MAGNETOHYDRODYNAMICS (MHD) BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER AN UNSTEADY STRETCHING SHEET

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ABSTRACT

The study of magnetohydrodynamics boundary layer problem has been linked to various engineering problems including petroleum industries, geothermal energy extractions and boundary layer control in the field of aerodynamics. The study on the boundary layer flow and heat generation flows in stretching sheet plays an important role in engineering applications that includes both metal and polymer sheets. The magnetohydrodynamics (MHD) boundary layer flow and heat transfer over an unsteady stretching sheet is studied. The similarity transformation is used to transform partial differential equations (PDE) to ordinary differential equations (ODE). To obtain the result for numerical solution, Maple software are used for Runge-Kutta method using shooting technique. The effect of parameter which are unsteadiness, Prandtl number and magnetic on heat transfer characteristics are presented graphically and discussed in detail. Increases in unsteadiness, Prandtl number, and magnetic variables are found to increase the rate of heat transfer at the surface.

Keywords: *Boundary Layer Flow, Heat Transfer, Magnetohydrodynamic, Stretching Sheet.*

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1. Introduction

The evolution of boundary layer problems in engineering and industry increases continuously each year as a new problem has been proposed by the researcher. The need to study in this field is important because the industrial and engineering approach for problem-solving gives advanced results. The increasing quality of the product with yet undiscoverable characteristics can be achieved by increasing the number of new findings in much less time. The study of the dynamics of electrically conducting fluids is known as magnetohydrodynamic (MHD) which refers to magnetofluid dynamics or hydromagnetic. Plasmas, liquid metals, and salt water or electrolytes are examples of such fluids. The term magnetohydrodynamic comes from the word magneto-, which means magnetic field, hydro-, which means liquid, and -dynamic, which means movement. The inclined channel of mixed convection and effects porosity on MHD two-phase fluid flow proposed by Hasnain et al. (2015). He found the fluid velocity and temperature distribution decreases and the magnetic, porosity, and third-grade increases. The upper phase of velocity field is high as opposed to the lower phase due to the presence of third-grade fluid. The fluid velocity and temperature field are also increased as we increase the height values and the viscosity ratio. Zin et al. (2017) investigated the free convection of unsteady MHD flow of rotating Jeffrey fluid over ramped wall temperature with porous medium. MHD nanofluids in a permeable channel with porosity was studied by Khan et al. (2019). He found that in the case of suction, the velocity of nanofluids increases with the volume fraction, radiation, and permeability parameter increases, and velocity of *Ag* nanofluids decreases with an increase in the magnetic material. The *Ag* nanofluids temperature decreased with nanoparticles increases. Finally, various forms of nanoparticles have various suction effects and injection on velocity and temperature. Yürüsoy et al. (2019) investigated MHD flow with perturbation solutions in a non-newtonian fluid over concentric cylinders. He found the magnitude of temperature and velocity is decreased as the MHD parameter is increased in both viscosity models.

A boundary layer is a thin layer of viscous fluid in contact with a moving stream whose flow rate differs within its thickness from zero at the wall where the flow sticks to the wall due to its viscosity at the boundary, which roughly within 1 percent error corresponds to the free stream velocity. Prandtl (1904) introduced the essential idea of the boundary layer, which describes it as a layer of fluid producing inflows with extremely high Reynolds Numbers Re, that is, with comparatively low viscosity when compared to inertia forces. The boundary layer parameters are determined by solving equations taken from the Navier–Stokes equations for viscous fluid motion, which are first greatly reduced to account for the boundary layer's thinness. Prandtl's solution is basically the first term of the Navier–Stokes equation's power series expansion, with the series expansion conducted for powers of the dimensionless parameter.

