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APPLICATION OF THE RUNGE-KUTTA FOURTH ORDER (RK4) AND RUNGE-KUTTA FIFTH ORDER (RK5) IN DYNAMIC SYSTEMS: A FOCUS ON ELECTRICAL CIRCUIT ANALYSIS

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Abstract

This study examines the application of the Runge–Kutta Fourth Order (RK4) and Fifth Order (RK5) numerical methods in modelling dynamic electrical systems, focusing on series and parallel RLC circuits. Both approaches are widely recognized for solving ordinary differential equations (ODEs) with accuracy and efficiency, making them valuable when analytical solutions are difficult to obtain. The research compares the performance of RK4 and RK5 in simulating transient and steady-state behaviours under different damping conditions, including underdamped, overdamped, and critically damped cases. Mathematica was used as the computational platform to evaluate each method in terms of accuracy, stability, and computational cost. Findings show that RK4 offers a practical balance between simplicity and performance, making it suitable for general circuit analysis. In contrast, RK5 demonstrates superior precision, especially in scenarios where high accuracy is essential. By applying both methods to different circuit configurations, the study provides a well-rounded understanding of their capabilities and limitations in real-world applications. Overall, the results highlight the trade-offs between RK4 and RK5, offering engineers, researchers, and students clearer guidance in selecting the most suitable method. Choosing the appropriate approach enhances both the accuracy and efficiency of electrical circuit modelling.

Keywords*:* RK4, RK5, Mathematica, Circuit analysis

Introduction

Numerical methods play a vital role in solving ordinary differential equations (ODEs), particularly when obtaining exact analytical solutions is challenging or impractical. In electrical engineering, these techniques are essential for analyzing the time-dependent behavior of circuits, especially during sudden changes or transient events. While Euler's method, one of the earliest approaches to solving differential equations, remains a foundation for modern numerical techniques (Euler, 1768; Burden & Faires, 2011; Sauer, 2018). It often lacks the accuracy required for complex or high-precision systems (Chen et al., 2024).

Runge–Kutta methods, particularly the fourth-order (RK4) and fifth-order (RK5) variants, provide significantly improved accuracy and stability. By estimating slopes at multiple points within each time step, these methods are well-suited for studying real-time circuit responses (Ashgi et al., 2021; Ezhilarasi, 2023). The several current studies apply RK4 and RK5 to compare booth method in

RLC circuits (Godswill *et al.* (2025) and also implementing Runge-Kutta RK4 vs RK5 in RLC Transient Responses (Kafle *et al.* (2021). Although both methods are widely used, detailed comparative studies remain limited. Using Mathematica as the computational platform, this research evaluates the accuracy and efficiency of each method, aiming to offer practical guidance for engineers in selecting the most appropriate approach—particularly for nonlinear circuits or systems with rapidly changing signals (Shaikh et al., 2022; Ying et al., 2023).

Methodology

Equation (1) is the formula of the Runge-Kutta fourth-order method given by (Shaikh et al., 2022b) as below:

$$y'' = f(x, y) \tag{1}$$

$y(x_0) = y_0$ and $y'(x_0) = y'_0$

$$y_{i+1} = y_i + \frac{1}{6}(R1 + 2(R2 + R3) + R4) + O(h^5)$$

where,

$$R1 = hf(x_n, y_n)$$

$$R2 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}R1\right)$$

$$R3 = hf\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}R2\right)$$

$$R4 = hf(x_n + h, y_n + R3)$$

h represents the step size and $\boldsymbol{y_i}$ are the values of the dependent variables at the current step. R1, R3, and R4 are intermediate calculations based on the derivative function.

There exist several RK5 formulations, like equation (2), each suited for different applications. Some are given by the RK5 formula (Kafle et al., 2021).

$$y_{i+1} = y_i + \frac{h}{90}(7k_1 + 32k_3 + 32k_5 + 7k_6) \tag{2}$$

where,

$$t_{n+1} = t_n + h$$

$$k_1 = hf(t_n, y_n)$$

$$k_2 = f\left(t_n + \frac{h}{4}, y_n + \frac{h}{4}k_1\right)$$

$$k_3 = f\left(t_n + \frac{h}{4}, y_n + \frac{h}{8}k_1 + \frac{h}{8}k_2\right)$$

$$k_4 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2 + hk_3\right)$$

$$k_5 = f\left(t_n + \frac{3h}{4}, y_n + \frac{3h}{16}k_1 + \frac{9h}{16}k_4\right)$$

$$k_6 = f\left(t_n + h, y_n - \frac{3h}{7}k_1 + \frac{2h}{7}k_2 + \frac{12h}{7}k_3 - \frac{12h}{7}k_4 + \frac{8h}{7}k_5\right)$$

