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## ANALYSIS OF WAITING LINES AND SERVICE FACILITIES: A STUDY USING M/M/1 AND M/D/1 QUEUING MODELS

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### Abstract

The increase in vehicles nowadays has led to many problems on the road, especially traffic congestion. Traffic congestion usually occurs during the busy hour, for example, in the morning and evening. This might happen because of the busy city and fewer people want to commute using public transport. Therefore, this project aims to analyse the waiting lines and service facilities by applying two queuing models, M/M/1 and M/D/1, to overcome traffic congestion. The project took place at the traffic light located in front of UiTM Seremban Campus, where the data is collected during the morning time, which is from 7.00 am to 8.00 am. The data collection is conducted for four types of traffic, assigned as Traffic A, B, C, and D. The project's primary focus is to analyse the performance of M/M/1 and M/D/1 models regarding waiting times and queue lengths. This project will compare the performance of both models under similar conditions, such as the duration of traffic lights. As a result, this project has proven that the M/D/1 model consistently outperformed another model, M/M/1, in reducing traffic delays. M/D/1 has displayed a lower average for vehicle count and waiting time in the system and queue. In the future, the consideration of vehicle type can be highlighted because it could directly influence the waiting time at traffic lights.

**Keywords:** Queuing models, M/M/1 model, M/D/1 model

### Introduction

Traffic congestion has been frequently observed to be the major problem related to mobility in every country, and it has led to many problems for everyone, such as waste of time and energy and fuel consumption [1]. Queuing theory might help overcome this problem. The queuing theory is used to analyze the queues, where Danish Engineer A.K. Erlang founded the queuing theory in 1913 [2]. Queuing is generally used to address a few issues, typically related to balancing economics and effectiveness, requiring waiting and delays during serving customers or the maintenance of equipment [3]. In 2022, Shanmugasundaram and Sivaram mentioned that in a few queuing models, some preparations needed to be done before initiating a service, where this preparation is done for customers that might happen in hospitals, processes of production, banks, etc. [4].

However, a roadway's traffic characteristics are influenced by various factors, such as the surface type, roadway width, driver skill, side activities, road maintenance, etc. [5]. Thus, this project uses M/M/ and M/D/1 queuing models in operations research and performance analysis to describe systems within a traffic light. The M/M/1 model consists of a single server, where arrivals follow a Poisson process (Markovian, or "M"), service times are exponentially distributed (Markovian), and there is only one server (1). The queue operates under a first-

come, first-served (FCFS) discipline, and there is no limit on queue length. The M/D/1 model extends this by having D parallel servers instead of just one, allowing multiple customers to be served simultaneously. The presence of multiple servers reduces waiting times and congestion compared to the M/M/1 system, making it more efficient for handling higher traffic loads.

Therefore, to go into detail about this model, the traffic light in front of UiTM Kampus Seremban was selected for this project. The data collection was done during peak hours in the morning. It involved four types of traffic: Traffic A, B, C, and D. This project aimed to demonstrate the average vehicle count and time of waiting in the system and the queue by utilising both models. Another purpose of this analysis is to compare the two models by considering waiting times and the quantity of vehicles passing through the traffic signal.

## Methodology

This section indicates the method implemented in this project, including data collection and model formulation. It also explains the illustration of data collection in detail.

### *Illustration of The Traffic Light System Project at Persiaran Seremban Tiga*

Figure 1.1 shows the traffic light system project at Persiaran Seremban Tiga. The traffic lights in front of the Universiti Teknologi MARA (UiTM) Campus Seremban have provided four traffic lights. The first traffic assigned is Traffic A, which meets towards Rasah Jaya. The second traffic, known as Traffic B, meets towards the entrance gate of UiTM Seremban for vehicles that arrive from the University Commercial Centre. The data for Traffic A and Traffic B were taken over a 100-meter distance. Next, the third traffic was assigned as Traffic C, measured for vehicles arriving from Rasah Jaya towards Unicity Mall. Finally, the fourth traffic is measured for vehicles arriving from Rasah Jaya and approaching the UiTM Seremban entrance gate. As for Traffic C and Traffic D, the data was taken for 50 meters. Therefore, in this project, the data was taken during busy hours from 7.30 a.m. to 8.00 a.m., and there were about 5 times of traffic lights that were taken for 30 minutes. The arrival rate of vehicles for Traffic A, B, C and D is assigned as  $\lambda_A, \lambda_B, \lambda_C, \lambda_D$  respectively. The service rate of vehicles for Traffic A, B, C, and D is assigned as  $\mu_A, \mu_B, \mu_C, \mu_D$  respectively.

