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HAMMING DISTANCE OF GENERALISED L-R INTUITIONISTIC FUZZY NUMBER AND ITS APPLICATION

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ABSTRACT

This paper proposes a distance measure for generalised L-R intuitionistic fuzzy number (GLRIFN) which is Hamming distance, aiming to enhance the theoretical and practical tools available for decision-making under uncertainty. The properties of the Hamming distance of generalised L-R intuitionistic fuzzy number are also discussed in this study. GLRIFN extends traditional L-R intuitionistic fuzzy number by incorporating confidence level for both membership and non-membership functions, making them more reliable in the evaluation process. To demonstrate the practical utility of the proposed measure, it is applied within the Generalised L-R Intuitionistic Fuzzy Technique for Order of Preference by Similarity to Ideal Solution (GLRIF-TOPSIS), a multi-criteria decision-making (MCDM) method. A real-world case study on river water pollution classification is conducted, wherein the proposed model effectively evaluates the pollution levels of different rivers by capturing the nuances of imprecise, vague, and conflicting environmental data. The results show that the River J_5 *is the cleanest river, while the River* J_1 *is the most polluted river. The integration of the Hamming distance with GLRIF-TOPSIS offers a structured and adaptable decision-making framework, capable of addressing complex multi-criteria problems across domains characterised by high levels of ambiguity. This contribution not only enriches the existing body of fuzzy set theory but also opens avenues for further applications in environmental assessment and other areas that require robust fuzzy modeling.*

Keywords: *Hamming Distance, Intuitionistic Fuzzy Number, L-R Type, Water Pollution.*

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1. Introduction

In recent years, fuzzy set theory has emerged as a vital tool for modeling and solving problems characterised by uncertainty and imprecision (Klir, 2006; Klir & Yuan, 1995). The real world often presents situations where exact data is unavailable or difficult to obtain, especially in areas such as environmental management, social sciences, and engineering (Ameen et al., 2019). Traditional mathematical models, which rely on precise input, are often inadequate in such scenarios. In contrast, fuzzy set theory provides a flexible means of representing and processing imprecise information, making it a powerful tool for complex decision-making processes (Tarmudi et al., 2024).

Among its extensions of fuzzy sets, the intuitionistic fuzzy set, introduced by Atanassov (1986), has gained significant traction due to its ability to capture membership and non-membership degrees. This triadic structure enables more nuanced evaluations and captures the uncertainty of information more effectively. The intuitionistic fuzzy set offers a more comprehensive evaluation for addressing complex decision-making and analysis tasks compared to fuzzy set (Atanassov, 1986).

Building on the foundation laid by intuitionistic fuzzy sets, researchers have developed more advanced representations to address increasingly complex decision environments. One such advancement is the Generalised L-R Intuitionistic Fuzzy Number (GLRIFN), as proposed by Shafie et al. (2023). GLRIFN extends the intuitionistic fuzzy number by introducing non-linear L-R type membership and non-membership functions. More importantly, they incorporate the confidence levels of decision-makers, allowing for a more realistic representation of uncertainty. This makes GLRIFN particularly suitable for real-world applications where expert opinions, preferences, and the credibility of information must be taken into account.

Despite the growing importance of GLRIFN, a critical component within this framework remains underdeveloped, which is the distance measure. The concept of distance measures is pivotal in fuzzy set theory, as it facilitates the comparison of fuzzy numbers and supports various applications, including pattern recognition (Khan et al., 2024; Naranjo et al., 2021; Zhu et al., 2024), decision-making (Ardil, 2023; Deli & Keleş, 2021; Wang et al., 2020), optimisation (Keikha & Sabeghi, 2024; Kumar & Sharma, 2023; Sahu et al., 2021), and et cetera. In the context of MCDM, distance measures help evaluate and rank alternatives based on their proximity to ideal solutions (Yazid et al., 2023).

While several distance measures have been proposed for classical fuzzy numbers and even for intuitionistic fuzzy sets (Aguilar-Peña et al., 2016; Guha & Chakraborty, 2010), very few have been specifically tailored for GLRIFN. This presents a significant gap in the literature, as the unique structure of GLRIFN, especially its non-linear functions and embedded confidence levels which demand a more suitable approach. A generic distance measure may fail to capture the intricacies of GLRIFN, leading to suboptimal or inaccurate outcomes in decision-making models.

Therefore, this study proposes a new Hamming distance measure designed specifically for GLRIFN. The Hamming distance, known for its simplicity and effectiveness, is adapted here to accommodate the characteristics of GLRIFN. This is due to the fact that when Hamming distance is adapted to the structure of GLRIFN, the Hamming distance effectively captures variations in membership and non-membership degrees while also accommodating the embedded confidence levels of decision-makers. This ensures that the distinctive characteristics of GLRIFN are fully preserved in comparative analysis. Hence, the proposed Hamming distance not only fills the methodological gap in measuring similarity among GLRIFNs but also enhances decision-making accuracy in environments where uncertainty, vagueness, and conflicting information are prevalent.

In addition to formally defining this measure, this study examines its mathematical properties and demonstrates its practical utility by integrating it into the Generalised L-R Intuitionistic Fuzzy Technique for Order Preference by Similarity to Ideal Solution (GLRIF-TOPSIS), a robust MCDM method. A real-world case study on river water pollution classification is presented to validate the model's applicability, showcasing how the proposed measure can effectively manage vague and conflicting data in environmental decision-making.

This study contributes to the growing body of knowledge in fuzzy decision-making by enhancing the theoretical foundation of GLRIFN and expanding their utility in solving complex, real-world problems. This paper is organised in the following manner. This section provides the introduction and background of the study. Next section defines and discusses some of the preliminaries along with the proposed method of Hamming distance of GLRIFN. The following section implements the proposed method in the river water pollution classification. The next section discusses the results and the discussion of the findings. Finally, the last section concludes this study.

