NUMERICAL METHODS FOR SOLVING HIGHER-ORDER ORDINARY DIFFERENTIAL EQUATIONS

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ABSTRACT

The higher-order ordinary differential equation (ODE) is essential for accurately describing complex systems that cannot be captured by first or second-order models. This research employs various numerical methods to solve higher-order ODEs for fourth- and fifth-order, including both homogeneous, non-homogeneous and stiff equations. The methods include Euler's Method, Heun's Method, the Fourth-Order Runge-Kutta method, Fifth-Order Runge-Kutta method and Runge-Kutta-Fehlberg method. To enhance the accuracy of approximate solutions for higher-order ODE, the numerical methods are combined with Richardson's and Aitken's extrapolation, respectively. The approximate solutions for each method are presented in tables and visualised through graphs. The error for each method is recorded in tables and illustrated through graphs to identify the most effective method, while the CPU time is measured to evaluate the computational cost. The results showed that the combination of Runge-Kutta fifth-order method and Runge-Kutta-Fehlberg method with Aitken's extrapolation gives the best result in solving the fourth-, fifth-order for both homogeneous and non-homogeneous. For stiff equation, Runge-Kutta-Fehlberg method with Aitken's extrapolation give the lowest error compared to Runge-Kutta fifth-order method with Aitken's extrapolation. For, the combination of Richardson's extrapolation can effectively reduce the computational cost but its accuracy is not good as combining with Aitken's extrapolation.

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