

# LOGIC AND ELEMENTARY PROVING METHODS

Shamsatun Nahar Ahmad

Rusnani Mohamad Khalid

Foo Fong Yeng

# TABLE OF CONTENTS

*List of Figures*

*List of Tables*

*Preface*

*Acknowledgements*

INTRODUCTION	INTRODUCTION	xv
LOGICAL IMPLICATIONS	LOGICAL IMPLICATIONS	ix
Conclusion by Truth Tables	Conclusion by Truth Tables	xii
Method	Method	xiii
Exercise 1.1	Exercise 1.1	1
Exercise 1.2	Exercise 1.2	2
Exercise 1.3	Exercise 1.3	3
Exercise 1.4	Exercise 1.4	4
Exercise 1.5	Exercise 1.5	5
THE SPECIALISATION	THE SPECIALISATION	1
Writing Analysis and Condensed Proof	Writing Analysis and Condensed Proof	2
Specialisation	Specialisation	3
Method	Method	4
Conclusion by Truth Tables	Conclusion by Truth Tables	5
Exercise 2.1	Exercise 2.1	6
Exercise 2.2	Exercise 2.2	7
Exercise 2.3	Exercise 2.3	8
Exercise 2.4	Exercise 2.4	9
THE LAW OF LOGIC	THE LAW OF LOGIC	10
Writing Analysis and Condensed Proof	Writing Analysis and Condensed Proof	11
Specialisation	Specialisation	12
Method	Method	13
Conclusion by Truth Tables	Conclusion by Truth Tables	14
Exercise 3.1	Exercise 3.1	15
Exercise 3.2	Exercise 3.2	16
Exercise 3.3	Exercise 3.3	17
Exercise 3.4	Exercise 3.4	18
THE LAW OF LOGIC	THE LAW OF LOGIC	19
Writing Analysis and Condensed Proof	Writing Analysis and Condensed Proof	20
Specialisation	Specialisation	21
Method	Method	22
Conclusion by Truth Tables	Conclusion by Truth Tables	23
Exercise 4.1	Exercise 4.1	24
Exercise 4.2	Exercise 4.2	25
Exercise 4.3	Exercise 4.3	26
Exercise 4.4	Exercise 4.4	27
Exercise 4.5	Exercise 4.5	28
Exercise 4.6	Exercise 4.6	29
Exercise 4.7	Exercise 4.7	30
Exercise 4.8	Exercise 4.8	31
Exercise 4.9	Exercise 4.9	32
Exercise 4.10	Exercise 4.10	33

## Chapter 1 SET THEORY

1.1	SETS: BASIC DEFINITIONS AND NOTATIONS	1
	Equal Set	2
	Empty Set	3
	Finite Set	4
	Infinite Set	4
	Exercise 1.1	5
1.2	SUBSETS	7
	Power Set	8
	Proper Subset	9
	Exercise 1.2	10
1.3	SET OPERATIONS	13
	Intersection of Sets	13
	Union of Sets	14
	Universal and Complement of Sets (Relative Complement Sets)	14
	Difference of Sets	16
	Exercise 1.3	19
1.4	LAWS OF SET THEORY	23
	Exercise 1.4	25
1.5	APPLICATIONS OF VENN DIAGRAM	26
	Exercise 1.5	33

# PREFACE

This book includes five chapters which aim to provide university-level students with a solid foundation in set theory, mathematical logic, and proof techniques. This edition seeks to equip students and educators with insightful, relevant information on the fundamentals of logical reasoning and introductory proof methods, accompanied by clear explanations of key concepts and principles underlying proof techniques.

We hope this book provides a better understanding for students and educators in teaching and learning set theory, logic, and proving techniques. To meet this objective, this book includes the following features:

- Detailed explanations about mathematical terms used in set operations.
- Discussion questions for each topic so students will reflect on what they have learnt, besides promoting active self-learning or group learning.
- Proper guidelines for selecting a proving method and definitions of terms related to proving methods. Based on our experience, students often struggle to prove propositions. To tackle this issue, we provide numerous examples accompanied by detailed explanations, including an analysis of each proof.
- In addition, this book provides many exercise questions at the end of each subtopic.
- We welcome any suggestions and comments on improving this book.