2. Materials and Methods

This section includes the preliminary concepts and theories on membership functions of generalised L-R intuitionistic fuzzy number (GLRIFN), Hamming distance for triangular fuzzy numbers, and Hamming distance for triangular intuitionistic fuzzy numbers. This section also discusses the proposed method of Hamming distance of GLRIFN.

2.1 Preliminaries

The definitions of the membership function of GLRIFN, the Hamming distance for triangular fuzzy numbers, and the Hamming distance for triangular intuitionistic fuzzy numbers are given as follows:

Definition 1 (Shafie et al., 2023) A generalised L-R intuitionistic fuzzy number $A=\left(p_a,q_a;p'_a,q'_a;\alpha_a,\beta_a;\alpha'_a,\beta'_a;\omega_a;\omega'_a\right)_{LR}$ defined by membership and non-membership functions $A=\left\{\left\langle x,\mu_A(x),\nu_A(x)\right\rangle \middle| x\in X\right\}$ with the condition $0\le\omega_a+\omega'_a\le 1$, where

$$\mu_A(x)=\begin{cases}\omega_a\cdot L\left(\dfrac{p_a-x}{\alpha_a}\right) & : \quad -\infty\le x\le p_a\\ \omega_a & : \quad p_a\le x\le n_a\\ \omega_a\cdot R\left(\dfrac{x-q_a}{\beta_a}\right) & : \quad q_a\le x\le+\infty\end{cases}$$
$$\nu_A(x)=\begin{cases}1-\left(1-\omega'_a\right)\cdot L\left(\dfrac{p'_a-x}{\alpha'_a}\right) & : \quad -\infty\le x\le p'_a\\ \omega_a' & : \quad p'_a\le x\le q'_a\\ 1-\left(1-\omega'_a\right)\cdot R\left(\dfrac{x-q'_a}{\beta'_a}\right) & : \quad q'_a\le x\le+\infty\end{cases} \tag{1}$$

such that $p_a,q_a,p'_a,q'_a,\alpha_a,\beta_a,\alpha'_a,\beta'_a\in\mathbb{R}$, $p_a\le q_a$, $p'_a\le q'_a$, $\omega_a\in(0,1]$, and $\omega'_a\in[0,1)$.

Definition 2 (Izadikhah, 2009) Let $A=\left(a_1,b_1,c_1\right)$ and $B=\left(a_2,b_2,c_2\right)$ be two triangular fuzzy numbers, then the Hamming distance is defined as

$$d(A,B)=\int_R\left|\mu_A(x)-\mu_B(x)\right|dx \tag{2}$$

Definition 3 (Aikhuele, 2021; Wan et al., 2017) Let $A=\left(a_1,a_2,a_3;w_a,u_a\right)$ and $B=\left(b_1,b_2,b_3;w_b,u_b\right)$ be two triangular intuitionistic fuzzy numbers. The Hamming distance between A and B is defined as

$$d\left(A,B\right)=\frac{1}{6}\left[\begin{array}{c}\left|\left(1+w_a-u_a\right)a_1-\left(1+w_b-u_b\right)b_1\right|+\\ \left|\left(1+w_a-u_a\right)a_2-\left(1+w_b-u_b\right)b_2\right|+\\ \left|\left(1+w_a-u_a\right)a_3-\left(1+w_b-u_b\right)b_3\right|\end{array}\right] \tag{3}$$

where the confidence level of triangular intuitionistic fuzzy number is $0\le w+u\le 1$.

2.2 Hamming Distance of Generalised L-R Intuitionistic Fuzzy Number

The Hamming distance is a metric for comparing two binary data (Norouzi et al., 2012; Zhang et al., 2013). The Hamming distance calculates the number of positions at which corresponding elements in the two GLRIFNs differ. The Hamming distance can be adapted to assess the distance in terms of the positions at which corresponding elements in two GLRIFNs differ. This distance measure provides a straightforward and interpretable way to compare GLRIFNs in such structured scenarios. Hence, the definition of Hamming distance of GLRIFNs is as follows:

Definition 4 Suppose two generalised L-R intuitionistic fuzzy numbers (GLRIFNs) $A=\left(p_a,q_a;p'_a,q'_a;\alpha_a,\beta_a;\alpha'_a,\beta'_a;\omega_a;\omega'_a\right)_{LR}$ and $B=\left(p_b,q_b;p'_b,q'_b;\alpha_b,\beta_b;\alpha'_b,\beta'_b;\omega_b;\omega'_b\right)_{LR}$ or can be written as $A=\left(p_a-\alpha_a,p_a,q_a,q_a+\beta_a;p'_a-\alpha'_a,p'_a,q'_a,q'_a+\beta'_a;\omega_a;\omega'_a\right)_{LR}$ and $B=\left(p_b-\alpha_b,p_b,q_b,q_b+\beta_b;p'_b-\alpha'_b,p'_b,q'_b,q'_b+\beta'_b;\omega_b;\omega'_b\right)_{LR}$. Thus, the Hamming distance between A and B can be calculated if $L_A\left(x\right)=L_B\left(x\right)$ and $R_A\left(x\right)=R_B\left(x\right)$ for the left and right reference functions respectively. The Hamming distance of GLRIFNs is defined as