Also, the RK5 formula constructed by Kazeem Iyanda et al. (2021) as equation (3):

$$Y_{i+1} = Y_i + \frac{1}{192}(23k_1 + 125k_3 - 81k_5 + 125k_6) \tag{3}$$

where,

$$k_1 = hf(t_i, Y_i)$$

$$k_2 = f\left(t_i + \frac{h}{3}, Y_i + \frac{k_1}{3}\right)$$

$$k_3 = f\left(t_i + \frac{2h}{5}, Y_i + \frac{1}{25}(4k_1 + 6k_2)\right)$$

$$k_4 = f\left(t_i + h, Y_i + \frac{1}{4}(k_1 - 12k_2 + 15k_3)\right)$$

$$k_5 = f\left(t_i + \frac{2h}{3}, Y_i + \frac{1}{81}(6k_1 + 90k_2 - 50k_3 + 8k_4)\right)$$

$$k_6 = f\left(t_i + \frac{4h}{5}, Y_i + \frac{1}{75}(6k_1 + 36k_2 + 10k_3 + 8k_4)\right)$$

h is the step size, $\boldsymbol{x_n}$ are the values of the independent variables and $\boldsymbol{y_n}$ are the values of the dependent variables. k1, k3, k4, k5, and k6 are intermediate calculations based on the derivative function. This requires the use of numerical techniques to solve the governing differential equations of the circuit. Parameters for the circuit are given as resistance (R), inductance (L), and capacitance (C).
Classification of circuit response:

The damping factor (ξ)using equation (4):

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}} \tag{4}$$

Classify the system's response based on the damping force:

$\xi > 1$: Overdamped

$\xi = 1$:Critically damped

$\xi < 1$: Underdamped

This classification is crucial as it determines the nature of the transient response, ranging from convergence to steady state in overdamped systems to oscillatory behaviour in systems

Absolute error is calculated using the formula in equation (5) from (Shaikh et al., 2022):

$$|V_A - V_E| \tag{5}$$

$$V_A = 12 - [12\cos(8660.25t) + 6.39\sin(8660.25t)]e^{-5000t}$$

Results and Discussion

Figure 1 displays the voltage-time response of a series RLC circuit under underdamped conditions. The graph shows that the voltage oscillates with decreasing amplitude over time, which is typical of an underdamped system. Both the RK4 and RK5 methods in this study successfully capture this oscillatory behaviour.

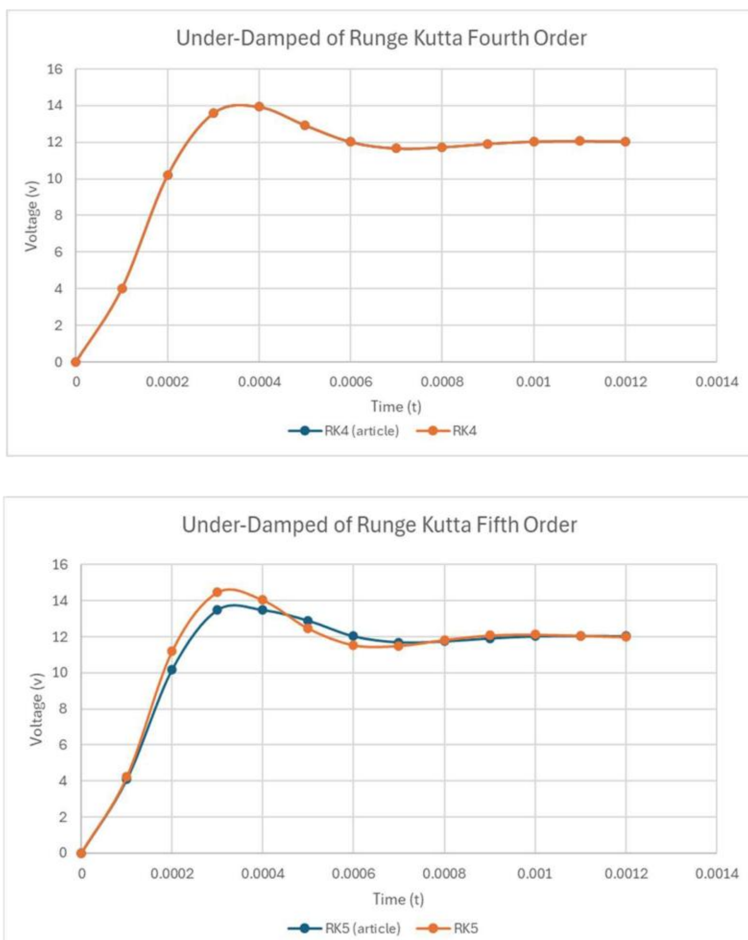

Figure 1: Voltage-Time Response of the Underdamped RLC Series Circuit

Table 1 shows the absolute error analysis for the underdamped RLC parallel circuit over a short simulation period from t=0.0000 to t=0.0012 seconds. The exact solution is compared against the RK4 and RK5 numerical methods at each time step to evaluate their accuracy.