***MATHEMATICS is not about numbers, equations, computations, or algorithms.  
It is about UNDERSTANDING.***

**Shamsatun Nahar Ahmad  
Rusnani Mohamad Khalid  
Foo Fong Yeng**

# CHAPTER 1 SET THEORY

The set on the left is  $\{1, 3, 6, 10\}$ . Notice that the elements are 1, 3, and 6. The set on the right is  $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ . Notice that the elements of that set, the elements are also 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. Therefore, both sets are equal.

## 1.1 SETS: BASIC DEFINITIONS AND NOTATIONS

A set is a collection of items, such as a group of numbers. The objects of a set are known as its elements. A set can be defined by listing its elements. For instance,  $\{1, 3, 6, 10\}$  represents a set containing 1, 3, 6, and 10. This is referred to as the set's list format. Take notice of the curly braces. When defining a set by listing, always use curly brackets,  $\{\}$ .

Sets are commonly denoted with capital letters  $A, B, C, D$ , and so on. The notation  $x \in A$  indicates that "x is an element of A" while  $x \notin A$  denotes that "x is not an element of A". Alternatively, "x belongs to A" or "A contains x" may be used.

You may have seen the sets **Z**, **N**, **W**, **Q** and **R** in earlier math classes as follows:

- The set of integers  $\mathbf{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- The set of even integers  $2\mathbf{Z} = \{\dots, 2, 4, 6, 8, 10, \dots\}$
- The set of natural numbers  $\mathbf{N} = \{1, 2, 3, 4, 5, 6, \dots\}$
- The set of whole numbers  $\mathbf{W} = \{0, 1, 2, 3, 4, \dots\}$
- The set of rational numbers  $\mathbf{Q} = \{\dots, -5, \dots, 0, \dots, \frac{2}{5}, \dots, 1.25, \dots, \sqrt{36}, \dots\}$
- The set of real numbers  $\mathbf{R}$  is for all numbers (i.e., rational, irrational, integers, whole, and natural numbers) except complex numbers.

The set  $\{2\}$  is regarded as being different from the number 2. A set of numbers is not a number.  $\{2\}$  is a set with only one element, which is the number 2. But a set is not the same as the object it contains:  $\{2\} \neq 2$ . The statement  $2 \in \{2\}$  is correct. The statement  $\{2\} \in \{2\}$  is wrong.

# LOGIC AND ELEMENTARY PROVING METHODS

This **Logic and Elementary Proving Methods** book is specially written for students of Universiti Teknologi MARA (UiTM) pursuing diploma and degree courses in Mathematics studies. This book is also suitable for students following off-campus courses and distance learning programs. Students from other higher learning institutes pursuing Mathematics courses can also use this book.



**Shamsatun Nahar Ahmad** is a senior lecturer at the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA (UiTM), Johor branch, Segamat campus. She received her Diploma in Computer Sc. from Institut Teknologi MARA (ITM), Shah Alam, Bachelor of Computer Sc. with Education (Mathematics), MSc. (Mathematics), and Ph.D. (Mathematics) from Universiti Teknologi Malaysia (UTM). She has written papers in national and international indexed journals and proceedings, newspaper articles, books, book chapters, and anthology books. In addition to Mathematical Logic and Proving Techniques, she has taught courses including Calculus, Linear Algebra 1, Discrete Mathematics, and Ordinary Differential Equations.



**Rusnani Mohamad Khalid** is a lecturer at the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA (UiTM), Johor branch, Segamat campus. She received her Diploma in Science (Physics) from UiTM Jengka Pahang, BSc. (Hons) Mathematics and MSc. (Applied Mathematics) from UiTM Shah Alam. She has written newspaper articles, chapters in books, and papers in national and international journals and proceedings. In addition to Mathematical Logic and Proving Techniques, she has also taught other courses, including Calculus, Linear Algebra 1, Introduction to Probability and Statistics, and Business Mathematics.



**Foo Fong Yeng** is a senior lecturer at the Faculty of Computer and Mathematical Sciences, Universiti Teknologi MARA (UiTM), Johor branch, Pasir Gudang campus. She received her BSc. (Industrial Mathematics), MSc. (Mathematics), and Ph.D (Mathematics) from Universiti Teknologi Malaysia (UTM). She has written proceeding papers and journal articles at national and international levels. In addition to Mathematical Logic and Proving Techniques, she has taught courses including Calculus, Linear Algebra 1, Introduction to Probability and Statistics, and Business Mathematics.