Liu et al. (2008) further extended research of the hydrodynamics of a flow in a thin liquid film driven by an unsteady stretching surface to a more generic form of the required temperature variation in year 2008. The combined effects of a non-uniform heat source or sink and a magnetic field on thin film flow and heat transmission over an unsteady stretched sheet were examined by Abel et al. (2019). They discovered that increasing the non-uniform heat source or sink and magnetic field parameters causes the local Nusselt number to increase. When viscous dissipation and temperature- dependent thermal conductivity are present, Vajravelu et al. (2012) investigated the flow and heat transfer in a power-law liquid layer on an unsteady porous stretched sheet. They discovered that the existence of viscous dissipation can raise the dimensionless free surface temperature for all values of the power-law index. Kumaran et al. (2010) studied the impacts of a magnetic field imposed and removed on a viscous flow across a stretching sheet on the transition effect of boundary layer flow. It is seen that the first paper on non-Newtonian with stretching sheet in nanofluids was introduced by Hamad et al. (2009) considering the problem of two-dimensional laminar mixed convection flow of a power-law non-Newtonian nanofluids over a vertical stretching sheet. Khan et al. (2010) studied the problem of laminar fluid flow over a stretching sheet in a nanofluid numerically. Arnold et al. (2010) extended the research of flow across a stretching sheet by looking at the viscoelastic fluid flow and heat transfer properties, as well as the impacts of viscous dissipation and internal heat generation or absorption.

2. Methodology

The equation that is being involved in Aziz et al. (2018) research are continuity, momentum, and energy. These three equations can be referred to as governing equation. The adjustment was rendered by adding the new parameter to the equation. The adjustment involves by adding a new parameter to the momentum equation, which is magnetic parameter. The equation governing in published article with addition magnetic parameter are as below:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial x}=0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{nf}}{\rho_{nf}}\frac{\partial^2 u}{\partial y^2} - \frac{\sigma {B_0}^2}{\rho}u \tag{2}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 u}{\partial y^2} \tag{3}$$

where ${B_0}^2$ is magnetic field and ρ is fluid density, μ_{nf} is the effective viscosity and ρ_{nf} is the effective density of the nanofluid. The boundary equation is:

$$u = U, v = 0,\ T = T_S\ at\ y = 0 \tag{4}$$

$$\frac{\partial u}{\partial y} = \frac{\partial T}{\partial y} = 0,\ v = \frac{dh}{dt}\ at\ y = h \tag{5}$$

The equations were reduced by converting the PDE to ODE using similarity transformation method. The similarity variable, η , dimensionless function, f , and dimensionless temperature, θ were introduced and obtained by Aziz et al. (2018). The variables of u, v, η, ψ and T are represented as below:

$$\psi = \beta x\left[\frac{v_f b}{1-\alpha t}\right]^{1/2} f(\eta),\ T = T_0 - T_{ref}\left[\frac{dx^2}{v_f(1-\alpha t)^{-\frac{3}{2}}}\right]\theta(\eta),$$

$$\eta = \frac{1}{\beta}\left[\frac{b}{v_f(1-\alpha t)}\right]^{1/2} y,\ u = \frac{\partial\psi}{\partial y} = \frac{bx}{(1-\alpha t)}f'(\eta),\ v = -\frac{\partial\psi}{\partial x} = -\left(\frac{vb}{1-\alpha t}\right)^{1/2}\beta\, f(\eta) \tag{6}$$

After these variables have been applied in governing equation, ODE reduced from PDE are written as below:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} = 0 \tag{7}$$

$$\frac{1}{(1-\phi)^{2.5}\left(1-\phi+\phi\frac{\rho_s}{\rho_f}\right)}f''' + \gamma\left(ff' - \frac{1}{2}S\eta f'' - (f')^2 - Sf' - Mf'\right) = 0 \tag{8}$$

$$\frac{\frac{k_{nf}}{k_f}}{Pr\left(1-\phi+\phi\frac{(\rho C_p)_s}{(\rho C_p)_f}\right)}\theta'' + \gamma\left(f\theta' - 2f'\theta - \frac{1}{2}S\eta\theta' - \frac{3}{2}S\theta\right) = 0 \tag{9}$$

The boundary conditions are transformed as follows:

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1\quad at\ \ y = 0 \tag{10}$$

$$f(1) = \frac{1}{2}S,\ f''(1) = 0,\ \theta'(1) = 0\quad at\ \ y = h \tag{11}$$

where prime denote differentiation with respect to η, the magnetic parameter is $M = \frac{\sigma B_0{}^2}{\rho}$, the Prandtl number is $Pr = \frac{v}{\alpha}$, the unsteadiness parameter is S, nanoparticles volume fraction is ϕ, ρ_f is the reference density of fluid fraction, ρ_s is the reference density of the solid fraction, k_{nf} is the effective thermal conductivity of the nanofluid, k_f is the thermal conductivity of the fluid and $\frac{(\rho C_p)_s}{(\rho C_p)_f}$ is the heat capacitance of the nanofluid.

