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Springback Analysis of Thin Tubes Under Torsional Loading

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ABSTRACT

Springback, the elastic recovery of material on the release of applied load, is the major factor in obtaining the accurate and consistent dimensions of the final part. The mechanics of springback is essential for its effective prediction and compensation. The aim of the paper is to present a theoretical analysis of the torsional springback in thin tubes of bi-linear work hardening material. The bi-linear behavior of the material is approximated by using modified Ludwik type stress-strain relation. The theoretical analysis is based on membrane and sand heap analogies. The analytical calculations establishes relationship for angle of twist to twisting moment and residual/springback angle of twist per unit length for thin tubes under plastic torsion in non-dimensionalized form. The theoretical analysis is supported by experiments performed on thin tubes of mild steel and aluminium with different geometry and mechanical properties. A comparison between the results obtained for thin tubes on bi-linear and non-linear work-hardening material loaded under torsion is also made.

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Keywords: *Metal forming, Springback, Torsional springback, Thin tubes, Non-linear work hardening materials*

Nomenclature

u, v, and w	Small displacements in x, y, and z direction, respectively
θ	Angle of twist per unit length of the tube
θ_θ, θ_R and θ_S	Elastic, residual and springback angle of twist per unit length respectively
$\bar{\theta}$	Non-dimensionalized angle of twist per unit length
$\bar{\theta}_R$, $\bar{\theta}_S$	Non-dimensionalized residual and springback angle of twist respectively
γ	Shear strain
τ_{xz}, τ_{yz}	Shear stress
G	Modulus of rigidity
Φ_θ	Elastic stress function
∇	Gradient
T	Twisting moment (torque)
T_θ	Elastic torque
T	Non-dimensionalized torque
ζ	Deflection of membrane
σ	Yield strength in tension
β	Numerical factor; 1/2 for Tresca and $1/\sqrt{3}$ for Mises yield criterion
$\varnothing_P$	Plastic stress function
A	Mean cross-sectional areas of the tube enclosed by the outer and inner boundaries
S	Length of the center line of ring section of the tube
τ	Thickness of the tube
τ_0, γ_0	Yield shear stress and strain respectively
α	Plastic modulus of rigidity
$\in_x$, $\in_y$	Longitudinal strain
μ	Poisson's ratio
I_α	Refers to the annular area over which $\iint \Phi_\theta$ is taken
I_1	Area enclosed by inner boundary
$\varnothing_{\theta 1}$	Constant value of stress functions along the internal boundary

Introduction

Metal forming, the backbone of modern manufacturing industry, contributes 20% of the Gross Domestic Product (GDP) of industrialized countries [1]. Sheet metal forming is one of the types of metal forming to produce light weight components. Light weight fabricated components are advantageous where mass is critical to enable the product function, such as aeronautical, automobile, and home appliances industries. In automobile industries, thin tubular components with circular, square and rectangular cross-sections are commonly used in chassis, frames, bumpers, panels, etc. The prime requirement in these components is finishing quality and stiffness i.e. component does not bend too much due to elasticity [2]. Hence the success of the forming process depends on the quality of final part shape that may be affected by the defects such as wrinkling, necking and subsequent fracture, surface deflections drawing grooves, orange peel and springback [3]. These defects may affect customer satisfaction and loyalty for the final product.

After completion of bending and upon removal of tooling during a sheet metal forming process, the tubular component springbacks due to elastic nature of material. This phenomenon is known as springback or elastic recovery of material. During a forming operation, the contour of the sheet section is elastically deformed and takes the die shape on the application of load. But on the removal of the applied load, elastic strain energy is released and elastic deformation disappears due to reduction in stress. Springback is inherent and is the main source of dimensional variation in sheet metal formed components. In sheet metal forming different types of springback may exist on the basis of part geometry and deformation area: bending, membrane, torsional and combined bending and membrane [4]. The torsional or twisting springback is the measure of elastic recovery of angle of twist on the removal of applied torque after twisting the section beyond elastic limit. It arises due to uneven elastic recovery of material in different directions. Springback has to be considered at the initial phases of design to achieve consistent and accurate dimensions of the final part. It may affect the whole product design & development process in terms of production delay, higher rejection rate and increment in cost for tooling revisions. It is more evident in the automobile industry where numerous parts are manufactured by plastic forming. All these factors reduce customer satisfaction and loyalty for the final product. US automobile industries alone bear a loss of more than $50 million per year because of the springback [5].