$$d^H_{GLRIFNs}\left(A,B\right)=\frac{1}{8}\left[\begin{array}{c}\left|\left(\omega_a\right)\left(p_a-\alpha_a\right)-\left(\omega_b\right)\left(p_b-\alpha_b\right)\right|+\\ \left|\left(\omega_a\right)\left(p_a\right)-\left(\omega_b\right)\left(p_b\right)\right|+\left|\left(\omega_a\right)\left(q_a\right)-\left(\omega_b\right)\left(q_b\right)\right|+\\ \left|\left(\omega_a\right)\left(q_a+\beta_a\right)-\left(\omega_b\right)\left(q_b+\beta_b\right)\right|+\\ \left|\left(1-\omega'_a\right)\left(p'_a-\alpha'_a\right)-\left(1-\omega'_b\right)\left(p'_b-\alpha'_b\right)\right|+\\ \left|\left(1-\omega'_a\right)\left(p'_a\right)-\left(1-\omega'_b\right)\left(p'_b\right)\right|+\\ \left|\left(1-\omega'_a\right)\left(q'_a\right)-\left(1-\omega'_b\right)\left(q'_b\right)\right|+\\ \left|\left(1-\omega'_a\right)\left(q'_a+\beta'_a\right)-\left(1-\omega'_b\right)\left(q'_b+\beta'_b\right)\right|\end{array}\right]. \tag{4}$$

The distance measure of $d^H_{GLRIFNs}\left(A,B\right)$ between A and B satisfies the following propositions.

Proposition 1 If both A and B are GLRIFNs, then the distance measurement $d^H_{GLRIFNs}\left(A,B\right)$ is identical to the Hamming distance.

Proof:

Suppose that both $A=\left(p_a-\alpha_a,p_a,q_a,q_a+\beta_a;p'_a-\alpha'_a,p'_a,q'_a,q'_a+\beta'_a;\omega_a;\omega'_a\right)_{LR}$ and $B=\left(p_b-\alpha_b,p_b,q_b,q_b+\beta_b;p'_b-\alpha'_b,p'_b,q'_b,q'_b+\beta'_b;\omega_b;\omega'_b\right)_{LR}$ are two GLRIFNs, then let $p_a-\alpha_a=p_a=q_a=q_a+\beta_a=p'_a-\alpha'_a=p'_a=q'_a=q'_a+\beta'_a=\overline{A}$, $p_b-\alpha_b=p_b=q_b=q_b+\beta_b=p'_b-\alpha'_b=p'_b=q'_b=q'_b+\beta'_b=\overline{B}$, $\omega_a=\omega_b=\omega=1$, and $\omega'_a=\omega'_b=\omega'=0$. The distance measurement $d^H_{GLRIFNs}\left(A,B\right)$ can be calculated as:

$$\begin{aligned} d^H_{GLRIFNs}\left(A,B\right)&=\frac{1}{8}\left[\begin{array}{c} \left|(\omega)\overline{A}-(\omega)\overline{B}\right|+\left|(\omega)\overline{A}-(\omega)\overline{B}\right|+ \\ \left|(\omega)\overline{A}-(\omega)\overline{B}\right|+\left|(\omega)\overline{A}-(\omega)\overline{B}\right|+ \\ \left|(1-\omega')\overline{A}-(1-\omega')\overline{B}\right|+\left|(1-\omega')\overline{A}-(1-\omega')\overline{B}\right|+ \\ \left|(1-\omega')\overline{A}-(1-\omega')\overline{B}\right|+\left|(1-\omega')\overline{A}-(1-\omega')\overline{B}\right| \end{array}\right] \\ &=\left|\overline{A}-\overline{B}\right|. \end{aligned}$$

Proposition 2 Two GLRIFNs $A=B$ if and only if $d^H_{GLRIFNs}\left(A,B\right)=0$.

Proof:

Let $A=\left(p_a-\alpha_a,p_a,q_a,q_a+\beta_a;p'_a-\alpha'_a,p'_a,q'_a,q'_a+\beta'_a;\omega_a;\omega'_a\right)_{LR}$ and $B=\left(p_b-\alpha_b,p_b,q_b,q_b+\beta_b;p'_b-\alpha'_b,p'_b,q'_b,q'_b+\beta'_b;\omega_b;\omega'_b\right)_{LR}$ be two GLRIFNs. If $A=B$, then $p_a-\alpha_a=p_b-\alpha_b$, $p_a=p_b$, $q_a=q_b$, $q_a+\beta_a=q_b+\beta_b$, $p'_a-\alpha'_a=p'_b-\alpha'_b$, $p'_a=p'_b$, $q'_a=q'_b$, $q'_a+\beta'_a=q'_b+\beta'_b$, $\omega_a=\omega_b$, and $\omega'_a=\omega'_b$. Thus, the distance between A and B is

$$d^H_{GLRIFNs}\left(A,B\right)=\frac{1}{8}\left[\begin{array}{c} \left|(\omega_a)(p_a-\alpha_a)-(\omega_b)(p_b-\alpha_b)\right|+\left|(\omega_a)(p_a)-(\omega_b)(p_b)\right|+ \\ \left|(\omega_a)(q_a)-(\omega_b)(q_b)\right|+\left|(\omega_a)(q_a+\beta_a)-(\omega_b)(q_b+\beta_b)\right|+ \\ \left|(1-\omega'_a)(p'_a-\alpha'_a)-(1-\omega'_b)(p'_b-\alpha'_b)\right|+ \\ \left|(1-\omega'_a)(p'_a)-(1-\omega'_b)(p'_b)\right|+ \\ \left|(1-\omega'_a)(q'_a)-(1-\omega'_b)(q'_b)\right|+ \\ \left|(1-\omega'_a)(q'_a+\beta'_a)-(1-\omega'_b)(q'_b+\beta'_b)\right| \end{array}\right]=0.$$