3. Results and Discussion

The results obtained by applying the boundary condition from Aziz et al. (2018) in the transformed equation based on Runge-Kutta method by using Maple software. Results will be discussed in parts, based on the involving parameter. The results of the skin friction coefficient and heat transfer rate are shown in Table 1, Table 2, and Table 3 respectively. For unsteadiness, Prandtl number and magnetic parameter, noted that the heat transfer rate is local Nusselt number. Figure 1, Figure 3, and Figure 5 plots the velocity boundary layer graphs of the different numbers of parameters with the presence of the other parameter. Other than that, the temperature boundary layer graphs of parameter *S*, *Pr* and *M* with the presence of the other parameter are plotted in Figure 2, Figure 4 and Figure 6 respectively.

3.1 Result of the skin friction coefficient and heat transfer rate

Table 1. Value of $f''(0)$ and $\theta'(0)$ for different value of unsteadiness *S*

S	**Skin friction coefficient, $-f''(0)$**	**Heat transfer rate, $\theta'(0)$**
0.8	1.0551	1.0705
1.0	1.1271	1.1697
1.2	1.1983	1.2620
1.4	1.2687	1.3521

Table 1 shows changes in skin friction coefficient and heat transfer rate when the *S* parameter value is varied, and other parameter are fixed which is the *Pr* and *M* value are constant at 1. The skin friction coefficient decreases when *S* increase. However, heat transfer rate increase following the *S* parameter.

Table 2. Value of $f''(0)$ and $\theta'(0)$ for different value of *Pr*

Pr	**Skin friction coefficient, $-f''(0)$**	**Heat transfer rate, $\theta'(0)$**
1.5	1.1271	1.4263
2	1.1271	1.6343
3	1.1271	1.9658
7	1.1271	2.8022

Table 2 shows changes in the coefficient of skin friction and the rate of heat transfer when *Pr* value is varied. The *S* and *M* parameters are constant at 1. As the value of *Pr* increase, skin friction coefficient remains at same value. This cause by uncoupled boundary layer equation. There is no existence of the Pr in the momentum equation. The heat transfer rate increases as Pr increase.

Table 3. Value of $f''(0)$ and $\theta'(0)$ for different value of M

M	Skin friction coefficient, $-f''(0)$	Heat transfer rate, $\theta'(0)$
0	0.7307	1.0458
1	1.1271	1.1698
2	1.3948	1.2002
4	1.7979	1.2137

Table 3 shows change in skin friction coefficient and heat transfer rate when the M value is varied. The S and Pr are constant at 1. The skin friction coefficient decreases as the M increase. Thus, heat transfer rate increases with increases in M.

