Research Article

Calculating Distance from Shah Alam to Ipoh by Using Least Square Method

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Abstract: *This study aims to calculate the distance from Shah Alam to Ipoh, but the uneven surface of the map makes it difficult to measure with mathematical calculations. To address this, a strategy is needed to generate a mathematical equation that corresponds to the map model, so that the distance can be calculated even in uneven conditions. The least squares method is used to find the best-fit line or curve that minimises the overall sum of the squared differences between observed data points and predicted values. It is commonly used in regression analysis to determine the relationship between variables and make predictions. The distance of a map from Shah Alam to Ipoh can be determined using the least squares method and the Gauss-Jordan elimination method. The distance of region 1 + region 2 + region 3 is 17.9541 units, meaning 1 unit = 12.5km. The total arc length is 224.43 km, as the distance from Tanjong Karang to Kuala Selangor is 15km and the plotted graph is 1.2 units. This study provides a guide to learning and applying the least-squares method of interpolation, providing insights into its strengths, weaknesses, and practical implications.*

Keywords: Least square method, Gauss-Jordan elimination method

1. INTRODUCTION

Arc length is the distance between two points along a section of a curve (Calculating Arc Length, n.d). The arc length provides a more precise measurement of the distance between two points along a curved path. When dealing with maps or any representation of a curved surface, measuring distances in a straight line can lead to significant inaccuracies. Arc length on a map refers to the length of a curved line connecting two points on a map, following the curvature of the Earth's surface.

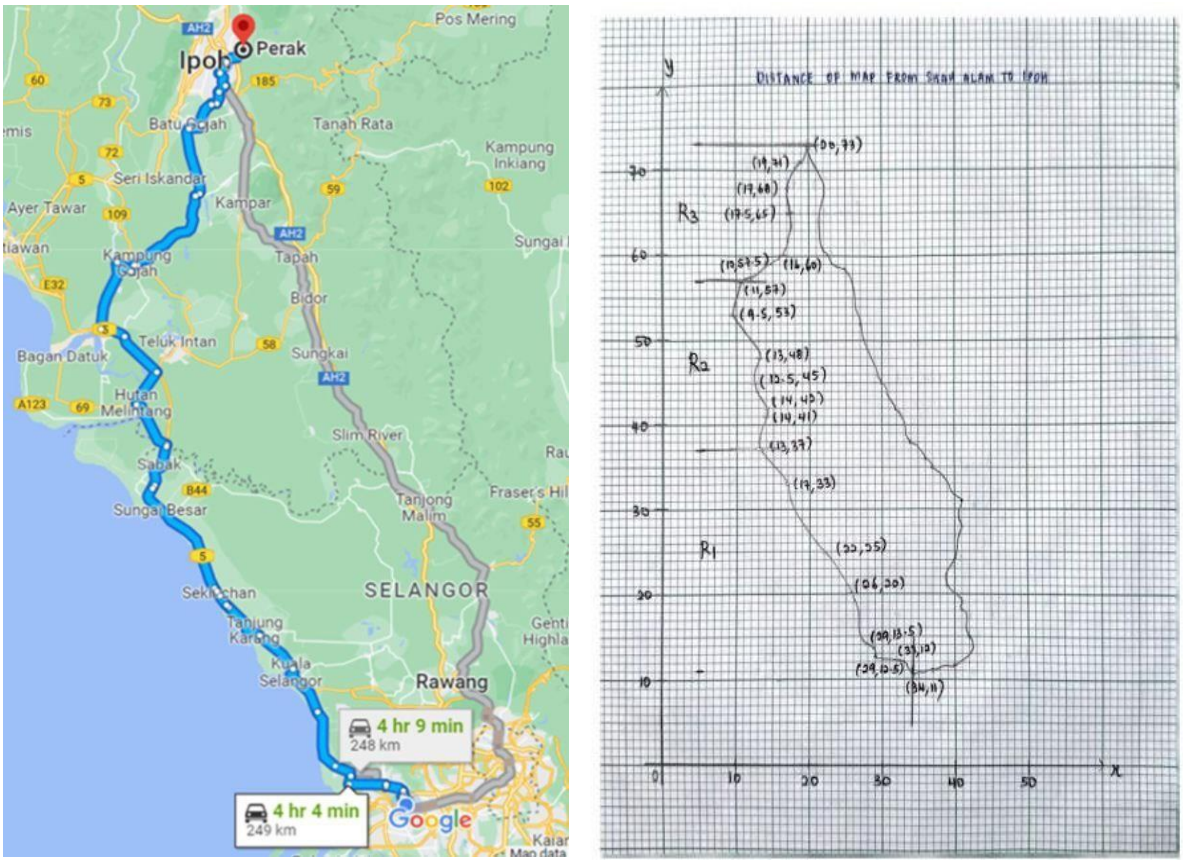
In this study, the arc length on a map is computed to determine the distance between Shah Alam and Ipoh. The arc length can be calculated using the least squares method. The least squares method is a mathematical regression analysis technique that helps identify the line of best fit for a given data set. It visually represents the relationship between the data points (Kenton, 2022). It is widely used in various fields, including statistics, economics, engineering, and data analysis. The Least Square Method (LSM) is widely utilised in physics and experimental sciences for fitting purposes. Its

popularity stems from its statistical properties, which allow it to obtain unbiased solutions with minimal variance, independent of the data's probability distribution function. Moreover, the solutions of LSM tend to converge asymptotically towards a multivariate normal distribution (Helene et al., 2016). The least squares method is the method commonly used for generating a polynomial equation from a given data set *(f(x)= a0 + a1x + a2x + ... + ax, where m < n)*.