In literature there are two ways in which research on this phenomenon have been made: effective prediction of springback; and springback compensation in tooling design. The essential requirement for effective prediction of springback is the understanding of its mechanics [6]. Numerous research studies have been published on this phenomenon in the last four decades and Most of them have focused on reduction of springback in the sheet bending operations through experiments [7]-[18] and simulation [19]-[26]. Almost all the researchers agreed that the final shape of the part depends upon the amount of elastic energy stored in the part during the sheet metal forming process [27]. This amount of elastic energy depends on a number of parameters related to material as well as process [28]. In recent years, much attention has been placed on springback of tubes [29]-[33]. Most of the researchers use simplified models and different material hardening laws in finding out the amount of springback in tube bending operations.

Springback prediction is problematic when a twisting mode is encountered. This factor is present in most springback scenarios, and as a result the prediction can be problematic [34]. Torsional springback of different cross-sections have been analyzed by Dwivedi et al. [35]-[40]. Narrow rectangular strips [35]-[36] and general cross-sections [37]-[39] of linear work hardening materials under torsional loading were estimated by using Ramberg-Osgood stress-strain relation and deformation theory of plasticity. Analytical solution was based on finite difference approximation. In a study [40], a mathematical model was proposed to analyze torsional springback in narrow rectangular bar made of non-linear work-hardening materials based on modified Ludwick type stress-strain relationship. Shigeki et al. [41] proposed a new method of torsional springback compensation by performing a benchmark test using S-rail by setting the appropriate heights of draw-beads on a die face. An optimization technique based on finite-element simulation was used to determine the appropriate heights of draw-beads. Recently, Choubey et al. [42] developed a theoretical expression for torsional springback in thin tubes with non-linear work hardening behavior of the material and compared them with experimental findings for mild steel case.

In this paper, the objective is to develop a theoretical model to predict springback in thin tubular sections for torsional loading application. The analytical model is based on the deformation theory of plasticity taking into consideration of work hardening behavior. The theoretical analysis is supported by experimental tests on thin tubes made of mild steel and aluminum having different thickness, lengths and cross-sections (square and rectangular). Experimental results are in good agreement with theoretical analysis. From the derived relationship, the angle of twist or the twisting moment that has to be provided to a strip of a particular material, thickness and length, can be calculated to obtain the desired residual angle of twist.

Mathematical Formulation

A prismatic bar undergoing torsion and elastic deformation is considered. Let u, v and w be the small displacements of the point (x, y, z) relative to its initial position, in the x, y and z directions respectively. At a section, where z is constant, the cross-section rotates about the z-axis. Thus, according to [43], [44] the values of u, v & w are given as,

$$u = -yz\theta, \quad v = xz\theta, \quad \text{and} \quad w = \theta f(x, y) \tag{1}$$

where θ is the angle of twist per unit length of the bar.

For elastic states of stress considered,

$$\frac{\partial \tau_{xz}}{\partial y} - \frac{\partial \tau_{yz}}{\partial x} = 2G\theta = \text{constant} \tag{2}$$

If a stress function $\varnothing_0$ is taken such that

$$\tau_{xz} = \frac{\partial \varnothing_0}{\partial y} \quad \text{and} \quad \tau_{xz} = \frac{\partial \varnothing_z}{\partial x} \tag{3}$$

the equilibrium equation

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = 0 \tag{4}$$

is automatically satisfied. Hence from equations (2) and (3)

$$\left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y}\right)\varnothing_\theta = \nabla^2 \varnothing_\theta = -2G\theta \tag{5}$$

Along a boundary curve $\tau_{yz} / \tau_{xz} = d_y / d_x$, the boundary is free from any stress, so

$$-\tau_{yz} dx + \tau_{xz} dy = \frac{\partial \varnothing_\theta}{\partial x} dx + \frac{\partial \varnothing_\theta}{\partial y} dy = d\varnothing_\theta = 0 \tag{6}$$

and along a boundary, the twisting moment T is given by

$$T = \iint \left(\tau_{yz} x - \tau_{xz} y\right) dx\, dy = 2 \iint \varnothing_\theta \, dx\, dy \tag{7}$$

According to Prandtl's membrane or soap film analogy [45] & [46], a thin membrane clamped around a bounding curve is considered identical to that of the cross-section of the twisted bar loaded by a constant lateral pressure. It can be shown that the deflection ξ of such a membrane satisfies a differential equation of the form

$$\nabla^2 \xi = \text{a constant} \tag{8}$$

Along the contour line of the surface, ξ is constant, that is equivalent to $\varnothing_\theta$ = constant. But along a contour line $\tau_{yz} / \tau_{xz} = d_y / d_x$ which means that the resulting shear stress direction is along the tangent to the contour line and the magnitude of the shear stress is proportional to the greatest slope of the surface at that point.