Conversely, if $d^H_{GLRIFNs}\left(A,B\right)=0$, then the distance between A and B is

$$d_{GLRIFNs}^{H}(A,B)=\frac{1}{8}\begin{bmatrix}\left|(\omega_a)(p_a-\alpha_a)-(\omega_b)(p_b-\alpha_b)\right|+\\ \left|(\omega_a)(p_a)-(\omega_b)(p_b)\right|+\left|(\omega_a)(q_a)-(\omega_b)(q_b)\right|+\\ \left|(\omega_a)(q_a+\beta_a)-(\omega_b)(q_b+\beta_b)\right|+\\ \left|(1-\omega'_a)(p'_a-\alpha'_a)-(1-\omega'_b)(p'_b-\alpha'_b)\right|+\\ \left|(1-\omega'_a)(p'_a)-(1-\omega'_b)(p'_b)\right|+\\ \left|(1-\omega'_a)(q'_a)-(1-\omega'_b)(q'_b)\right|+\\ \left|(1-\omega'_a)(q'_a+\beta'_a)-(1-\omega'_b)(q'_b+\beta'_b)\right|\end{bmatrix}=0.$$

It follows that $p_a-\alpha_a=p_b-\alpha_b$, $p_a=p_b$, $q_a=q_b$, $q_a+\beta_a=q_b+\beta_b$, $p'_a-\alpha'_a=p'_b-\alpha'_b$, $p'_a=p'_b$, $q'_a=q'_b$, $q'_a+\beta'_a=q'_b+\beta'_b$, $\omega_a=\omega_b$, and $\omega'_a=\omega'_b$. Therefore, two GLRIFNs A and B are identical.

Proposition 3 If A and B are two GLRIFNs. The distance $d_{GLRIFNs}^{H}(A,B)=d_{GLRIFNs}^{H}(B,A)$.

Proof:

Let $A=(p_a-\alpha_a,p_a,q_a,q_a+\beta_a;p'_a-\alpha'_a,p'_a,q'_a,q'_a+\beta'_a;\omega_a;\omega'_a)_{LR}$ and $B=(p_b-\alpha_b,p_b,q_b,q_b+\beta_b;p'_b-\alpha'_b,p'_b,q'_b,q'_b+\beta'_b;\omega_b;\omega'_b)_{LR}$ be two GLRIFNs. The distance of $d_{GLRIFNs}^{H}(A,B)$ is equal to the distance of $d_{GLRIFNs}^{H}(B,A)$. It is obvious that

$$\begin{gathered}(\omega_a)(p_a-\alpha_a)-(\omega_b)(p_b-\alpha_b)\neq(\omega_b)(p_b-\alpha_b)-(\omega_a)(p_a-\alpha_a),\\ (\omega_a)(p_a)-(\omega_b)(p_b)\neq(\omega_b)(p_b)-(\omega_a)(p_a),\ (\omega_a)(q_a)-(\omega_b)(q_b)\neq(\omega_b)(q_b)-(\omega_a)(q_a),\\ (\omega_a)(q_a+\beta_a)-(\omega_b)(q_b+\beta_b)\neq(\omega_b)(q_b+\beta_b)-(\omega_a)(q_a+\beta_a),\\ (1-\omega'_a)(p'_a-\alpha'_a)-(1-\omega'_b)(p'_b-\alpha'_b)\neq(1-\omega'_b)(p'_b-\alpha'_b)-(1-\omega'_a)(p'_a-\alpha'_a),\\ (1-\omega'_a)(p'_a)-(1-\omega'_b)(p'_b)\neq(1-\omega'_b)(p'_b)-(1-\omega'_a)(p'_a),\\ (1-\omega'_a)(q'_a)-(1-\omega'_b)(q'_b)\neq(1-\omega'_b)(q'_b)-(1-\omega'_a)(q'_a),\text{ and}\\ (1-\omega'_a)(q'_a+\beta'_a)-(1-\omega'_b)(q'_b+\beta'_b)\neq(1-\omega'_b)(q'_b+\beta'_b)-(1-\omega'_a)(q'_a+\beta'_a).\end{gathered}$$

But,

$$\begin{gathered}\left|(\omega_a)(p_a-\alpha_a)-(\omega_b)(p_b-\alpha_b)\right|=\left|(\omega_b)(p_b-\alpha_b)-(\omega_a)(p_a-\alpha_a)\right|,\\ \left|(\omega_a)(p_a)-(\omega_b)(p_b)\right|=\left|(\omega_b)(p_b)-(\omega_a)(p_a)\right|,\\ \left|(\omega_a)(q_a)-(\omega_b)(q_b)\right|=\left|(\omega_b)(q_b)-(\omega_a)(q_a)\right|,\\ \left|(\omega_a)(q_a+\beta_a)-(\omega_b)(q_b+\beta_b)\right|=\left|(\omega_b)(q_b+\beta_b)-(\omega_a)(q_a+\beta_a)\right|,\\ \left|(1-\omega'_a)(p'_a-\alpha'_a)-(1-\omega'_b)(p'_b-\alpha'_b)\right|=\left|(1-\omega'_b)(p'_b-\alpha'_b)-(1-\omega'_a)(p'_a-\alpha'_a)\right|,\\ \left|(1-\omega'_a)(p'_a)-(1-\omega'_b)(p'_b)\right|=\left|(1-\omega'_b)(p'_b)-(1-\omega'_a)(p'_a)\right|,\\ \left|(1-\omega'_a)(q'_a)-(1-\omega'_b)(q'_b)\right|=\left|(1-\omega'_b)(q'_b)-(1-\omega'_a)(q'_a)\right|,\text{ and}\\ \left|(1-\omega'_a)(q'_a+\beta'_a)-(1-\omega'_b)(q'_b+\beta'_b)\right|=\left|(1-\omega'_b)(q'_b+\beta'_b)-(1-\omega'_a)(q'_a+\beta'_a)\right|.\end{gathered}$$

Hence,

$$d^{H}_{GLRIFNs}\left(A,B\right)=d^{H}_{GLRIFNs}\left(B,A\right)$$

Therefore, $d^{H}_{GLRIFNs}\left(A,B\right)=d^{H}_{GLRIFNs}\left(B,A\right)$ has been proved.

