analysis, and numerical analysis are the steps to accomplish the analysis of mathematical models to ensure the accuracy of the model to the real data. The stability analysis of model depends on the basic reproduction

number, R_0 by performing the most use method of the next-generation matrix (NGM). The model is locally stable at disease free equilibrium point when $R_0 < 1$ and at endemic equilibrium point when $R_0 > 1$. Furthermore, the basic epidemic model of SIR (susceptible, infected, and recovered) is sometimes extended in several studies such as the extended SIR model of SEIR consists of exposed individuals, and SIRV where V referred to vaccinated individuals. Hence, the epidemic model of SIR used depends on the requirement of compartmental classes in research. Several studies also conduct a comparison between different types of models used and has found that SEIR is mostly used in research since it is the most suitable model for epidemics compared to other models.

5. Conclusion

The study of SLR is conducted to obtain a comprehensive overview of the related topics which are the COVID-19, Fractional Order Differential Equation (FODE) model, and Ordinary Differential Equation (ODE) model. The analysis of 16 articles is developed which were already published starting from the year 2005 until 2023 retrieved from reliable sources and PRISMA is being used to conduct the analysis of choosing reliable and suitable articles for this study. This study has finally accomplished in achieving the objectives proposed which are to analyze the systematic literature review table of the proposed study and to provide a comprehensive summary and synthesis of the existing knowledge from the articles. It is proven that SLR assists in making a comprehensive overview of the study since SLR's purpose itself is to identify, select, evaluate, and synthesize existing research findings. Moreover, the themes of studies which are the purpose of studies, type of articles, research area, and finding from studies created in this study have provided useful information for each article in making SLR. Hence, several steps must be followed to accomplish the analysis of SLR in the study. It is concluded that a lot of information can be retrieved from one article and summarization can be made when different articles investigate the same topic approached.

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8. Conflict of Interest

The authors have no conflicts of interest to declare.

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