t	Exact Solution	Abs Error	
		RK4	RK5
0.0000	0	0	0
0.0001	4.0828	0.44349	0.1447
0.0002	10.1925	1.03632	1.0215
0.0003	13.4920	0.0869	0.9744
0.0004	13.8375	0.0952	0.1951
0.0005	12.8952	0.0401	0.4289
0.0006	12.0276	0.0153	0.4899
0.0007	11.6923	0.0350	0.1972
0.0008	11.7481	0.0240	0.0778
0.0009	11.9152	0.00452	0.01588
0.0010	12.0260	0.00718	0.0934
0.0011	12.0517	0.00829	0.0033
0.0012	12.0310	0.00387	0.0389

Table 1: Error Analysis of RK4 and RK5 of RLC Series Circuit (Case I: Underdamped)

Figure 2 illustrates a visual comparison, which plots the voltage response of Euler, RK3, BRK5, RK4, and RK5 against time. The graph clearly shows that the RK5 curve remains closer to the reference (BRK5) throughout, indicating higher fidelity and better error control.

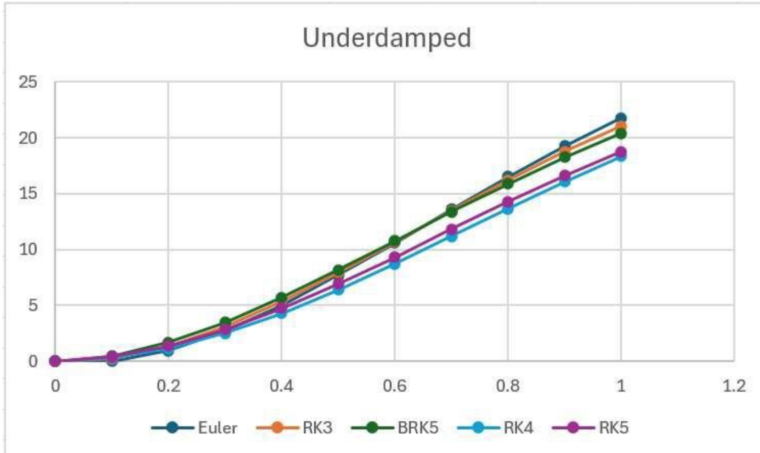

Figure 2: Voltage-Time Response of the Underdamped RLC Parallel Circuit.

Table 2 presents the absolute error analysis for the RLC parallel circuit under underdamped conditions (Case II). The errors were calculated by comparing the RK4 and RK5 results with the exact analytical solution over the time interval t=0 to t=1.0 seconds.

t	Exact Solution	Abs Error	
		RK4	RK5
0.0	0	0	0
0.1	0.73755	0.44349	0.34137
0.2	2.18195	1.03632	0.81197
0.3	4.15289	1.65848	1.30695
0.4	6.48369	2.21999	1.75112
0.5	9.02491	2.66072	2.09789
0.6	11.64659	2.94854	2.32655
0.7	14.23940	3.07625	2.43851
0.8	16.71490	3.05767	2.45295
0.9	19.00490	2.92302	2.40189
1.0	21.06029	2.71405	2.32534

Table 2 Absolute Error Analysis of RK4 and RK5 of RLC Parallel Circuit (Case II: Underdamped).

Conclusion

This study successfully applied and evaluated the Runge–Kutta Fourth Order (RK4) and Fifth Order (RK5) methods in solving differential equations that model the dynamic behavior of electrical circuits, with a focus on RLC configurations. Through simulations of transient and steady-state responses under underdamped, critically damped, and overdamped conditions, the results confirmed that both RK4 and RK5 are reliable and effective tools for circuit analysis.

RK4 emerged as a computationally efficient method, delivering accurate results with lower complexity—making it particularly suitable for real-time simulations and simpler circuit models. In contrast, RK5 consistently provided higher accuracy, proving advantageous in scenarios with rapid transients, stringent precision requirements, or nonlinear system dynamics. Despite its higher computational cost, RK5 demonstrated the ability to minimize absolute error over extended time intervals.

Overall, the findings highlight that RK4 is well-suited for fast, less resource-intensive applications, while RK5 is the method of choice for high-accuracy modeling of complex or sensitive electrical systems. This comparative insight can guide engineers and researchers in selecting the most appropriate approach based on the trade-off between computational efficiency and precision.

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