Proposition 4 If A, B, and C are three GLRIFNs. The distance between A and C is $d^{H}_{GLRIFNs}\left(A,C\right)\le d^{H}_{GLRIFNs}\left(A,B\right)+d^{H}_{GLRIFNs}\left(B,C\right)$.

Proof:

Let
$A=\left(p_a-\alpha_a,p_a,q_a,q_a+\beta_a;p'_a-\alpha'_a,p'_a,q'_a,q'_a+\beta'_a;\omega_a;\omega'_a\right)_{LR}$,
$B=\left(p_b-\alpha_b,p_b,q_b,q_b+\beta_b;p'_b-\alpha'_b,p'_b,q'_b,q'_b+\beta'_b;\omega_b;\omega'_b\right)_{LR}$,
$C=\left(p_c-\alpha_c,p_c,q_c,q_c+\beta_c;p'_c-\alpha'_c,p'_c,q'_c,q'_c+\beta'_c;\omega_c;\omega'_c\right)_{LR}$ and be three GLRIFNs.
Since $A-C=\left(A-B\right)+\left(B-C\right)$, it is obvious that

$$\left|\left(\omega_a\right)\left(p_a-\alpha_a\right)-\left(\omega_c\right)\left(p_c-\alpha_c\right)\right|\le\left|\left(\omega_a\right)\left(p_a-\alpha_a\right)-\left(\omega_b\right)\left(p_b-\alpha_b\right)\right|+\left|\left(\omega_b\right)\left(p_b-\alpha_b\right)-\left(\omega_c\right)\left(p_c-\alpha_c\right)\right|,$$

$$\left|\left(\omega_a\right)\left(p_a\right)-\left(\omega_c\right)\left(p_c\right)\right|\le\left|\left(\omega_a\right)\left(p_a\right)-\left(\omega_b\right)\left(p_b\right)\right|+\left|\left(\omega_b\right)\left(p_b\right)-\left(\omega_c\right)\left(p_c\right)\right|,$$

$$\left|\left(\omega_a\right)\left(q_a\right)-\left(\omega_c\right)\left(q_c\right)\right|\le\left|\left(\omega_a\right)\left(q_a\right)-\left(\omega_b\right)\left(q_b\right)\right|+\left|\left(\omega_b\right)\left(q_b\right)-\left(\omega_c\right)\left(q_c\right)\right|,$$

$$\left|\left(\omega_a\right)\left(q_a+\beta_a\right)-\left(\omega_c\right)\left(q_c+\beta_c\right)\right|\le\left|\left(\omega_a\right)\left(q_a+\beta_a\right)-\left(\omega_b\right)\left(q_b+\beta_b\right)\right|+\left|\left(\omega_b\right)\left(q_b+\beta_b\right)-\left(\omega_c\right)\left(q_c+\beta_c\right)\right|,$$

$$\left|\begin{matrix}\left(1-\omega'_a\right)\left(p'_a-\alpha'_a\right)-\\ \left(1-\omega'_c\right)\left(p'_c-\alpha'_c\right)\end{matrix}\right|\le\left|\begin{matrix}\left(1-\omega'_a\right)\left(p'_a-\alpha'_a\right)-\\ \left(1-\omega'_b\right)\left(p'_b-\alpha'_b\right)\end{matrix}\right|+\left|\begin{matrix}\left(1-\omega'_b\right)\left(p'_b-\alpha'_b\right)-\\ \left(1-\omega'_c\right)\left(p'_c-\alpha'_c\right)\end{matrix}\right|,$$

$$\left|\left(1-\omega'_a\right)\left(p'_a\right)-\left(1-\omega'_c\right)\left(p'_c\right)\right|\le\left|\left(1-\omega'_a\right)\left(p'_a\right)-\left(1-\omega'_b\right)\left(p'_b\right)\right|+\left|\left(1-\omega'_b\right)\left(p'_b\right)-\left(1-\omega'_c\right)\left(p'_c\right)\right|,$$

$$\left|\left(1-\omega'_a\right)\left(q'_a\right)-\left(1-\omega'_c\right)\left(q'_c\right)\right|\le\left|\left(1-\omega'_a\right)\left(q'_a\right)-\left(1-\omega'_b\right)\left(q'_b\right)\right|+\left|\left(1-\omega'_b\right)\left(q'_b\right)-\left(1-\omega'_c\right)\left(q'_c\right)\right|,$$

$$\text{and }\left|\begin{matrix}\left(1-\omega'_a\right)\left(q'_a+\beta'_a\right)-\\ \left(1-\omega'_c\right)\left(q'_c+\beta'_c\right)\end{matrix}\right|\le\left|\begin{matrix}\left(1-\omega'_a\right)\left(q'_a+\beta'_a\right)-\\ \left(1-\omega'_b\right)\left(q'_b+\beta'_b\right)\end{matrix}\right|+\left|\begin{matrix}\left(1-\omega'_b\right)\left(q'_b+\beta'_b\right)-\\ \left(1-\omega'_c\right)\left(q'_c+\beta'_c\right)\end{matrix}\right|.$$