The objective of this study is to answer for the coefficients *a0, a1, a2...* using the least-squares strategy by solving this system of equations using some of the techniques such as Gauss Elimination, Gauss-Jordan elimination, and Cramer's Rule.

2. METHOD & MATERIAL

In this study, the distance from Shah Alam to Ipoh will be investigated using arc length. The arc length will be obtained from the map as shown in Figure 1A. The map was plotted in a graph paper as in Figure 1B.



A: Original Map B: Map plotted in Graph paper

Figure 1: Whole Map

The least square method was used to find arc length of a map from Shah Alam to Ipoh. The least square method is a mathematical technique used to find the best-fit line or curve that minimises the overall sum of the squared differences between the observed data points and the predicted values. This method was selected because it is commonly used in regression analysis to determine the relationship between variables and make predictions. Linear regression or Linear least-squares provides a powerful technique for fitting a best line given set of data, but a quadratic or higher degree

of polynomial may be preferable. The calculation process of regression analysis begins with a set of data points to be plotted on x-axis and y-axis. Next, the most suitable model to fit the data points was determined, whether a linear function, quadratic or cubic is considered as the most appropriate. The calculation process was divided into five steps as demonstrated in Figure 2.

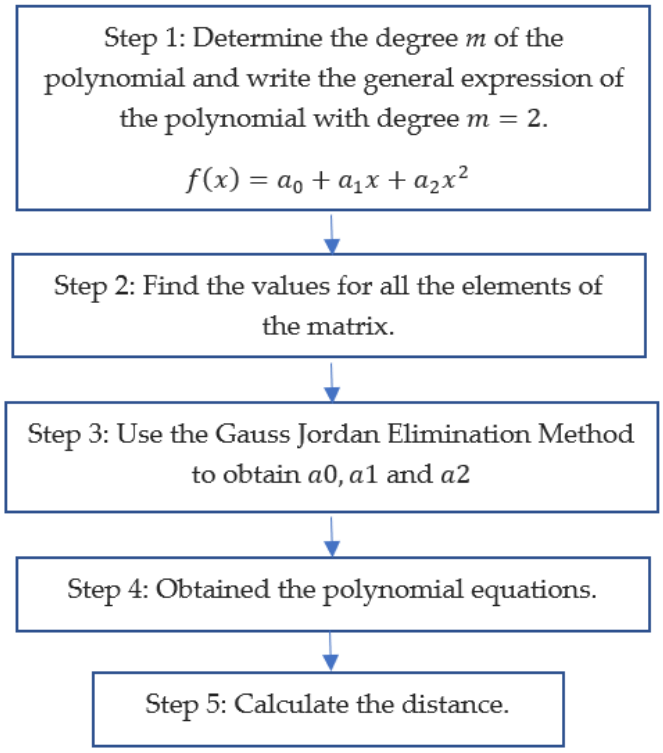


Figure 2. Flowchart of the Procedure.

3. FINDINGS

The findings will be explained in this section. In this study, the distance from Shah Alam to Ipoh is the main focus. The distance from Shah Alam to Ipoh is chosen as a sample destination between two locations for the purpose of this study. The distance was plotted on a graph paper and the calculations obtained are between coordinates (34,11) until (20,77) as in Figure 1B. The regions were divided into three parts. The calculation starts at region 1. The distance is calculated by taking R1 from (34,11) until (13,37) as region 1.

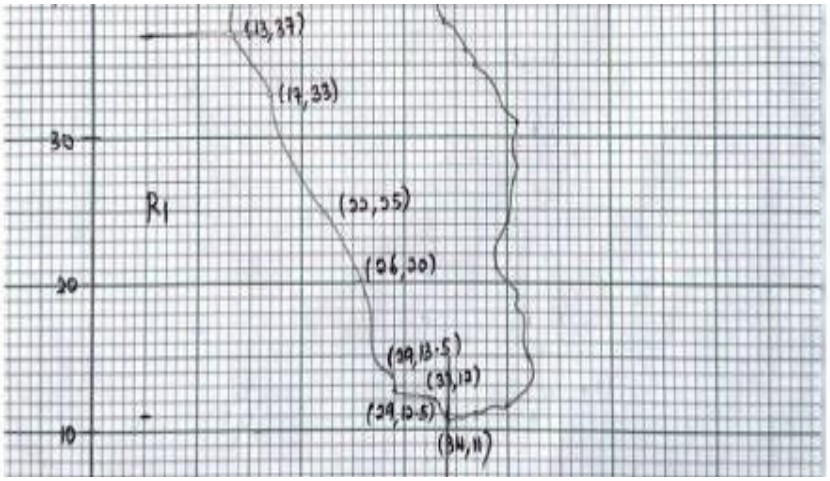


Figure 3. Map for Region 1

3.1 Solution

As the first step, the degree m of the polynomial was determined, and the general expression of the polynomial was written with degree $m = 2$.