For a prismatic bar with hollow section [44], twisting moment

$$T = 2\left[\varnothing_{\theta 1}\, I_1 + I_{\infty}\right] \tag{9}$$

where, I_{∞} refers to the annular area over which $\iint \varnothing_{\theta}\, dx\, dy$ is taken and I_1 is the area enclosed by the internal boundary of the section, $\varnothing_{\theta 1}$ being constant value of stress function along the internal boundary.

The membrane analogy may be applied to hollow sections with slight modification. The first term of the right hand side of equation (9) clearly represents a cylindrical prism formed beneath the 'hollow' and second term is the volume directly under the film. If a light rigid plate having the shape of the inner boundary is constrained to move vertically by any amount, the soap film between the plate and the outer boundary is stretched and the plate finds its own height due to air-pressure beneath. However, in case of a thin tube the variation of the slope of the membrane is negligible along the thickness and the slope may be taken as constant.

In case of plastic deformation [45],

$$\tau_{xz}^2 + \tau_{yz}^2 = \left(\frac{\alpha}{\beta}\right)^2 \tag{10}$$

where α may be taken, as current yield strength in tension and β is a factor equal to 1/2 for the Tresca and $1/\sqrt{3}$ for the Mises yield criterion. Equation (10) with the help of equation (3) may be written as

$$\left.\begin{array}{c} \nabla\varnothing_p + \nabla\varnothing_p = \left(\dfrac{\alpha}{\sigma}\right) = [\tau(\gamma)]^2 \\ \text{or} \\ |\nabla\varnothing_p| = \tau(\gamma) \end{array}\right\} \tag{11}$$

Where $\varnothing_p$ is the plastic stress function, $\tau(\gamma)$ is the current yield shear strength expressed as a function of shear strain γ. For a perfectly-plastic material,

$$|\nabla\varnothing_p| = \tau = \text{constant} \tag{12}$$

From equation (12) it is clear that if the material is elastic-perfectly plastic theFrom equation (12) it is clear that if the material is elastic-perfectly plastic then $\varnothing_p$ forms a surface of constant slope. So to determine the limiting torque in such a case, Nadai's Sand heap analogy [45] & [46] may be adopted.

However, when the material is strain hardened, such a procedure is not feasible. But, in such cases the roof representing the plastic state of stress will be an ever-changing one with the slope at a point increasing with increasing θ for a particular value of θ. The roof and the membrane representing the plastic

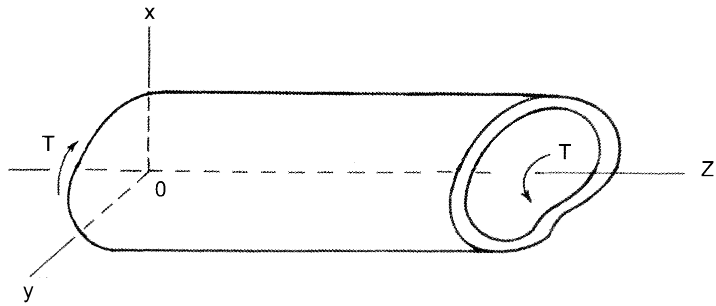

Figure 1: Geometry of the Problem

region will be touching each other, i.e., they will have the same slope at a point undergoing plastic deformation.

Consider a hollow tube of arbitrary section having constant thickness τ under torsion (Figure 1). The tube dimensions are such that there is no chance of buckling. Let it be assumed that the shear stress-strain curve for the material of the tube is bi-linear (Figure 2), which follows the modified Ludwik (47) equation and is given by

$$\tau = \begin{cases} G\gamma & \tau \le \tau_0 \text{ or } \gamma \le (\tau_0 / G) = \gamma_0 \\ \tau_0\left(1 - \dfrac{\alpha}{G}\right) + \alpha\gamma & \tau \ge \tau_0 \text{ or } \gamma \ge \gamma_0 \end{cases} \tag{13}$$

Where τ_0 and γ_0 are the yield stress and strain and α is the plastic modulus of rigidity given by the slope of the stress- strain curve for $\tau \ge \tau_0$. Elastic analysis of the tube under torsion using soap film analogy [43] & [44] gives,

$$\tau = \frac{\tau}{2A\tau} \tag{14}$$

$$\theta = \frac{TS}{4A^2GT} \tag{15}$$

also,

$$|\tau| = \frac{2A}{S} G\theta \tag{16}$$

Where A is the area enclosed by the outer and inner boundaries of the cross-section of the tube and S is the length of the center line of the ring concentration section of the tube. The effect of re-entrant corner, if any, produces stress concentration but its influence in torque deformation characteristic is negligible.