Hence,

$$
\frac{1}{8}\begin{bmatrix}
\left|(\omega_a)(p_a-\alpha_a)-(\omega_c)(p_c-\alpha_c)\right|+ \\
\left|(\omega_a)(p_a)-(\omega_c)(p_c)\right|+\left|(\omega_a)(q_a)-(\omega_c)(q_c)\right|+ \\
\left|(\omega_a)(q_a+\beta_a)-(\omega_c)(q_c+\beta_c)\right|+ \\
\left|(1-\omega'_a)(p'_a-\alpha'_a)-(1-\omega'_c)(p'_c-\alpha'_c)\right|+ \\
\left|(1-\omega'_a)(p'_a)-(1-\omega'_c)(p'_c)\right|+ \\
\left|(1-\omega'_a)(q'_a)-(1-\omega'_c)(q'_c)\right|+ \\
\left|(1-\omega'_a)(q'_a+\beta'_a)-(1-\omega'_c)(q'_c+\beta'_c)\right|
\end{bmatrix}
$$

$$
\leq\frac{1}{8}\begin{bmatrix}
\left|(\omega_a)(p_a-\alpha_a)-(\omega_b)(p_b-\alpha_b)\right|+ \\
\left|(\omega_a)(p_a)-(\omega_b)(p_b)\right|+\left|(\omega_a)(q_a)-(\omega_b)(q_b)\right|+ \\
\left|(\omega_a)(q_a+\beta_a)-(\omega_b)(q_b+\beta_b)\right|+ \\
\left|(1-\omega'_a)(p'_a-\alpha'_a)-(1-\omega'_b)(p'_b-\alpha'_b)\right|+ \\
\left|(1-\omega'_a)(p'_a)-(1-\omega'_b)(p'_b)\right|+ \\
\left|(1-\omega'_a)(q'_a)-(1-\omega'_b)(q'_b)\right|+ \\
\left|(1-\omega'_a)(q'_a+\beta'_a)-(1-\omega'_b)(q'_b+\beta'_b)\right|
\end{bmatrix}
+\frac{1}{8}\begin{bmatrix}
\left|(\omega_b)(p_b-\alpha_b)-(\omega_c)(p_c-\alpha_c)\right|+ \\
\left|(\omega_b)(p_b)-(\omega_c)(p_c)\right|+\left|(\omega_b)(q_b)-(\omega_c)(q_c)\right|+ \\
\left|(\omega_b)(q_b+\beta_b)-(\omega_c)(q_c+\beta_c)\right|+ \\
\left|(1-\omega'_b)(p'_b-\alpha'_b)-(1-\omega'_c)(p'_c-\alpha'_c)\right|+ \\
\left|(1-\omega'_b)(p'_b)-(1-\omega'_c)(p'_c)\right|+ \\
\left|(1-\omega'_b)(q'_b)-(1-\omega'_c)(q'_c)\right|+ \\
\left|(1-\omega'_b)(q'_b+\beta'_b)-(1-\omega'_c)(q'_c+\beta'_c)\right|
\end{bmatrix}.
$$

Considering the above inequalities, the $d^H_{GLRIFNs}(A,C)\leq d^H_{GLRIFNs}(A,B)+d^H_{GLRIFNs}(B,C)$ is obtained.

Example 5.2 Let $A=(1,2,3,4;1,2,3,4;0.7;0.2)_{LR}$ and $B=(4,5,6,7;4,5,6,7;0.6;0.3)_{LR}$ be two GLRIFNs, then $d^H_{GLRIFNs}(A,B)$ is

$$
\begin{aligned}
d^H_{GLRIFNs}(A,B)&=\frac{1}{8}\begin{bmatrix}
\left|(0.7)(1)-(0.6)(4)\right|+\left|(0.7)(2)-(0.6)(5)\right|+ \\
\left|(0.7)(3)-(0.6)(6)\right|+\left|(0.7)(4)-(0.6)(7)\right|+ \\
\left|(1-0.2)(1)-(1-0.3)(4)\right|+\left|(1-0.2)(2)-(1-0.3)(5)\right|+ \\
\left|(1-0.2)(3)-(1-0.3)(6)\right|+\left|(1-0.2)(4)-(1-0.3)(7)\right|
\end{bmatrix} \\
&=\frac{1}{8}[13.6] \\
&=1.7
\end{aligned}
$$

3. Implementation of the Proposed Methodology in the River Water Pollution Classification

Five alternatives (rivers) have been selected for further evaluation: the River J_1, River J_2, River J_3, River J_4, and River J_5 for the year 2021. The six criteria (parameters) are K_1, K_2, K_3, K_4, K_5, and K_6 were used for evaluating the five possible alternatives.

Using the generalised L-R intuitionistic fuzzy TOPSIS (GLRIF-TOPSIS) developed by Shafie et al. (2023), the Euclidean distance has been used to calculate the distance of each alternative from generalised trapezoidal L-R intuitionistic fuzzy positive ideal solution (GTrLRIF-PIS) and generalised trapezoidal L-R intuitionistic fuzzy negative ideal solution (GTrLRIF-NIS). This study replaces the Euclidean distance with the Hamming distance to enhance the accuracy and reliability of the decision-making process, as the Hamming distance provides a straightforward and interpretable approach for comparing GLRIFNs in structured scenarios. By comparing with the results in Shafie et al. (2023), the Euclidean distance effectively reflects the geometric closeness between intuitionistic fuzzy numbers by emphasising larger deviations, making it suitable for evaluating overall magnitude differences. In contrast, the Hamming distance provides a simpler, equal-weighted assessment of inconsistencies, which is beneficial when uniform treatment of all attributes is desired. Therefore, the weighted normalised GTrLRIF decision matrix for the year 2021 is shown in Table 1 followed by GTrLRIF-PIS and GTrLRIF-NIS for the year 2021 in Table 2. GTrLRIF-PIS and GTrLRIF-NIS calculated using the Hamming distance are shown in Table 3. Table 4 shows the closeness coefficient and order of alternatives with Hamming distance of GLRIFNs using GLRIF-TOPSIS.

Table 1. Weighted Normalised GTrLRIF Decision Matrix for the Year 2021.

