

UNIVERSITI TEKNOLOGI MARA

**A COMPARATIVE STUDY OF NONLINEAR
SECOND ORDER ORDINARY DIFFERENTIAL
EQUATION BY USING FOURTH-ORDER RUNGE-
KUTTA METHOD AND DIFFERENTIAL
TRANSFORM METHOD**

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ABSTRACT

This study performs a comparative analysis of two numerical methods, the Fourth-order Runge Kutta Method (RK4) and the Differential Transform Method (DTM), for solving second order ordinary differential equations (ODE). Nonlinear equations are often difficult to solve analytically due to their complexity. Therefore, numerical methods was used to obtain numerical solution. This study aims to solve nonlinear second order ODE using RK4 and DTM. RK4 method is known for its high accuracy in solving initial value problems (IVP) and DTM is based on Taylor series expansion, provides a semi analytical method that makes it easy to find numerical solution. There is limited study on solving nonlinear second order ODEs by using these methods. In this study, three nonlinear problems were selected from past research and have been solved using RK4 and DTM methods with a fixed step size, $h = 0.1$ for RK4 and series is restricted to a finite number of terms for DTM. Solutions were evaluated by comparing their results with the exact solutions through absolute error analysis. The results showed that both methods are capable on solving second order nonlinear ODE. However, in term of accuracy, RK4 gives better approximation for all problem compared to DTM especially when the value of the independent variable, x increases. For future study, it is recommended that the researchers conduct their research in solving the nonlinear higher-order differential equations with different step sizes. The accuracy and efficiency of both methods can be systematically analysed based on the relative errors, computational work, or on the execution time.

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