UNIVERSITI TEKNOLOGI MARA

ROUGH NEUTROSOPHIC MULTISETS

SURIANA BINTI ALIAS

PhD

May 2024

ABSTRACT

Multiple uncertainty conditions exist in real life with incomplete knowledge gathered by human observation. These conditions are about vagueness, imprecision, ambiguity, and inconsistency which need to be taken into consideration. It has not, however, been satisfactorily modeled by the classical and existing mathematical concept for uncertainty theory. To capture all the associated uncertainty factors, this research aims to develop a new concept of uncertainty with more representable defined boundary sets. A rough neutrosophic multisets theory which is a fusion between neutrosophic multisets and rough sets theory was introduced as a novel finding of this research. It is a more appropriate and representable theory for a real problem in resolving multiple uncertainty information with boundary regions involving lower and upper approximation. Another significant finding of the research is the basic rough neutrosophic multisets operations, such as complement, union, intersection, contained, and equality, are defined with numerical examples. The "AND" and "OR" rough neutrosophic multisets operators are shown. The algebraic operations over rough neutrosophic multisets, including addition, subtraction, multiplication, and scalar multiplication, are also introduced. It also determined the level of relevant uncertainty factors in every situation. Other findings indicated that a roughness measure is defined as a principal for the development of rough neutrosophic multisets in equivalence relations. As a result, rough neutrosophic multisets are derived concerning relations properties for single and two universal sets, four types of vector similarity measures and three types of distance measures, two types of distance-based similarity measures, two types of entropies measures, and three types of aggregation operator. Finally, this research contributed a procedure of rough neutrosophic multisets operators as a novel approach to improving the representation of the evaluation process involving multiple-criteria decisions-making. The highlighted medical diagnosis problem is used for illustrative purposed to explain the applicability of the procedure. In addition, the rough neutrosophic multisets with multi-criteria decision-making is applicable to be used in solving problems of a similar nature.

ACKNOWLEDGEMENT

All praises be to Allah, the Most Gracious of the Merciful. Thank you, God, for providing me with the opportunity to embark on a PhD. and for helping me complete this challenging and long journey successfully. I have been very fortunate to have had the guidance and assistance of many people throughout the writing and completion of this thesis, and I will never forget to thank them.

I wish to extend my deepest gratitude to my supervisor, Prof. Dr Daud Mohamad, for his support of my PhD study and related research, and his patience, motivation, and in-depth knowledge. My deep admiration and inspiration for him come from his vision, sincerity, and motivation. I learned to present my research as clearly as possible from him. Throughout the research process and the writing of this thesis, his guidance was extremely helpful. I also wish to express my sincere thanks to my co-supervisor Associate Prof. Dr Adibah Shuib for all the guidance and encouragement she provided me throughout my PhD studies. The friendship, empathy, and great sense of humor she showed me were greatly appreciated. My PhD study could not have been more fortunate to have you both as my advisor and mentor.

Thank you so much to the Dean and staff, the Faculty of Computer and Mathematical Sciences, and the Institute of Graduate Study, Universiti Teknologi MARA for their support. My sincere gratitude goes to Universiti Teknologi MARA, Universiti Teknologi MARA Kelantan Campus, and the Ministry of Higher Education, Malaysia, for crediting my research and providing me with financial support during my PhD studies.

My beloved husband, Wan Rusydi Hamzah, deserves a huge thank you for his love, prayers, sacrifices, caring, and continued support, and has made it possible for me to complete this study and fulfill this ambition. I am also grateful to have my children's love, understanding, prayers, and support throughout this research: Wan Umairah Afifah, Wan Umar Abid, Wan Ali Musaddad, Wan Ufairah Afia, and Wan Uthman Shah. It has been an incredible experience to study with my children. I am very thankful to have them during this time of study. I also like to thank my parent-in-law, Hj. Hamzah Ismail and , my siblings, brother, and sister-in-law for supporting me spiritually through writing this thesis, and for my life in general. My thesis is dedicated to my late parents, En. Alias Mahmood and
who I loved very much, for the love, prayers, and vision to educate me. My deep gratitude and love for them cannot be adequately expressed.

Last but not least, my friends and research group have contributed immensely to this thesis, and I cannot accomplish it without their support and dedication. Thank you for your kindness and willingness to spend time with me. I greatly appreciate it. Alhamdulillah.

TABLE OF CONTENTS

		Page
CONFIRMATION BY PANEL OF EXAMINERS		**ii**
AUTHOR'S DECLARATION		**iii**
ABSTRACT		**iv**
ACKNOWLEDGEMENT		**v**
TABLE OF CONTENTS		**vi**
LIST OF TABLES		**xi**
LIST OF FIGURES		**xiii**
LIST OF SYMBOLS		**xiv**
LIST OF ABBREVIATIONS		**xvi**
CHAPTER 1 INTRODUCTION		**1**
1.1	Research Background	1
1.2	Problem Statement	6
1.3	Research Questions	8
1.4	Research Objectives	9
1.5	Scope and Limitation	10
1.6	Significance of the Research	11
1.7	Thesis Organization	12
1.8	Summary	15
CHAPTER 2 LITERATURE REVIEW		**16**
2.1	Introduction	16
2.2	Generalization of Uncertainty Information for Single-valued Data	18
	2.2.1 Relationship Among Fuzzy Set, Rough Set, Intuitionistic Fuzzy Set, Neutrosophic Set, and Their Generalization	21
2.3	Multisets and Its Variants	24
	2.3.1 Relationship Among Multisets Theory and Their Generalization	25
2.4	On Generalised Rough Neutrosophic Multisets	27

CHAPTER 1
INTRODUCTION

1.1 Research Background

In recent decades, set theory has developed further as a means of handling critical thinking in areas like games (win, fair, or losing), voting (vote, blank vote, and against), and decision-making (right, neutral, or wrong). Such critical thinking opinions always come with uncertainties that are dependent upon observations and decisions made by experts (Kishishita, 2020). Uncertainties may be visualization by hierarchical data, time-series and stochastic, and in linguistic terms (Hägele et al., 2022). Nevertheless, since there are uncertainties, the collected information is vague, imprecise, ambiguous, and inconsistent condition. According to George (2006) definition of information, uncertainty arises when there is a lack of information, whereas the capacity to lessen uncertainty is there when there is information. In this way, uncertainty and information are inextricably linked. Thus, to model uncertainty information, a set theory must be capable of interpreting all the lacking information accurately as having unclear limits information leads to vagueness, imprecision is an outcome of different observers' knowledge that cannot be measured, and ambiguity occurs because of inadequate knowledge or mistakes in the acquisition, and inconsistency results from conflicting values. Furthermore, to model the uncertainty information with different types of data, the introductory uncertainty set theory is further discussed with a limited interpretation for an accurate result (Yang et al., 2009).

Information can be categorized into single-valued data and multiset data (Blizard, 1988). In the reviewed research by Seoni et al. (2023), the uncertainty involved in single-valued and multiset data always occurs in healthcare. Nowadays, experts have prepared machine learning or artificial intelligence (AI) to overcome the uncertainty condition. An example would be the information collected from the patient who saw a doctor because they had symptoms related to Coronavirus (COVID-19), such as cough, fever, runny nose, sore throat, muscle pain, and headaches. Single-valued data can only contain information concerning the appointment date. Nevertheless, as an epidemic virus, the data must not remain relevant for only one time. COVID-19 virus inspection should take place within one week or a minimum of three days (Hashmi et al., 2020).