River	K_1	K_2	K_3	K_4	K_5	K_6
J_1	(.1863, .1863; .0966, .0966; .0248, .0337; .0148, .0148; .9100; .0900)$_{LR}$	(.1443, .1735; .0108, .0130; .0330, .0165; .0009, .0039; .8900; .1100)$_{LR}$	(.1423, .1423; .0146, .0146; .0177, .0177; .0016, .0021; .9100; .0900)$_{LR}$	(.0294, .0494; .0101, .0169; .0000, .0207; .0030, .0000; .9100; .0900)$_{LR}$	(.0751, .0758; .1176, .1186; .0008, .0004; .0007, .0014; .9300; .0700)$_{LR}$	(.1162, .1500; .0002, .0003; .0216, .0000; .0000, .0001; .9100; .0900)$_{LR}$
J_2	(.0927, .0941; .1913, .1942; .0012, .0000; .0000, .0025; .8500; .1500)$_{LR}$	(.0099, .0105; .1788, .1900; .0000, .0003; .0056, .0000; .8300; .1700)$_{LR}$	(.0130, .0151; .1375, .1600; .0000, .0018; .0145, .0000; .8300; .1700)$_{LR}$	(.0896, .1113; .0045, .0056; .0102, .0205; .0007, .0007; .7600; .2400)$_{LR}$	(.1189, .1193; .0747, .0749; .0003, .0007; .0004, .0002; .8600; .1400)$_{LR}$	(.0022, .0024; .0150, .0164; .0002, .0004; .0021, .0016; .8500; .1500)$_{LR}$
J_3	(.0917, .0924; .1948, .1962; .0008, .0000; .0000, .0017;	(.0129, .0133; .1413, .1462; .0007, .0015; .0139, .0088;	(.0189, .0243; .0857, .1102; .0000, .0039; .0119, .0000;	(.0809, .0898; .0055, .0061; .0123, .0246; .0012, .0011;	(.0974, .0974; .0915, .0915; .0012, .0013; .0012, .0012;	(.0003, .0005; .0682, .1200; .0000, .0002; .0185, .0000;

	.8800; .1200)$_{LR}$	.8800; .1200)$_{LR}$	.8800; .1200)$_{LR}$	.8500; .1500)$_{LR}$	.8600; .1400)$_{LR}$	.8800; .1200)$_{LR}$
J_4	(.1074, .1114; .1616, .1676; .0046, .0026; .0037, .0075; .9300; .0700)$_{LR}$	(.0166, .0177; .1066, .1134; .0010, .0020; .0107, .0072; .9300; .0700)$_{LR}$	(.0327, .0345; .0603, .0637; .0014, .0028; .0046, .0029; .9300; .0700)$_{LR}$	(.0932, .1124; .0044, .0053; .0238, .0476; .0013, .0018; .9300; .0700)$_{LR}$	(.0798, .0808; .1103, .1117; .0015, .0008; .0011, .0021; .9400; .0600)$_{LR}$	(.0125, .0125; .0029, .0029; .0014, .0014; .0003, .0004; .9300; .0700)$_{LR}$
J_5	(.0824, .0833; .2161, .2185; .0006, .0000; .0000, .0015; .9300; .0700)$_{LR}$	(.0111, .0113; .1669, .1690; .0000, .0002; .0022, .0000; .9300; .0700)$_{LR}$	(.0130, .0172; .1209, .1600; .0000, .0037; .0214, .0000; .9300; .0700)$_{LR}$	(.0031, .0047; .1057, .1600; .0000, .0021; .0329, .0000; .9100; .0900)$_{LR}$	(.0755, .0759; .1175, .1180; .0005, .0003; .0004, .0008; .9400; .0600)$_{LR}$	(.0002, .0002; .1477, .1500; .0000, .0000; .0023, .0000; .9300; .0700)$_{LR}$

Table 2. GTrLRIF-PIS and GTrLRIF-NIS for the Year 2021.

Ideal Solution	K_1	K_2	K_3	K_4	K_5	K_6
A^+	(.0818, .0824; .0833, .0833; .2161, .2161; .2185, .2200; .9300; .0700)$_{LR}$	(.1113, .1443; .1735, .1900; .0099, .0108; .0130, .0169; .8300; .1700)$_{LR}$	(.1246, .1423; .1423, .1600; .0130, .0146; .0146, .0167; .8300; .1700)$_{LR}$	(.0793, .0932; .1124, .1600; .0031, .0044; .0053, .0063; .7600; .2400)$_{LR}$	(.0743, .0751; .0758, .0761; .1170, .1176; .1186, .1200; .9400; .0600)$_{LR}$	(.0947, .1162; .1500, .1500; .0002, .0002; .0003, .0004; .8500; .1500)$_{LR}$
A^-	(.1615, .1863; .1863, .2200; .0818, .0966; .0966, .1114; .8500; .1500)$_{LR}$	(.0099, .0099; .0105, .0109; .1732, .1788; .1900, .1900; .9300; .0700)$_{LR}$	(.0130, .0130; .0151, .0169; .1230, .1375; .1600, .1600; .9300; .0700)$_{LR}$	(.0031, .0031; .0047, .0068; .0728, .1057; .1600, .1600; .9300; .0700)$_{LR}$	(.1186, .1189; .1193, .1200; .0743, .0747; .0749, .0751; .8600; .1400)$_{LR}$	(.0002, .0002; .0002, .0002; .1454, .1477; .1500, .1500; .9300; .0700)$_{LR}$

Table 3. GTrLRIF-PIS and GTrLRIF-NIS Using Hamming Distance.