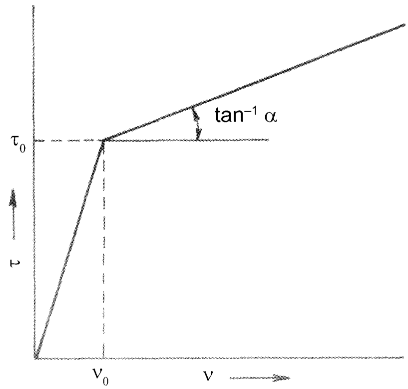

Figure 2: Bi-linear Shear Stress-strain Curve

From equation (13) the shape of shear stress-strain curves $d\tau/d\gamma$ (modulus in plastic range) can be obtained and given by,

$$\frac{d\tau}{d\gamma} = \begin{cases} G & \tau \leq \tau_0 \\ \alpha & \tau \geq \tau_0 \end{cases} \tag{17}$$

In case of torsion of thin tubes with uniform thickness, stress at ant torque (T) is uniformly distributed i. e. τ is constant. The stress τ is either in elastic or plastic zone and depends only on T. we also find in such cases $\int \tau ds = \tau s$. Now let us proceed to find θ while loading upto a torque T, $(T > T_0)$ and then let us find θ_θ which stands for elastic recovery. Hence the residual angle of twist is given by,

$$\theta_R = \theta - \theta_\theta \tag{18}$$

If $d\tau/d\gamma$ is constant then from equation (14),

$$d\tau = \frac{d\tau}{2A\tau} \tag{19}$$

on integrating equation (19), we get

$$\tau = \int_0^{\tau_0} d\tau + \int_{\tau_0}^{\tau} d\tau$$

or

$$\tau = \tau_0 + \int_{T_0}^{T} \frac{dT}{2At}$$

or

$$\tau = \frac{T_0}{2At} + \frac{1}{2At}\left[T - T_0\right] = \frac{1}{2At} \tag{20}$$

In view of equation (17),

$$\begin{cases} \text{While } \tau \text{ is elastic} & d\tau = \dfrac{2GA}{S} d\theta \\ \text{and While } \tau \text{ is elastic} & d\tau = \dfrac{2\alpha A}{S} d\theta \end{cases} \tag{21}$$

So, τ corresponding to T, ($T > T_0$) is given by

$$\tau = \int_0^{\tau_0} d\tau + = \int_{\tau_0}^{\tau} d\tau = \tau_0 + \frac{2AG\theta_0}{S} + \frac{2A\alpha}{S}(\theta - \theta_0)$$

or

$$\tau = \frac{2A\alpha\theta_0}{S}\left(\frac{G}{\alpha} - 1\right) + \frac{2A\alpha\theta}{S} \tag{22}$$

So, from equation (20) and equation (22)

$$\frac{2A\alpha\theta_0}{S}\left(\frac{G}{\alpha} - 1\right) + \frac{2A\alpha\theta}{S} = \frac{T}{2At}$$

or

$$\theta = \frac{TS}{4A^2\alpha t} - \alpha\theta_0\left(\frac{G}{\alpha} - 1\right) \tag{23}$$

Now, since θ_θ represents elastic recovery,

$$\theta_\theta = \frac{TS}{4A^2Gt} \tag{24}$$

Finally from equations (18), (23) and (24)

$$\theta_R = \frac{TS}{4A^2Gt}\left(\frac{G}{\alpha} - 1\right) - \frac{\alpha_0 S}{2GA}\left(\frac{G}{\alpha} - 1\right)$$

or

$$\theta_R = \frac{S}{2AG}\left(\frac{G}{\alpha} - 1\right)\left[\frac{T}{2At} - \tau_0\right] \tag{25}$$

Writing in non-dimensional form,

$$\overline{\theta} = \frac{\theta}{\theta_0} = \frac{\theta}{T_0 S / 2AG} \text{ , and} \tag{26}$$

$$\overline{T} = \frac{T}{T_0} = \frac{T}{4A^2tG\theta_0 / S} = \frac{T}{2At\theta_0} \tag{27}$$

Again from equation (25) non-dimensionalized residual angle of twist θ_R can be obtained as,

$$\overline{\theta}_R = \frac{\theta_R}{\theta_0} = \left(\frac{G}{\alpha} - 1\right)\left[\frac{T}{2At_0} - 1\right] \tag{28}$$

Spring back percentage in twist ($\overline{\theta}_S$), can be obtained as

$$\%\overline{\theta}_S = \left[1 - \frac{\overline{\theta}_R}{\theta}\right] \times 100 \tag{29}$$

Experiments

The mild steel and aluminum tubular bars of square (25 mm × 25 mm) and rectangular (30 mm × 20 mm) cross-section, of different lengths (178-280 mm) and of different wall thickness (1 and 2 mm) are subjected to torsion test on "Avery Torsion Testing Machine" that has accuracy up to 0.1 degree. The angle of twist, torque and residual angle of twist was noted. Experimental points were plotted along with the theoretical curves drawn for different values of work-hardening index. Details of the experiments and material properties are given in Appendix A.