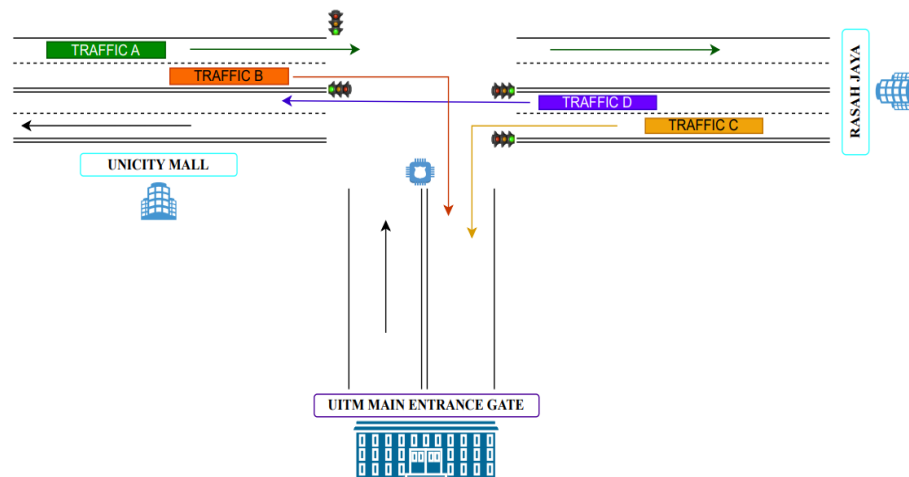


Figure 1.1 Illustration of Traffic Light in Front of UiTM Seremban Campus

**The Data collection**

Table 1.1 displays the data obtained during the morning session for an hour in Traffic A.

Table 1.1 Data Collection of Traffic A

Traffic A	Arrival number of vehicles, $\lambda_A$	Service number of vehicles, $\mu_A$	Waiting time (minutes)	Service time (minutes)
1	3	7	0.17	0.29
2	1	4	0.17	0.47
3	2	7	0.17	0.52
4	1	3	0.17	0.36
5	1	7	0.17	0.50

Table 1.2 displays the data observed during the one-hour morning session at Traffic B.

Table 1.2 Data Collection of Traffic B

Traffic B	Arrival number of vehicles, $\lambda_B$	Service number of vehicles, $\mu_B$	Waiting time (minutes)	Service time (minutes)
1	12	15	0.49	0.42
2	15	18	0.49	0.44
3	10	17	0.49	0.42
4	15	19	0.49	0.51
5	13	20	0.49	0.55

Table 1.3 below displays the data from an hour of observation in the morning session at Traffic C.

Table 1.3 Data Collection of Traffic C

Traffic C	Arrival number of vehicles, $\lambda_C$	Service number of vehicles, $\mu_C$	Waiting time (minutes)	Service time (minutes)
1	4	12	0.50	0.44
2	5	8	0.50	0.30
3	4	7	0.50	0.44
4	3	7	0.50	0.44
5	3	6	0.50	0.23

Table 1.4 exhibits the data obtained during the morning session for an hour at Traffic D.

Table 1.4 Data Collection of Traffic D

Traffic D	Arrival number of vehicles, $\lambda_D$	Service number of vehicles, $\mu_D$	Waiting time (minutes)	Service time (minutes)
1	12	16	1	0.45
2	7	9	1	0.28
3	11	11	1	0.40
4	9	11	1	0.43
5	9	9	1	0.41

Thus, Tables 1.1 to 1.4 show that the data collected are applied using the two queuing models, M/M/1 and M/D/1, to determine the waiting times and queue lengths.



### The Formulation of the Mathematical Model

Table 2.1 below summarises the formulas for the M/M/1 and M/D/1 models.

