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Optimal Parameter Estimation of MISO System Based on Fuzzy Numbers

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Abstract

A great deal of information for many real-world problems arising in multivariable control systems is provided by human experts. Often, the decision parameters of the systems are described verbally through vague, uncertain or imprecise statements. This situation can be taken into account by using fuzzy numbers, which has been proved to be a very useful tool. Thus, this paper discusses the development of a Fuzzy State Space algorithm for optimal parameter estimation in multiple-input single-output (MISO) system based on fuzzy numbers. Briefly, the procedure involved fuzzification of all the input parameters, processing of the fuzzified parameters in the fuzzy environment and defuzzification of the processed data. The optimal input parameters are determined by the Modified Optimized Defuzzified Value Theorem. As an illustration, this algorithm is applied to a system with three input parameters.

Keywords: Fuzzy State Space algorithm, Parameter estimation, Modified Optimized Defuzzified Value Theorem, Multivariable Control System.

1. Introduction

Many of the real-world control systems for handling parameter decision problems in multivariable dynamic systems are far from simple. Furthermore, a great deal of information for many real-world systems is provided by human experts, who describe the system verbally through vague, uncertain or imprecise statements. According to Jamshidi (1997), the most relevant information about any system comes in one of three ways, that is, a mathematical model, sensory input and output data or measurement, and human expert knowledge. The common factor in all these three sources is knowledge. Apart from a mathematical model where utilization is clear, numerical input-output data can be used to develop an approximate model as well as a controller, based on the best available knowledge to treat uncertainties in the system. A typical example of techniques that make use of human knowledge and deductive processes is fuzzy modeling. Besides, fuzzy sets (Zadeh, 1965) also provide a tool for handling ill-conditioned or ill-posed problems, which exist as a result of combining measurements with engineering models. The inverse problem (Hensel, 1991), or more precisely the inverse modeling, is one type of ill-conditioned or ill-posed problems. In inverse modeling, the desired responses are given and a model is used to estimate the input parameters. Traditionally, such inverse problems have been addressed by repeated simulation of forward problems, for example Ordys *et* al. (1994), Ram and Patel (1998). However, this method requires excessive computer time and thus can be very costly.

The objective of this paper is to present the development of a Fuzzy State Space algorithm for estimating the optimal input parameters of a multiple-input single-output (MISO) system. The imprecise or uncertain parameters in the system are represented by fuzzy numbers (Kaufmann and Gupta, 1985) with their membership function derived from expert knowledge. Thus, uncertainty in the formulation of this algorithm refers to the imperfect knowledge of the input and output parameters in the mathematical model of the decision-making problem. It is related to an expert who gives the description of the uncertainty.

The paper is organized as follows. After this introduction, section 2 describes briefly about fuzzy numbers. Calculations with fuzzy numbers are performed using operation rules based on α-cuts (Kaufmann and Gupta, 1985). The development of the Fuzzy State Space algorithm for determining the optimal parameter estimation in MISO system are explained in Section 3. The validity of this algorithm is shown by implementing it to the state space model of a furnace system with three input parameters, which is presented in section 4. Finally, section 5 draws some conclusions from the presented work.

2. Fuzzy Numbers

The concept of a fuzzy number arises from the fact that many quantifiable phenomena cannot be characterized in terms of absolutely precise numbers. A fuzzy number is described in terms of a number word and a linguistic modifier, such as "approximately", "nearly", "about" or "around". For example, "about three o'clock" is fuzzy, because it includes some number values on either side of its central value of three. In this study, triangular fuzzy numbers are use as they have an intuitive appeal and are easily specified by experts (Pedrycz, 1994). A triangular fuzzy number (TFN) is a special type of fuzzy number with three parameters: smallest possible value (a_1), most desirable value (a_2), and largest possible value (a_3). The membership functions for TFN can be represented by their breaking points A = $(a_1; a_2; a_3)$, which is defined as

$$\mu_A(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & x \in [a_1, a_2] \\ 1 & x = a_2 \\ \dfrac{a_3-x}{a_3-a_2}, & x \in [a_2, a_3] \\ 0 & otherwise \end{cases}$$

The α-cuts of TFN is the interval, $[(a_2 - a_1)\alpha + a_1, (a_2 - a_3)\alpha + a_3]$, $\forall \alpha \in (0,1]$. The graph of a typical TFN is shown in Figure 1.