Results and Discussion

The amount of springback in terms of angle of twist, depends on

- the point on $T - \theta$ curve from where unloading is initiated
- the slope of the elastic unloading line.

Since, the material is strain hardened, the amount of springback will be a function of the angle of twist, the elastic modulus of rigidity and the work hardening index. This has been obtained theoretically in equations (28 & 29) and verified experimentally.

Figure 3 and 4 shows the variation in torque as the angle of twist is increased for square and rectangular cross-sections respectively. It was observed that for the same angle of twist, the torque is greater in case of rectangular cross-section. It was also seen that for same values of angle of twist the torque is more for higher thickness in both the materials. Figure 5 and 6 shows the variation of springback /residual angle of twist in percentage with the variation in angle of twist for square and rectangular cross-sections respectively. It is found from the figures that upto a certain angle of twist, the residual angle of twist increases rapidly but after this, the variation is slow and it is almost constant at higher values. This is quite expected because initially for smaller values of twist, bulk of deformation is elastic and, as such, recoverable percentage of deformation is high. However, as the angle of twist increases the share of non-recoverable plastic deformation increases and the recoverable elastic deformation (springback) as percentage of total deformation decreases. It is also clear that the variation of springback / residual angle of twist in percentage are not dependent on cross-section.

From Figures 3-6 it is evident that the experimental results are well in agreement with the theoretical predictions conforming the validity of the theoretical analysis made. However, as can be observed that from Figure 5 & 6, the values of the springback /residual angle of twist in percentage for aluminum show some deviation from the experimental values obtained. This deviation is not due to any error in α, because if this were so, the deviation would be there

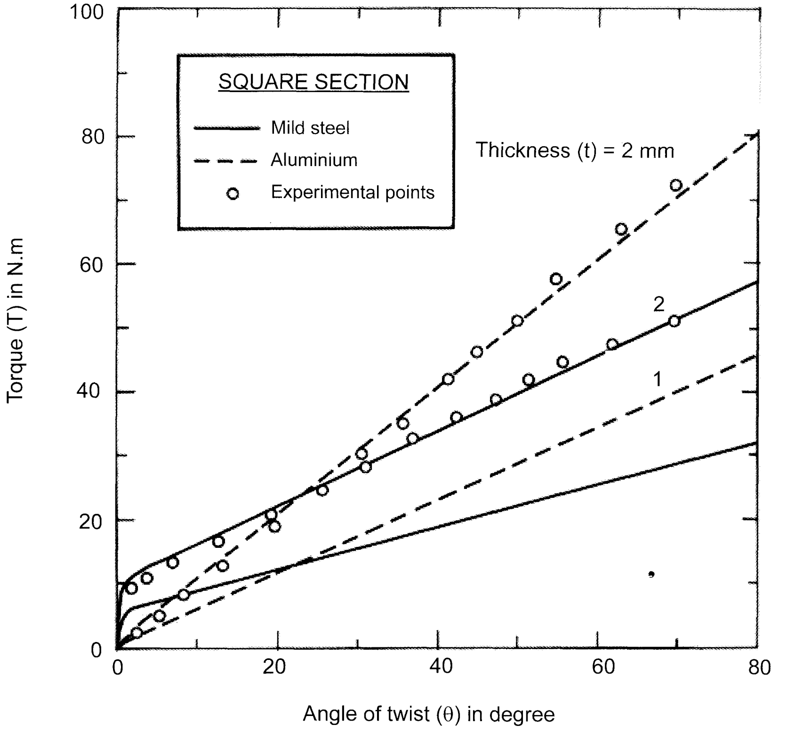

Figure 3: Curves Showing the Variation in Torque as Angle of Twist is Increased for Square Section