Terms	M/M/1	M/D/1
Average output rate, $\rho$	$\rho = \frac{\lambda}{\mu}$	$\rho = \frac{\lambda}{\mu}$
Average Vehicles Count in the System, $L_s$	$L_s = \frac{\lambda}{\mu - \lambda}$	$L_s = \frac{2\lambda\mu - \lambda^2}{2\mu(\mu - \lambda)}$
Average Vehicles Count in the Queue, $L_q$	$L_q = \frac{\lambda^2}{\mu(\mu - \lambda)}$	$L_q = \frac{\lambda^2}{2\mu(\mu - \lambda)}$
Average Time of Waiting in the System, $W_s$	$W_s = \frac{1}{\mu - \lambda}$	$W_s = \frac{2\mu - \lambda}{2\mu(\mu - \lambda)}$
Average Time of Waiting in the Queue, $W_q$	$W_q = \frac{\lambda}{\mu(\mu - \lambda)}$	$W_q = \frac{\lambda}{2\mu(\mu - \lambda)}$

### Results and Discussions

In this section, the table below provides the average vehicle count in the system,  $L_s$  Traffic A, B, C, and D were calculated using the M/M/1 and M/D/1 queuing models.

#### Average Vehicle Count in Queue, $L_q$

The average vehicle count in the system for both queuing models for all four traffic lights has been compared, as displayed in Table 3.1.

Traffic	M/M/1	M/D/1
Traffic A	2.81	3.04
Traffic B	2.70	1.22
Traffic C	0.59	0.58
Traffic D	0.54	0.47

According to Table 3.1, Traffic A had the most significant number of vehicles on average, 2.81 in M/M/1, whereas 3.04 in the M/D/1. Traffic B likewise shows a relatively high value of 2.70 in M/M/1 for the vehicle count on average but a lower value of 1.22 in M/D/1. Following that, there are fewer vehicles in Traffics C and D, with Traffic C recording 0.59 and 0.58 for M/M/1 and M/D/1, respectively. The lowest average, 0.54 in M/M/1 and 0.47 in M/D/1 was obtained by Traffic D. These results indicate how the M/D/1 queuing model can reduce the vehicle quantity in the system by handling traffic more effectively than the M/M/1 model.



### *Average Vehicle Count in Queue, $L_q$*

Table 3.2 provides the average vehicle count in a queue,  $L_q$  for Traffic A, B, C, and D.

Table 3.2 Average Vehicle Count in Queue,  $L_q$

<b>Traffic</b>	<b>M/M/1</b>	<b>M/D/1</b>
Traffic A	2.15	1.84
Traffic B	2.00	0.59
Traffic C	0.23	0.16
Traffic D	0.20	0.11

Based on Table 3.2, Traffic A records an average of 2.15 vehicles in the M/M/1 model, whereas it records 1.84 vehicles in the M/D/1 model. The smaller number of cars indicated in M/D/1 compared to M/M/1 shows that M/D/1 has a constant service rate. Nevertheless, there is a noticeable difference between the two models in Traffic B. After applying the M/M/1 model, it acquired an average of 2.00 vehicles; however, only 0.59 vehicles were in the M/D/1 model. Thus, the M/D/1 model has presented a lower average vehicle count, which might cause this model to have a deterministic rate of service that shortens the queue length for Traffic B. Compared to Traffic A and B, there are typically fewer cars queued up in Traffic C and D. Following Traffic C, the vehicle count average of 0.16 in M/D/1 and 0.23 in M/M/1 was obtained. However, in Traffic D, it obtained the value for vehicle number average of 0.20 and 0.11 in M/M/1 and M/D/1, respectively. After observing all the results, M/D/1 still indicates that it has obtained a lower average vehicle count in the queue than the M/M/1 model. Consequently, it is proven that the M/D/1 model consistently displays, in the queue, that it has fewer vehicles waiting than the M/M/1 model across all four traffic types. In conclusion, Traffic B for M/D/1 has a significantly lower number of vehicles in the queue.

### *Average Time of Waiting in System, $W_s$*

Table 3.3 shows the average time of waiting in the system,  $W_s$  for Traffic A, B, C and D.

Table 3.3 Average Time of Waiting in System,  $W_s$

<b>Traffic</b>	<b>M/M/1</b>	<b>M/D/1</b>
Traffic A	0.32	0.25
Traffic B	0.10	0.05
Traffic C	0.08	0.08
Traffic D	0.06	0.05

Table 3.3 shows that Traffic A experienced a longer waiting period than M/M/1, with an average waiting time of 0.32 seconds. However, Traffic A obtained a slightly shorter waiting time in M/D/1, 0.25 seconds. For Traffic B, the average difference is quite significant compared to Traffic A, where in M/D/1, it indicates a much shorter waiting time of 0.05 seconds; however, the M/M/1 model only obtained an average of 0.10 seconds. For Traffic C, both M/M/1 and M/D/1 models obtained the same waiting time of 0.08 seconds, respectively. However, for Traffic D, there was not much difference in average waiting time between the two models. Therefore, due to its deterministic character, the M/D/1 model has consistently shown that it can also reduce the waiting time in all traffic.