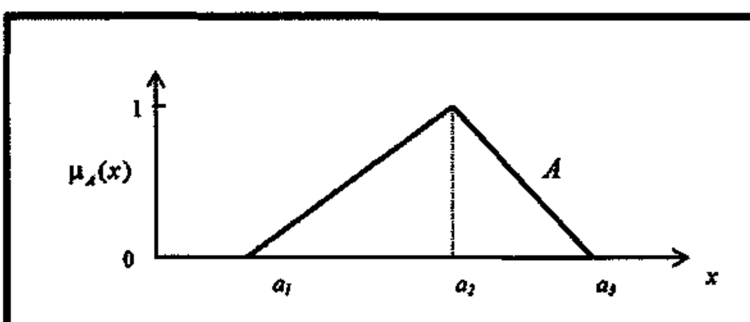

Figure 1 A triangular fuzzy number

It is assumed that the membership function $\mu_A(x)$ is a monotonically increasing function with $\mu_A(x) = 0$ and $\lim_{x\to\infty} \mu_A(x) = 0$ for $x \geq a_2$ and monotonically decreasing function with $\mu_A(x) = 1$ and $\lim_{x\to\infty} \mu_A(x) = 0$ for $x \geq a_2$. In many respects, fuzzy numbers depict the physical world more realistically than single-valued numbers, as the concept takes into account the fact that all phenomena have a degree of uncertainty.

3. Fuzzy State Space Algorithm

In formulating the Fuzzy State Space algorithm, the approach introduced in Ahmad (1998) is modified by considering the state space representation of the system. In his work, he had developed a fuzzy algorithm for optimization of geometrical and electrical parameters of microstrip lines using algebraic equations.

Given an input u_i that takes values in set I_i, and let preferences for different values of u_i be expressed by a fuzzy set F_{Ii} on I_i. For each $x \in I_i$, the value $F_{Ii}(x)$ designates the degree of desirability of using the particular value x within the given set of values I_i. Thus, set F_{Ii} is referred to as the set of desirable values of parameter I_i, and $F_{Ii}(x)$ is viewed as the grade of membership of value x in this set. Index i is used here to distinguish different input parameters.

The fuzzy sets expressing preference for all input parameters are employed for calculating the associated fuzzy sets for performance parameters. The target values of performance parameters are specified by functional requirements. Performance parameters, resulting from calculations with uncertain or vague input parameters, will also be represented by fuzzy preference functions. Similarly, the output parameter is represented by a range and a preference function.

It is assumed that all the fuzzy sets F_{Ii} expressing preferences of all input parameters $u_i \in I_i \subset \Re \ (i \in \aleph)$ are determined, normalized and convex. I is a close interval of real numbers. S_{gF} is a performance parameter based on the Fuzzy State Space Model (Razidah, 2005a) whereby all input parameters are considered as its variables and can be presented within a fuzzy set F_{Sg}. The Fuzzy State Space algorithm is formulated based on three phases of a fuzzy system as follows.

Phase 1: Fuzzification

1a:	Let $S_{gF} : \Re^n \to \Re$. S_{gF} is the performance parameter such that $r = S_{gF}(u_1, u_2, u_3, \ldots, u_n)$.
1b:	Select appropriate values for α-cut such that $\alpha_1, \alpha_2, \alpha_3, \ldots, \alpha_k \in [0, 1]$ which are equally spaced.
1c:	To fuzzify the input, determine all the α_k-cuts for all $F_{Ii} \ (i \in \aleph)$.
1d:	Generate all 2^n combinations of the endpoints of intervals representing α_k-cuts for all $F_{Ii} \ (i \in \aleph)$. Each combination is an n-tuple $(u_1, u_2, u_3, \ldots, u_n)$.
1e:	Determine $r_j = S_{gF}(u_1, u_2, u_3, \ldots, u_n)$ for each n-tuple, $j \in 1, 2, \ldots, 2^n$.
1f:	Set $(F_{ind})_l = [\min(r_j), \max(r_j)]$ for all $j \in 1, 2, \ldots, 2^n$
1g:	Determine all the α_k-cuts for the desired or preferred performance parameter, F_{SgFl}

Phase 2: Processing in the Fuzzy Environment

2a:	Set $[F_{ind} \wedge F_{SgF}]$.
2b:	Determine $f^* = \sup[F_{ind} \wedge F_{SgF}]$ and find the $S_{gF}{}^*$, the S_{gF} value for f^*.

