

**MHD WILLIAMSON BOUNDARY LAYER FLOW  
OVER STRETCHING SHEET IN  
DOUBLE STRATIFICATION MEDIUM**

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## ABSTRACT

This research examines the behaviour of Magnetohydrodynamic (MHD) Williamson boundary layer flow over stretching sheet in double stratification medium. The study begins with a review of existing research to identify gaps in understanding MHD and Williamson fluid dynamics. A step-by-step approach was followed, starting with defining the governing equations and boundary conditions and ending with analysing results from numerical simulations. The Williamson parameter was incorporated into the momentum equation, and similarity transformations were used to convert partial differential equations into ordinary differential equations. These equations were solved using the Runge-Kutta method, with computations and visualizations performed in Maple software. The study demonstrates that incorporating the Williamson parameter into the momentum equation, in conjunction with various parameter combinations, significantly influences the skin friction coefficient, heat transfer rate, and mass transfer rate. The effects of the Williamson parameter, along with the magnetic parameter, porosity parameter, Prandtl number, Schmidt number, and double stratification, are analysed and presented through graphical illustrations, accompanied by a detailed discussion. Based on the result obtained, it is found that the Williamson parameter caused the momentum boundary layer thickness to decrease. Meanwhile the thermal boundary layer thickness and concentration boundary layer increases with the increase of Williamson parameter.

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