### *Average Time of Waiting in Queue, $W_q$*

Table 3.4 shows the queue's time of waiting,  $W_q$  for Traffic A, B, C, and D, which were calculated using a two-queuing model named M/M/1 and M/D/1.

Table 3.4 Average Time of Waiting in Queue,  $W_q$

<b>Traffic</b>	<b>M/M/1</b>	<b>M/D/1</b>
Traffic A	0.24	0.17
Traffic B	0.07	0.02
Traffic C	0.03	0.02
Traffic D	0.02	0.01

Table 3.4 indicates that the M/D/1 model continues to give consistently reduced average queue waiting times compared to the M/M/1 model for all types of traffic. The difference between the models indicates that the M/D/1 model features a deterministic service time, which is more effective and predictable than the random character of M/M/1 service times. Higher arrival rates in Traffic A resulted in the longest waiting times, with 0.24 seconds in M/M/1 and 0.17 seconds in M/D/1. Traffic D had the lowest waiting times, at 0.02 seconds in M/M/1 and 0.01 seconds in M/D/1. This could be due to improved service effectiveness and decreased traffic intensity. On the other hand, Traffic B, C, and D gradually display lower waiting times. For Traffic D, the slight difference in waiting times between the two models shows how the impact of the model selection decreases with lower traffic intensities.

### *Comparison of M/M/1 and M/D/1*

Table 3.5 summarizes M/M/1 and M/D/1 to determine the best model.

Table 3.5 Comparison of Total Average of Parameters for M/M/1 and M/D/1 Models

<b>Categories</b>	<b>M/M/1</b>	<b>M/D/1</b>
Total Average Vehicles Count in the System, $L_s$	1.66	1.33
Total Average Vehicles Count in the Queue, $L_q$	1.15	0.68
Total Average Time of Waiting in the System, $W_s$	0.14	0.11
Total Average Time of Waiting in the Queue, $W_q$	0.09	0.06

Table 3.5 shows that in the M/M/1 model, the total average vehicle count in the system decreases from 1.66 to 1.33 in the M/D/1 model. These changes have demonstrated that M/D/1, which has a feature of constant time of service, achieved a better traffic flow compared to the M/M/1 model. In the queue, the total average vehicle count in M/D/1 also displayed a lower average of 0.68, while 1.15 was on the M/M/1, which is still higher than the M/D/1. The lower average vehicle count in queues for M/D/1 has highlighted the consequences of constant service rates in reducing congestion, which has successfully decreased the vehicle count lining up in traffic. Applying the M/D/1 model also obtained a lower average of 0.11 seconds, whereas the M/M/1 model obtained a higher average of 0.14 seconds; thus, there is little difference. However, the M/D/1 model still shows a lower average value up to this point. This indicates that the constant time of service at random arrival rates, which the M/D/1 model has improved efficiency in handling vehicles within the system. Next, in the queue, as for the total average time of waiting,  $W_q$  also have a lesser value in M/D/1, which is 0.06 seconds; meanwhile, in



the M/M/1 model, it obtained an average of 0.09 seconds, slightly higher than the value for the waiting time that M/D/1 has acquired. The results obtained from these values have indicated that the model of M/D/1 consistently shows a capability to reduce delays and ensure smoother traffic flow.

As a summary, the queueing model of M/D/1 has consistently outperformed the M/M/1 model in solving traffic congestion and in all the metrics evaluated. M/D/1 queueing is characterized by a deterministic service time that has successfully decreased the average vehicle count in the system and queue, along with the average time of waiting in the system and queue. Therefore, it might help reduce the congestion at the traffic light in front of UiTM Kampus Seremban.

## Conclusion

This research analyses traffic congestion at the traffic light in front of UiTM Kampus Seremban using two queueing models, M/M/1 and M/D/1. Data was collected during peak morning hours across four traffic directions (A, B, C, and D) to evaluate vehicle count and waiting time. Results show that the M/D/1 model consistently outperformed M/M/1, as its constant service time led to lower waiting times and improved traffic flow compared to the random service times of M/M/1. In the future, it recommends implementing traffic management strategies based on M/D/1, such as consideration of vehicle type, which can be highlighted because it could directly influence the waiting time at traffic lights. Besides, promoting public transportation and investing in dedicated student transport can also alleviate traffic.

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