Phase 3: Defuzzification

3a:	Find the endpoints of interval for each input F_{Ii} where $i = 1, 2, \ldots, n$.
3b:	Generate all 2^n combinations of the endpoints of intervals representing f^*- cuts for all F_{Ii} $(i \in \aleph)$. Each combination is an n-tuple $(u_1{}^*, u_2{}^*, u_3{}^*, \ldots, u_n{}^*)$.
3c:	Determine $r^* = S_{gF}{}^*(u_1, u_2, u_3, \ldots, u_n)_{f^*(opt)}$. which gives the optimal input parameters estimation.by using the Modified Optimized Defuzzified Value Theorem Let $S_g : R^n \rightarrow R$ where S_g is a performance parameter based on the Fuzzy State Space Model. If $S_g{}^* = r_j{}^* = \max r_j$ such that $\mu(r_j{}^*) = f^*$ for all $(r_j, f^*) \in F_{ind}$, then $r_j{}^* = S_g{}^* = \max[S_g(g_1{}^*, g_2{}^*, \ldots, g_m{}^*)]$ where $\mu(g_p{}^*) = f^*$.

The Modified Optimized Defuzzified Value Theorem forms the important part of the final phase of defuzzification. It is a modification of the Optimized Defuzzified Value Theorem proposed in Ahmad (1998). The proof of this theorem is published in Ismail *et* al. (2004). This theorem indicates that if the fuzzy desired parameter intersects on the maximum side of the fuzzy induced parameter, then the set of optimized parameters is the set for the maximum of the induced values. It has been shown that all normal and convex fuzzy sets F_{Ii} , expressing preferences of all input parameters $g_i \in I_i \subset R^+ (i \in N)$ are mapped by the Fuzzy State Space Model into the normal and convex induced fuzzy sets (Razidah *et* al., 2002).

4. An Illustrative Example

For illustrating the Fuzzy State Space algorithm, a first order three-input one-output State Space Model of a furnace system (Razidah, 2005b) is considered. The input parameters are w_F (fuel flow to the furnace in kg/s), w_A (air flow to the furnace in kg/s), and w_G (exhaust gas flow from the gas turbine in kg/s). The state variable is EG (density of exhaust gas from the boiler in kg/m^3) and the output parameter is Q_{is} (heat transferred to the superheater in J/s). A semi-automated approach using Matlab® M-file is used for the computations involved in this algorithm. The implementation of this algorithm is discussed according to the three phases of fuzzy system.

Phase 1: Fuzzification

Each of the input parameters of the furnace system is represented by a TFN, $A = (a_1; a_2; a_3)$. For the furnace system, these values are specified in Table 1.

Table 1 Input parameters specification

Input Parameters	$A = (a_1; a_2; a_3)$
Fuel flow: w_F	(10; 12; 16)
Air flow: w_A	(60; 65; 70)
Exhaust gas flow: w_G	(20; 22; 25)

In this illustration, α-cuts with an increment of 0.2 are used to calculate F_{ind}, the fuzzy values of induced output or performance parameters S_{gF}. For a better resolution, α-cuts with a much smaller increment can be used. Each output parameter can be expressed as a linear combination of the input parameters. From the state space model of the furnace system and using the steady state operating data (Ordys *et* al., 1994), the input-output equation is determined to be

$$Q_{is} = 3.523226 \quad 10^4 (w_F + w_A + w_G)$$

Combinations of the endpoints of intervals for all input parameters with respect to each particular value α-cuts cut are determined. The number of combinations increases with a smaller value of the α-cuts. Each of these values is substituted in input-output equation so as to obtain the corresponding performance parameter. The induced performance parameter F_{ind} is determined by taking the maximum and minimum value of each performance parameter. These values are used to plot the graph of F_{ind}.

The desired output parameter for Q_{is} is 3.6417 10^6 J/s, which is the value published in Ordys *et* al.(1994). The desired value and its domain [3.2 10^6, 3.8 10^6] are used to calculate the preferred or desired output parameters. α-cuts with increment of 0.2 as in the fuzzification of input parameters are used to calculate F_{SgF}, the fuzzy values of preferred or desired output parameters. Combinations of the endpoints of intervals for all output parameters with respect to each particular value of α-cuts are determined. These values are used to plot the graph of F_{SgF}.

Phase 2: Fuzzy Environment

The intersection of the fuzzy preferred output parameter and the fuzzified performance parameter is determined by superimposing the two graphs in order to obtain the f^*- value. If there are more than one intersection points, the largest fuzzy membership value, f_j^* is taken as the intersection point.