3.2 Result on velocity and temperature profiles

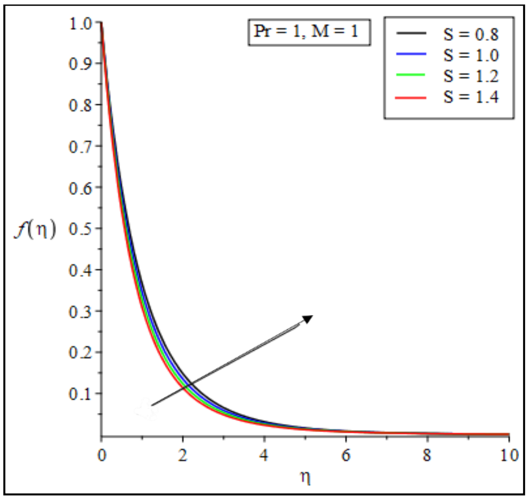

Figure 1. Velocity profiles for the various value of unsteadiness S

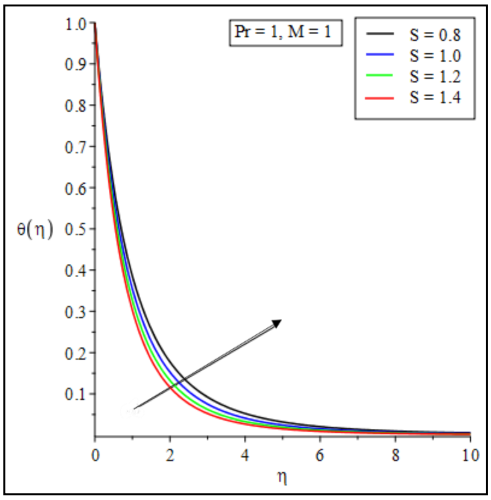

Figure 2. Temperature profiles for the various value of unsteadiness S

Figure 1 and Figure 2 show the effect of unsteadiness parameter on velocity profiles and temperature profiles respectively. As shown from Figure 1 and Figure 2, for various value of the (S = 0.8, 1.0, 1.2, 1.4) and the other parameters are constant at 1, however parameter ϕ is 0. Based on Figure 1, the velocity profiles thickness increase with the decreasing value of S and velocity profiles gradient at surface increase. Therefore, the skin friction coefficient at surface decrease with increasing of S. For Figure 2, the temperature profiles increase with the increasing S. As the S increases, the exact value of the temperature profile gradient increases at the surface. As a result, as the S is increased, the heat transfer rate at the surface increases. Table 2 shows the same conclusion, indicating that as the S increases, the skin friction coefficient decreases and the heat transfer rate at the surface increases.

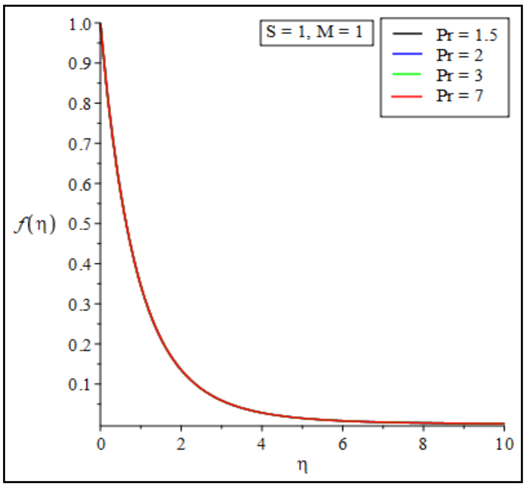

Figure 3. Velocity profiles for the various value of *Pr*

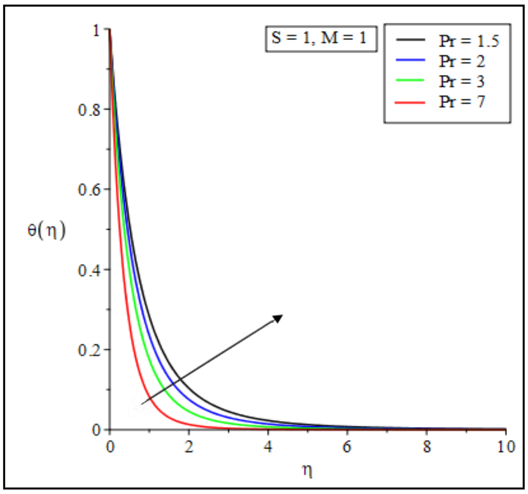

Figure 4. Temperature profiles for the various value of *Pr*

Figure 3 and Figure 4 show the effect of *Pr* on velocity profiles and temperature profiles. As seen from Figure 3 and 4, for the various valueof the (*Pr* = 1.5, 2, 5, 7) and the other parameters are constant value at 1 while parameter ϕ is 0. Based on Figure 3, there is no effect of *Pr* on velocity profiles. This phenomenon caused by uncoupled boundary layer equation. This shows that there is no existence of the *Pr* in the momentum equation. Figure 4 shows the temperature profiles thickness increase with increasing *Pr*. As the *Pr* increases, the exact value of the temperature profile gradient increases at the surface. As a result, as the *Pr* is increased, the heat transfer rate at the surface increases. Table 3 shows the same conclusion, indicating that as the *Pr* increases, the skin friction coefficient has no effect and the heat transfer rate at the surface increase.