Distance Values		***K_1***	***K_2***	***K_3***	***K_4***	***K_5***	***K_6***
J_1	d^*	.1064	.0050	.0063	.0260	.0010	.0038
	d^-	.0073	.0971	.0787	.0591	.0276	.0849
J_2	d^*	.0201	.1306	.1072	.0032	.0393	.0598
	d^-	.0555	.0072	.0061	.0691	.0011	.0659
J_3	d^*	.0170	.1158	.0848	.0054	.0228	.0933
	d^-	.0574	.0247	.0288	.0673	.0107	.0316
J_4	d^*	.0364	.1016	.0658	.0088	.0056	.0497
	d^-	.0417	.0398	.0452	.0678	.0237	.0730
J_5	d^*	.0000	.1316	.1083	.0951	.0004	.1230
	d^-	.0706	.0121	.0090	.0028	.0274	.0034

Table 4. Closeness Coefficient and Order of Alternatives with Hamming Distance of GLRIFNs.

River	**Hamming Distance**	
	CC_i	**Rank**
J_1	.7075	1
J_2	.3625	4
J_3	.3940	3
J_4	.5208	2
J_5	.2145	5
Order	$J_1 \succ J_4 \succ J_3 \succ J_2 \succ J_5$	

4. Result and Discussion

Classifying river water pollution proved to be a difficult and intricate task due to the need to simultaneously consider multiple factors, along with the inherent subjectivity and uncertainty in the classification process. This study assessed five rivers: River J_1, River J_2, River J_3, River J_4, and River J_5 for the year 2021, using parameters K_1, K_2, K_3, K_4, K_5, and K_6. To determine the most polluted river, the GLRIF-TOPSIS method was applied, utilising the Euclidean distance to calculate the separation measures between each alternative and the GTrLRIF-PIS and GTrLRIF-NIS. This approach enabled a more objective ranking of the rivers based on the weighted performance of each parameter, ultimately identifying the river with the highest level of pollution in 2021.

The results show that the ranking of alternatives using Hamming distances is $J_1 \succ J_4 \succ J_3 \succ J_2 \succ J_5$. It shows that the River J_1 is the most polluted, followed by River J_4, River J_3, River J_2, and River J_5. There are several factors that lead to polluted river water at River J_1. Based on the river data, the dissolved oxygen (DO) parameter at River J_1 is too low due to the industrial wastewater discharge, especially in 2019. DO in rivers reflects the breathing of aquatic life (Zhi et al., 2021). The changes in DO concentration in the river can affect the biochemical oxygen demand (BOD) and chemical oxygen demand (COD) in the river water. The high concentration of BOD and COD in River J_1 is also due to the discharge of industrial wastewater containing organic materials. Additionally, River J_1 has a high concentration of ammoniacal nitrate (AN) due to wastewater discharged from a nearby fertiliser industry. Such findings are instrumental for environmental management and policy decisions, enabling targeted interventions to address pollution in high-risk areas and preserve cleaner river water. Moreover, the integration of GLRIFNs in the evaluation process helps manage uncertainty and enhances the reliability of the classification, offering a valuable tool for sustainable water quality monitoring due to the consideration of confidence level in the evaluation.

The consistency in the rankings derived from both Euclidean and Hamming distance measures demonstrates the robustness and reliability of the GLRIF-TOPSIS method in handling imprecise and uncertain data. This suggests that the model is well-suited for complex environmental decision-making scenarios, where data ambiguity is often a challenge. Furthermore, the prioritisation of pollution levels among the rivers highlights the practical potential of this method in supporting local authorities and stakeholders in allocating resources efficiently. By identifying River J_1 as the most polluted, the model underscores the urgency for immediate remedial action in that area.

5. Conclusion

The Hamming distance of GLRIFN is introduced in this study along with its mathematical properties. This research addresses a notable gap in the current literature; the limited development of distance measures specifically tailored for generalised L-R intuitionistic fuzzy numbers (GLRIFNs). By proposing a structured Hamming distance within this framework, the study strengthens the theoretical foundation of GLRIFNs and extends their applicability in complex decision-making scenarios.

To demonstrate practical relevance, the proposed distance measure was implemented in the GLRIF-TOPSIS method to compute the distance of each alternative from the generalised L-R intuitionistic fuzzy positive ideal solution (GTrLRIF-PIS) and negative ideal solution (GTrLRIF-NIS). The case study on river water pollution classification revealed that River J_1 is the most polluted, while River J_5 is the cleanest. These findings validate the utility of the proposed Hamming distance in environmental data evaluation and contribute to improving decision-making tools in real-world applications.

However, this study has some limitations. The proposed Hamming distance measure assumes that decision-makers' preferences and the shapes of membership/non-membership functions are known and fixed. In real-world scenarios, such information may be imprecise or dynamic, potentially affecting the accuracy of the results. Additionally, the computational complexity associated with GLRIFNs, particularly when dealing with large-scale datasets, was not thoroughly analysed, which may influence the model's scalability.

Future research could explore adaptive or learning-based techniques to estimate membership and non-membership functions dynamically, allowing for greater flexibility and realism. Moreover, the development of efficient algorithms to handle large datasets within the GLRIFN framework could enhance the model's practical applicability. Integrating this approach with machine learning or data-driven fuzzy inference systems could also offer promising directions for advancing decision-making under uncertainty. The results proved that GLRIFN is a trustworthy technique for classifying pollution in river water. Given its broad advantages, GLRIFN is a valuable method not only for river water pollution classification but also for potential applicability in various other fields such as healthcare diagnostics, financial risk assessment, supply chain optimisation, and social sciences.

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Author Contribution

Author 1: Conceptualisation, methodology, formal analysis, investigation and writing-original draft; Author 2: Conceptualisation, methodology, and supervision; Author 3: Conceptualisation, formal analysis, and validation; Author 4: Conceptualisation, writing-review and editing, and validation.

Conflict of Interest

The authors have no conflicts of interest to declare.

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