

LOGIC AND ELEMENTARY PROVING METHODS

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PREFACE

This book includes five chapters which aim to provide university-level students with a solid foundation in set theory, mathematical logic, and proof techniques. This edition seeks to equip students and educators with insightful, relevant information on the fundamentals of logical reasoning and introductory proof methods, accompanied by clear explanations of key concepts and principles underlying proof techniques.

We hope this book provides a better understanding for students and educators in teaching and learning set theory, logic, and proving techniques. To meet this objective, this book includes the following features:

- Detailed explanations about mathematical terms used in set operations.
- Discussion questions for each topic so students will reflect on what they have learnt, besides promoting active self-learning or group learning.
- Proper guidelines for selecting a proving method and definitions of terms related to proving methods. Based on our experience, students often struggle to prove propositions. To tackle this issue, we provide numerous examples accompanied by detailed explanations, including an analysis of each proof.
- In addition, this book provides many exercise questions at the end of each subtopic.
- We welcome any suggestions and comments on improving this book.

***MATHEMATICS is not about numbers, equations, computations, or algorithms.
It is about UNDERSTANDING.***

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CHAPTER 1

SET THEORY

1.1 SETS: BASIC DEFINITIONS AND NOTATIONS

A set is a collection of items, such as a group of numbers. The objects of a set are known as its elements. A set can be defined by listing its elements. For instance, $\{1, 3, 6, 10\}$ represents a set containing 1, 3, 6, and 10. This is referred to as the set's list format. Take notice of the curly braces. When defining a set by listing, always use curly brackets, $\{\}$.

Sets are commonly denoted with capital letters A, B, C, D , and so on. The notation $x \in A$ indicates that “ x is an element of A ” while $x \notin A$ denotes that “ x is not an element of A ”. Alternatively, “ x belongs to A ” or “ A contains x ” may be used.

You may have seen the sets **Z**, **N**, **W**, **Q** and **R** in earlier math classes as follows:

- The set of integers $\mathbf{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- The set of even integers $2\mathbf{Z} = \{\dots, 2, 4, 6, 8, 10, \dots\}$
- The set of natural numbers $\mathbf{N} = \{1, 2, 3, 4, 5, 6, \dots\}$
- The set of whole numbers $\mathbf{W} = \{0, 1, 2, 3, 4, \dots\}$
- The set of rational numbers $\mathbf{Q} = \{\dots, -5, \dots, 0, \dots, \frac{2}{5}, \dots, 1.25, \dots, \sqrt{36}, \dots\}$
- The set of real numbers \mathbf{R} is for all numbers (i.e., rational, irrational, integers, whole, and natural numbers) except complex numbers.

The set $\{2\}$ is regarded as being different from the number 2. A set of numbers is not a number. $\{2\}$ is a set with only one element, which is the number 2. But a set is not the same as the object it contains: $\{2\} \neq 2$. The statement $2 \in \{2\}$ is correct. The statement $\{2\} \in \{2\}$ is wrong.

LOGIC AND ELEMENTARY PROVING METHODS

This **Logic and Elementary Proving Methods** book is specially written for students of Universiti Teknologi MARA (UiTM) pursuing diploma and degree courses in Mathematics studies. This book is also suitable for students following off-campus courses and distance learning programs. Students from other higher learning institutes pursuing Mathematics courses can also use this book.



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