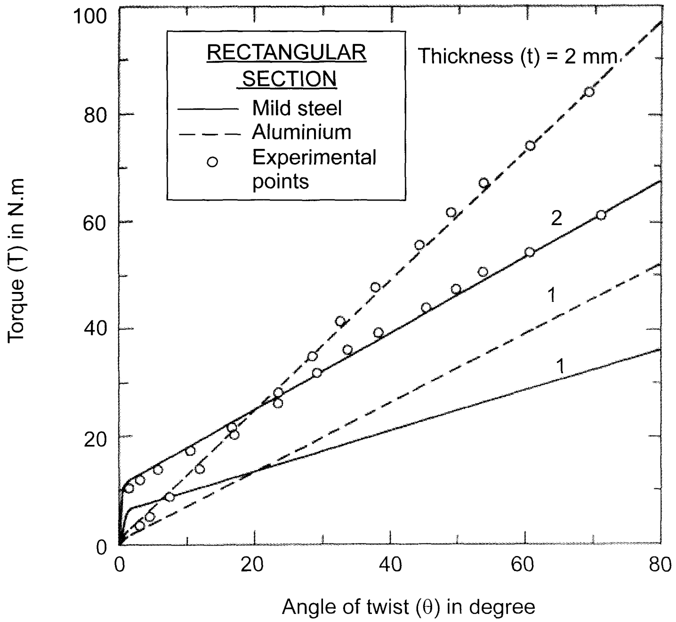

Figure 4: Curves Showing the Variation in Torque as Angle of Twist is Increased for Rectangular Section

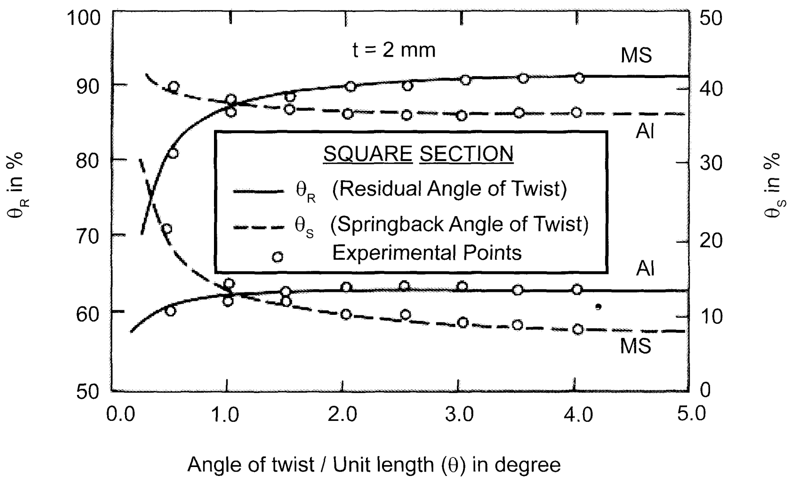

Figure 5: Springback/Residual Angle in % vs. Angle of Twist in Degree for Square Section

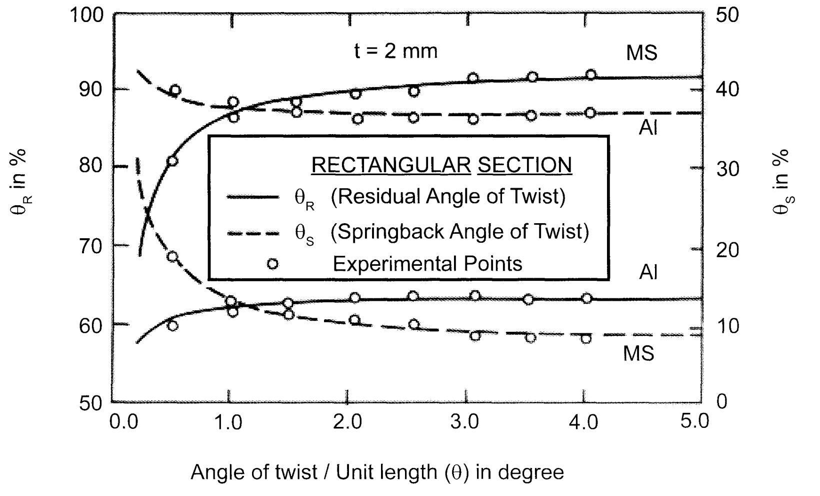

Figure 6: Springback/Residual Angle in % vs. Angle of Twist in Degree for Rectangular Section

in $\overline{T}-\overline{\theta}$ curve, which is not the case. Hence, this may due to the deviation of actual elastic unloading line in tension from the theoretically calculated line of unloading. Figure 7 shows the variation of torque ($\overline{T}$) with angle of twist ($\overline{\theta}$) for different values of α / G. It is evident from the figure that for particular angle of twist the required torque is more for higher values of α / G and it decreases as α / G decreases. Also, the nature of variation of springback ($\overline{\theta}_S$)/ residual angle ($\overline{\theta}_R$) in percentage with angle of twist ($\overline{\theta}$) for different values of of α / G is shown in Figure 8. It is observed from the figure that as the values of α / G decreases, springback percentage ($\overline{\theta}_S$) decreases.