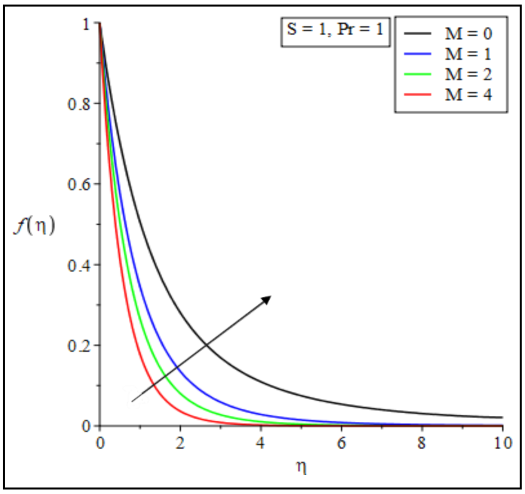

Figure 5. Velocity profiles for various value of *M*

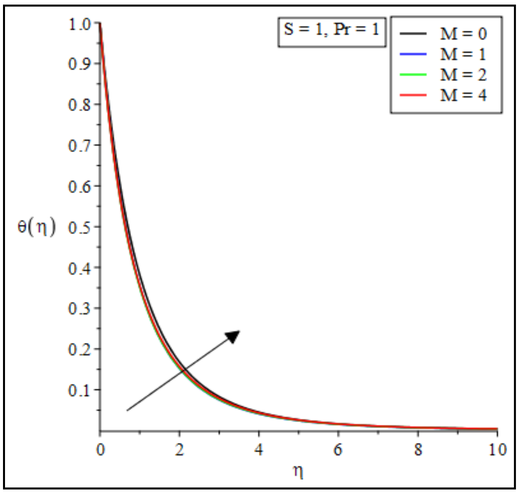

Figure 6. Temperature profiles for various value of *M*

Figure 5 and Figure 6 show the effect of M on velocity profiles and temperature profiles respectively. Based on Figure 5 and Figure 6, for various value of (M = 0, 1, 2, 4) and the other parameters are constantvalue at 1 while parameter ϕ is 0. As seen from Figure 5, the velocity profiles thickness increases with increasing of M. Therefore, the absolute value of velocity profiles gradient increase at surface with increasing of M and the skin friction coefficient decrease.

As for Figure 6, the temperature profiles thickness increases with the increasing of M. The absolute value of temperature profiles gradient increase at surface with increasing M. Therefore, the heat transfer rate decreases.Based on Table 4, the same result can be seen that when skin friction coefficient decreases, the heat transfer rate increases with increasing of M. This shows that the result in both table and graph are accurate.

4. Conclusion and Recommendations

In this research, a new modification has been made based on article flow and heat transfer in a nanofluid thin film over an unsteady stretching sheet by Aziz et al. (2018) was done by adding magnetic parameter in the previous research. The effect of the unsteadiness, Prandtl number, and magnetic parameter on skin friction coefficient and heat transfer rate was perceived. This shows that the addition of magnetic parameter in the research of magnetohydrodynamics boundary layer and heat transfer over an unsteady stretching sheet will decrease the skin friction coefficient and increase heat transfer rate when the value of magnetic parameter increases. The same effect goes with unsteadiness parameter. As the value of unsteadiness parameter increase, skin friction coefficient decreases and heat transfer rate increase. For Prandtl number, skin friction coefficient has no effect however, heat transfer rate increase as the value of Prandtl number increase. This research, magnetohydrodynamics (MHD) boundary layer and heat transfer flows over an unsteady stretching sheet is studied. In future research, the researcher can add permeability parameter into the boundary layer problem. The latest problem discovered is the magnetic effect of magnetohydrodynamic (MHD) boundary layer in an unsteady stretching sheet. The researcher can add new parameter which is permeability parameter where the ability for fluids to flow through boundary layer. A comparison can be made with other research to check the accuracy and validity of the research.

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References

Abel, M. S., Tawade, J., & Nandeppanavar, M. M. (2009). Effect of non-uniform heat source on MHD heat transfer in a liquid film over an unsteady stretching sheet. *International Journal of Non-Linear Mechanics*, 44(9), 990-998.

Arifin, N. M., Nazar, R., & Pop, I. (2011). Viscous flow due to a permeable stretching/shrinking sheet in a nanofluid. *Sains Malaysiana*, 40(12), 1359-1367.

Arnold, J. C., Asir, A. A., Somasundaram, S., & Christopher, T. (2010). Heat transfer in a viscoelastic boundary layer flow over a stretching sheet. *International journal of heat and mass transfer*, 53(5-6), 1112-1118.

Aziz, R. C., & Hashim, I. (2010). Liquid film on unsteady stretching sheet with general surface temperature and viscous dissipation. *Chinese Physics Letters*, 27(11), 110202.