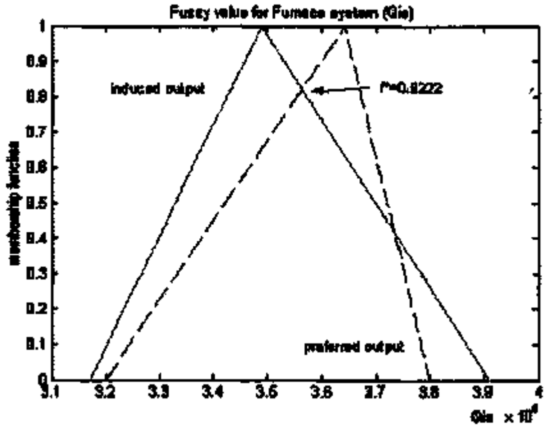

Figure 2 Fuzzy value for Furnace system (Q_{is})

Phase 3: Defuzzification

With the f^*- value obtained, $f^* = 0.8222$, the steps in the defuzzification process are carried out to calculate the best possible combination of the input parameters in order to accommodate all the constraints defined in the process of fuzzification. The endpoints of interval for each input parameter are tabulated in Table 2. With three imprecise or uncertain input parameters, there are eight possible combinations of the endpoints of interval. Each of these combinations is then substituted in the performance parameter. The optimal input parameters are determined by using the Modified Optimized Defuzzified Value Theorem.

Table 2 Endpoints of interval for input parameters (Q_{is})

Output Parameter		Input Parameters		
Q_{is}		w_F	w_A	w_G
$f^* =$ 0.8222	minimum	10.8222	61.6444	22.4666
	maximum	15.1778	68.3556	23.3556

The results of the implementation of the Fuzzy State Space algorithm for a MISO furnace system are shown in Table 3, where the optimal input parameters estimation are $w^F = 15.1778$ kg/s, $w^A = 68.3556$ kg/s and $w^G = 23.3556$ kg/s. These values differ from the desired values with an error of about 26.48%, 5.16% and 6.16% respectively. At the same time, the percentage error for each of the output parameters of the furnace system is computed as shown in Table 4.

Table 3 Optimized input parameters

$f^* = 0.8222$	Calculated Values	Desired Values	Error (%)
w_F	15.1778	12	26.48
w_A	68.3556	65	5.16
w_G	23.3556	22	6.16

Table 4 Calculated output parameters for Furnace System

$f^* =$ 0. 82 22	Calculated Values	Desired Values	Error (%)
Q_{tr}	2.7724×10^7	2.6846×10^7	3.27
Q_{is}	3.7659×10^6	3.6417×10^6	4.09
Q_{es}	1.2883×10^6	1.2465×10^6	3.35
p_G	1.0474×10^5	1.0130×10^5	3.39
Q_{rs}	3.2837×10^6	3.1749×10^6	3.42

Subsequently, a comparison is made between the optimal input parameters obtained using the Fuzzy State Space algorithm and the result obtained through simulation carried out by Ordys *et* al. (1994). The percentage error is calculated and tabulated in Table 5. The aim of this comparison is to highlight the difference between inverse modeling by utilizing fuzzy sets and a widely accepted forward modeling based on simulation. With the TFN used in modeling the uncertainty, the obtained result should have the same value as the result in Ordys *et* al. (1994) with no uncertainty consideration. It is observed that the values of the input parameters w_F (fuel flow to the furnace in kg/s), w_A (air flow to the furnace in kg/s), and w_G (exhaust gas flow from the gas turbine in kg/s) differ with an error of 7.77%, 6.65% and 0.81% respectively. In order to properly model the uncertainties and further improve the results, the parameters of the fuzzy numbers which are used to model uncertainties in this study, need to be adjusted based on the historical data or human experience. For a better resolution,α-cuts with much smaller increment can be used.

Table 5 Comparison of optimized input parameters

Input Parameters	Ismail's	Ordys'	Error (%)
w_F	15.1778	14.083	7.77
w_A	68.3556	64.093	6.65
w_G	23.3556	23.168	0.81

The good results obtained in this application show that this approach may become an interesting tool for decision-makers. Besides, it is relatively easy to take into account experts knowledge and considerations for establishing the membership functions. However, we anticipate obtaining better results with a reduction in computation time through the implementation of the Fuzzy State Space algorithm with multiple-input multiple-output (MIMO) structure.

5. Conclusion

The formulation of the Fuzzy State Space algorithm for MISO system was presented. Briefly, the procedure involved fuzzification of all the input parameters to create fuzzy environment. This is then processed to produce the induced output parameters. The best input parameters were extracted through defuzzification using an important theorem, Modified Optimized Defuzzified Value Theorem. The determination of the optimal input parameters estimation subjected to the desired output parameters can be obtained in a few computer runs, as compared to several hundred computer runs that are required for the commonly-accepted forward simulation approach. Besides that, the performance of these algorithms can be further improved by changing the initial input parameters or by reducing the $\hat{\alpha}$-cut increment. In general, this new technique for determination of optimal input parameters gives broader and useful information and provides a faster and innovative tool for decision-makers.

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