From the analysis, it is possible to ascertain theoretically the value of angle of twist of tubular section bars or the torque applied to the general cross-section bars after knowing the value of α / G. The required values can be determined either from equations (27) and (28) or from these figures. Usually, in practical situation for given value of ($\overline{\theta}$), the value of ($\overline{\theta}_R$) is to be determined if the loading is kinematic and ($\overline{T}$) is to be determined if loading is kinetic.

Figure 9 shows the comparison of torque vs. angle of twist curves for the non-linear work-hardening index n = 1/3 and the bi-linear work-hardening approximation $\alpha / G = 0.074$. It is clear from the figure that the experimental values have excellent match when non-linear material behavior is taken though the experimental points fall between these two approximations. Bi-linear behavior approximated (with $\alpha / G = 0.074$) is not found to be satisfactory.

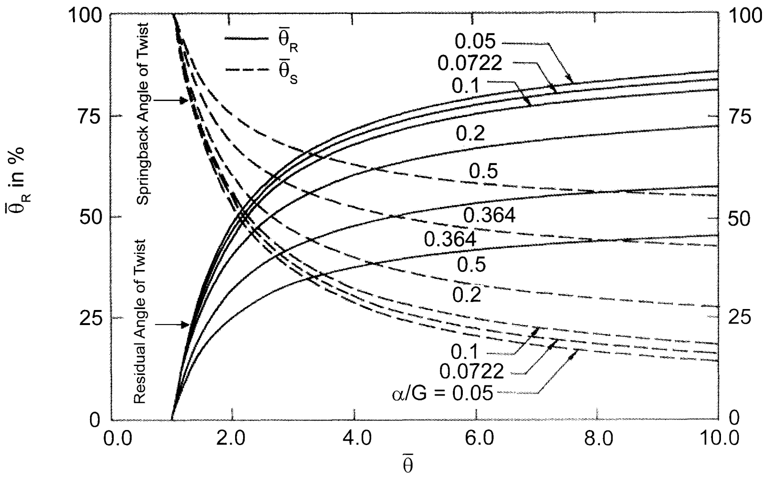

Figure 7: Non-Dimensionalized Springback ($\bar{\theta}_S$) in % vs. Non-Dimensionalized Angle of Twist ($\bar{\alpha}$) for Different Values of (α / G)

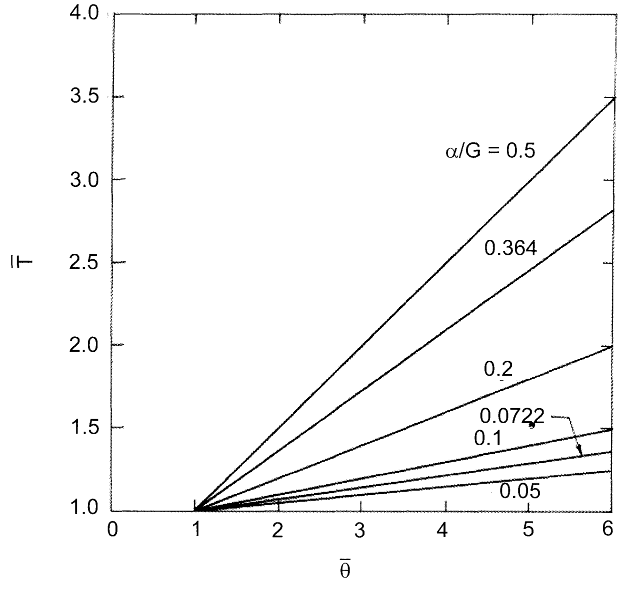

Figure 8: Non-Dimensionalized Torque ($\bar{T}$) in % vs. Non-Dimensionalized Angle of Twist ($\bar{\theta}$) for Different Values of (α / G)

