UNIVERSITI TEKNOLOGI MARA

HIGHER ORDER EXTENDED COMPACT FINITE DIFFERENCE SCHEME FOR GOURSAT PARTIAL DIFFERENTIAL EQUATION PROBLEM

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ABSTRACT

The Goursat partial differential equation (PDE) problem of the second-order hyperbolic type is important for modeling a wide range of phenomena across scientific and engineering fields, including economic dynamics, global optimal scheduling, geoscience, biomedical engineering, and Nordström-like black hole studies. Traditional numerical methods for solving Goursat PDE problems include Runge-Kutta methods, Newton-Cotes integration, reduction differential transform methods, fuzzy transforms, iterative regularization, signature Kernel methods, and finite difference methods (FDM). While these methods have been widely used, challenges remain in preserving linearity, achieving effective linearization, and maintaining explicitness. This study introduces a novel approach that employs higher order extended compact FDM integrated with Taylor series expansions. FDM are chosen due to their widespread use and effectiveness in solving PDEs. They are known for their straightforward implementation and ability to handle complex problems efficiently. In this study, new higher order schemes for solving Goursat PDE problems have been constructed. These schemes are specifically designed to preserve linearity in linear Goursat problems, facilitate accurate linearization for nonlinear Goursat problems, and maintain an explicit structure that avoids iterative procedures. Numerical experiments show that the new higher order finite difference schemes provide improved accuracy, enhance reliability, and simplify the solution process by avoiding complex iterative calculations. Given the limited number of existing studies on nonlinear Goursat problems, this research addresses the gap by employing a linearization technique to effectively explore and solve both linear and nonlinear hyperbolic Goursat PDE problems. These advancements enhance mathematical modeling and problem-solving capabilities across various scientific disciplines.

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CHAPTER 1
INTRODUCTION

1.1 Overview

This section provides an overview of the study. It presents the background of the problem and motivation for conducting out the research. This chapter discusses seven criteria of the study, which are the research background, problem statement, research objectives, scope and limitations, significance, research design, and thesis outline.

1.2 Research Background

The partial differential equation (PDE) describes many problems that naturally occurs in numerous fields of study such as engineering, physics, and applied mathematics. Much attention has been given by many mathematicians and scientists to the extensive effectiveness of these equations. The Goursat equations are second-order hyperbolic PDEs with two independent variables, while the Goursat problem is an initial value problem (Nasir, 2010) that concerns with solving or finding a solution to the Goursat equation given the initial conditions that are defined on certain surfaces. The Goursat problem takes its name from the French mathematician Edouard Jean-Baptiste Goursat (Oussaeif & Bouziani, 2018). The problem can take the form of linear or nonlinear as well as homogeneous or inhomogeneous problem. The standard form of the Goursat problem (Wazwaz, 1993) is defined as

$$
\begin{aligned}
&\text{''xv} = f(x,y,u,u_x,u_y),\\
&u(x,0) = {}_{gl}(x), \quad u(0,y) = g_2(y), \quad {}_{\text{gl}}(0) = \text{g}_2\ (0) = \text{«}(0,0), \qquad (1.1)\\
&0 < \text{x} < \text{c}, \qquad 0<y<d,
\end{aligned}
$$

where u_{xy} is the mixed derivative in space x and y while the term $f(x,y,u,u_x,u_y)$ is a function of independent variables x and y, dependent variable u, and the derivative terms u_x and u .