$$f(x) = a_0 + a_1 x + a_2 x^2 \quad (1)$$

In the second step, the values for all the elements of the matrix were decided. The total number of points of n need to be determined first, which is 8. Then, a table as illustrated in Table 1 was constructed:

Table 1. Table for all elements.

x	y	x^2	x^3	x^4	xy	x^2y
34	11	1156	39304	1336336	374	12716
33	12	1089	35937	1185921	396	13068
29	12.5	841	24389	707281	362.5	10512.5
29	13.5	841	24389	707281	391.5	11353.5
26	20	676	17576	456976	520	13520
22	25	484	10648	234256	550	12100
17	33	289	4913	83521	561	9537
13	37	169	2197	28561	481	6253
203	164	5545	159353	4740133	3636	89060

Third step, the normal equation will be written in matrix form (3x3) as in equation (2).

$$\begin{bmatrix} n & \sum x & \sum x^2 \\ \sum x & \sum x^2 & \sum x^3 \\ \sum x^2 & \sum x^3 & \sum x^4 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{matrix} \sum y \\ \sum xy \\ \sum x^2 y \end{matrix} \quad (2)$$

Using the values as in Table 1, equation (3) is obtained.

$$\begin{bmatrix} 8 & 203 & 5545 \\ 203 & 5545 & 159353 \\ 5545 & 159353 & 4740133 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 164 \\ 3636 \\ 89060 \end{bmatrix} \quad (3)$$

Next, the values of *a0, a1* and *a2* from equation (3) can be obtained by using Gauss Jordan Elimination method as follow.

$a0 = 64.2255$

$a1 = 2.2536$

$a2 = 1.9470 \times 10^{-3}$

Step 4. The value of *a0, a1* and *a2* obtained in Step 3 will be substituted into equation (1) to obtain equation (3). Hence, to obtain equation (5) and (6), the same steps were repeated for region 2 and region 3.

$$f(x) = 64.2255 - 2.2536x + (1.9470 \times 10^{-3})x^2 \quad (4)$$

$$f(x) = 33.6429 + 6.0020x + (-3.9606 \times 10^{-1})x^2 \quad (5)$$

$$f(x) = 78.0183 - 3.9910x + 0.1888x^2 \quad (6)$$

Step 5. Now, equations (4), (5), and (6) were applied separately to arc length equation to find the distance for each region. The formula for arc length given as in equation (6).

$$Arc\ length = \int_a^b \sqrt{1 + [f'(x)]^2}dx \quad (6)$$

3.2 Result

As a result, the total distance of a map from Shah Alam to Ipoh can be determined by adding arc length for region 1, region 2 and region 3 as follow.

$$\begin{aligned} Total\ distance\ &= Arc\ length\ of\ Region\ 1 + Arc\ length\ of\ Region\ 2\ + Arc\ length\ of\ Region\ 3 \\ &= 10.00516 + 3.1214 + 4.8275 = 17.9541\ unit \end{aligned}$$

Since the map shows that from Tanjong Karang to Kuala Selangor is 15km while in the plotted graph is 1.2 unit, hence 1 unit = 12.5km. So, it can be concluded that the total arc length is 224.43 km.

4. DISCUSSION

The distance from Shah Alam to Ipoh has been successfully calculated manually. However, it is time consuming to calculate it one by one. To solve this issue, it is recommended that a Graphical User Interface (GUI) will be developed in the future so that the uneven surface of the map can be measured easily with mathematical formula. The GUI is expected to be very useful for everyone since it is a system. All users need to do is just to upload the image of a map so that the system will be able to measure the distance from one place to its destination. Calculating the distance from a map can be useful in everyday life since it assists the potential users of this GUI system in decision making, especially when determining the shortest path from a region to another region.

5. CONCLUSION

In conclusion, the least squares method and the Gauss-Jordan elimination method were used to determine the distance of a map from Shah Alam to Ipoh. The total arc length is 224.43 km. The use of the least squares method and the Gauss-Jordan elimination method is considered effective to determine the distance of a map from Shah Alam to Ipoh. These methods are both powerful mathematical tools that can be applied to solve a variety of problems. By using the least-squares method of interpolation, it helps in finding the best line and curve, as well as the arc length of the curve. Overall, this study provides a detailed guide to learning and applying the least squares method of interpolation. It provides useful insights for researchers and practitioners into the strengths, weaknesses, and practical implications of this effective technique in a variety of domains.

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