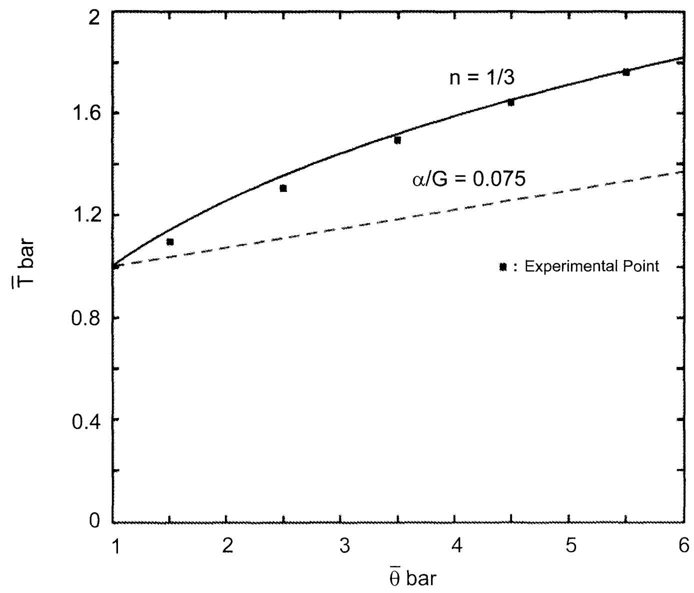

Figure 9: ($\bar{T}$) vs. ($\bar{\theta}$) for Bilinear Work Hardening $\alpha / G = 0.074$ and Non-linear Work Hardening n = 1/3 Case

Conclusions

An analysis relating the angle of twist to the twisting moment and the angle of twist to the Residual/Springback angle of twist for thin tubes under plastic torsion has been presented in non-dimensionalized form, so that same curves could be used for different material. The resulting formulae are in good agreement with experimental data of different section bars for mild steel and aluminum tubes.

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Appendix A

Table A: Mechanical Properties of Tube Materials

Material	Modulus of elasticity (E) (N / mm^2)	Modulus of rigidity (G) (N / mm^2)	Yield shear stress (τ_0) (N / mm^2)	Plastic modulus of rigidity (α) (N / mm^2)	α/G
Mild steel	2.1×10^5	8.24×10^4	108	6.18356×10^3	0.074
Aluminum	7×10^4	2.8×10^4	12	1.4×10^4	0.50

Experimental Procedure

The present work is concerned with experimentally quantifying the torque, springback and residual angle of twist per unit length of mild steel and aluminum tubes of square (25 mm × 25 mm) and rectangular cross-section (30mm × 20 mm), of different lengths (178-280 mm) and of different wall thickness (1 and 2 mm). Figure A shows the pictorial view of "Avery Torsion Testing Machine". The strain was applied to the specimen by worm and spur gearing was so arranged that the full load might be applied by hand without undue effort. The load was transmitted from the weighing spindle by means of a horizontal

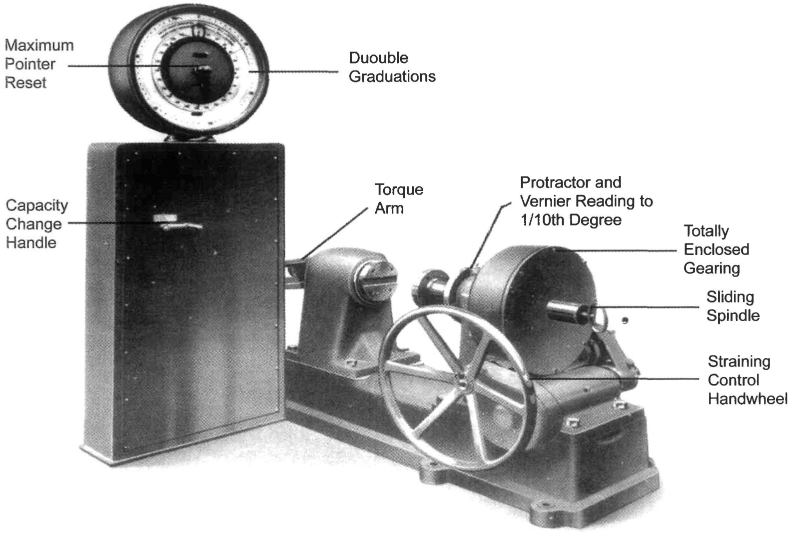

Figure A: Avery Torsion Testing Machine

torque arm, mounted on antifriction bearings, which transmitted a vertical pull to the indicating unit. The load indicator was of self indicating cam-resistant type and the die carried two sets of graduations with pointer and chart edge to edge to avoid parallax. Capacity could be controlled by means of hand lever and a maximum load pointer mounted on a separate spindle provided a record of breaking loads. For the purpose of testing and gripping of tubes, a holder had been designed and fabricated which could be fitted in four-jaw type self gripping chuck and attached with the face plate holder.

With the help of hand wheel, a small torque was applied to the specimen that was noted directly from the indicator and corresponding angle of twist was also noted with the help of Vernier and protractor fitted in the machine. The torque was gradually increased up to elasto-plastic regions and corresponding angle of twists were noted. Now the torque was released slowly to zero and the residual angles of twist (θ_R) of the deformed tubes were noted. These deflection readings were again checked with the help of combination set just after setting the torque to zero. The procedure was repeated for number of specimens. Springback percentage and residual angle to twist per unit length in percentage were calculated for each specimen with the help of equations (28) and (29).