Aziz, R. C., Hashim, I., & Alomari, A. K. (2011). Thin film flow and heat transfer on an unsteady stretching sheet with internal heating. *Meccanica*, *46*, 349-357.

Aziz, R. C., Hashim, I., & Abbasbandy, S. (2018). Flow and heat transfer in a nanofluid thin film over an unsteady stretching sheet. *Sains Malaysiana*, 47(7), 1599-1605.

Dandapat, B. S., Singh, S. K., & Maity, S. (2017). Thin film flow of bi-viscosity liquid over an unsteady stretching sheet: an analytical solution. *International Journal of Mechanical Sciences*, 130, 367-374.

Hasnain, J., Abbas, Z., & Sajid, M. (2015). Effects of porosity and mixed convection on MHD two phase fluid flow in an inclined channel. *PloS one*, *10*(3), e0119913.

Khan, I., & Alqahtani, A. M. (2019). MHD nanofluids in a permeable channel with porosity. *Symmetry*, 11(3), 378.

Khan, W. A., & Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International journal of heat and mass transfer*, 53(11-12), 2477-2483.

Kumaran, V., Kumar, A. V., & Pop, I. (2010). Transition of MHD boundary layer flow past a stretching sheet. *Communications in Nonlinear Science and Numerical Simulation*, 15(2), 300-311.

Hamad, M. A. A., & Bashir, M. A. (2009). Boundary-layer flow and heat transfer of a power-law non-Newtonian nanofluid over a vertical stretching sheet. *World Applied Sciences J (Special Issue for Applied Mathematics)*, 7, 172-178.

Liu, I. C., & Andersson, H. I. (2008). Heat transfer over a bidirectional stretching sheet with variable thermal conditions. *International journal of heat and mass transfer*, 51(15-16), 4018-4024.

Megahed, A. M. (2015). Effect of slip velocity on Casson thin film flow and heat transfer due to unsteady stretching sheet in presence of variable heat flux and viscous dissipation. *Applied Mathematics and Mechanics*, 36, 1273-1284.

Noor, N. F. M., & Hashim, I. (2010). Thermocapillarity and magnetic field effects in a thin liquid film on an unsteady stretching surface. *International Journal of Heat and Mass Transfer*, 53(9-10), 2044-2051.

Puttkammer, P. P. (2013). *Boundary layer over a flat plate* (Bachelor's thesis, University of Twente).

Sheikholeslami, M., Ganji, D. D., & Rashidi, M. M. (2016). Magnetic field effect on unsteady nanofluid flow and heat transfer using Buongiorno model. *Journal of Magnetism and Magnetic Materials*, 416, 164-173.

Singh, T. D., & Singh, B. (2006). Elsevier Geo-Engineering Book 5: *Tunnelling In Weak Rocks* (Vol. 5). Elsevier.

Uddin, M. J., Al Kalbani, K. S., Rahman, M. M., Alam, M. S., Al-Salti, N., & Eltayeb, I. (2016). Fundamentals of nanofluids: evolution, applications and new theory, International Journal of Biomathematics and Systems Biology. *International Journal of Biomathematics and Systems Biology*, 2(1), 1-32.

Vajravelu, K., Prasad, K. V., & Ng, C. O. (2012). Unsteady flow and heat transfer in a thin film of Ostwald–de Waele liquid over a stretching surface. *Communications in Nonlinear Science and Numerical Simulation*, 17(11), 4163-4173.

Waheed, S. E. (2016). Flow and heat transfer in a Maxwell liquid film over an unsteady stretching sheet in a porous medium with radiation. *SpringerPlus*, 5, 1-14.

Wang, C. (2006). Analytic solutions for a liquid film on an unsteady stretching surface. *Heat and Mass Transfer*, 42, 759-766.

Yurusoy, M., & Guler, O. F. (2019). Perturbation solutions for magnetohydrodynamics (Mhd) flow of in a non-Newtonian fluid between concentric cylinders. *International Journal of Applied Mechanics and Engineering*, 24(1), 199-211.

Zin, N. M., Khan, I., & Shafie, S. (2017, September). Unsteady MHD free convection flow of rotating Jeffrey fluid embedded in a porous medium with ramped wall temperature. In *Journal of Physics: Conference Series* (Vol. 890, No. 1, p. 012043). IOP Publishing.