

LINEAR ALGEBRA ESSENTIALS

A WORKBOOK FOR UNDERGRADUATES

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Preface

Linear Algebra Essentials: A Workbook for Undergraduates provides notes, samples of questions and solutions, and tutorials for six chapters. This workbook is designed to support and enhance the understanding of linear algebra fundamentals at the undergraduate level. It also offers a comprehensive approach in solving questions related to linear algebra. In addition, the use of Python is introduced at the end of the book.

The section in each chapter consists of:

1. Concise notes: The key concepts are presented in a clear and concise manner to assist in quick understanding. The notes serve as a quick reference for revising the fundamental ideas.
2. Sample questions and solutions: Examples are provided to illustrate the applications of concepts in solving problems in linear algebra. It has all the topics in each chapter. Detailed solutions are also provided in each sample problem.
3. Tutorials: The tutorials are designed to guide students through problems to enhance their analytical skills and conceptual understanding.

The goal of this workbook is to make linear algebra accessible and engaging. Regular practice and engagement with the content will help develop students' confidence and proficiency in linear algebra. In addition, using Python for solving linear algebra problems provides students with valuable computational skills that are essential in today's data-driven world.

We hope this workbook becomes an important resource for students in their mathematical journey.

Let's learn together!

Chapter 1

MATRICES

NOTES

INTRODUCTION TO MATRICES

Matrix

A matrix is a rectangular arrangement of m by n numbers that is enclosed within a bracket.

The numbers in a matrix are called its **elements** or **entries**. The entries or elements of the matrix are denoted by a_{ij} with the i^{th} rows and j^{th} column. In general, an $m \times n$ matrix A can be written as follows.

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} = (a_{ij})_{m \times n}$$

Transpose of a Matrix

For an $m \times n$ matrix A , its transpose, which is the matrix of $n \times m$ is

$$A^T = \begin{pmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{mn} \end{pmatrix}$$

Linear Algebra Essentials: A Workbook for Undergraduates is designed to make learning linear algebra enjoyable. It provides easy-to-understand notes, practical sample questions with step-by-step solutions, and helpful tutorials to build problem-solving skills. The workbook also introduces Python for elementary linear algebra problems while giving students valuable computational experiences. The workbook features interactive exercises and visual aids to enhance understanding. It also offers guidelines and strategies for effective studying. The aim is to boost confidence and proficiency in linear algebra through practice and engagement, making it accessible and relevant to students future careers. This comprehensive approach ensures that students grasp the fundamental concepts and appreciate the practical significance of elementary linear algebra.



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