Available online at
https://mijuitm.com.my

Mathematical Sciences and Informatics Journal

e-ISSN: 2735-0703

Mathematical Sciences and Informatics Journal 6(1) 2025, 65 – 81.

Enhancing MCDM with q-Rung Orthopair Fuzzy Dombi Generalized Heronian Mean

Muhammad Mukhlis Kamarul Zaman[1], Zahari Md Rodzi[1*], Yusrina Andu[2]

[1]College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, Negeri Sembilan Branch, Seremban Campus, 70300 Seremban, Negeri Sembilan, Malaysia
[2]College of Computing, Informatics and Mathematics, Universiti Teknologi MARA, Negeri Sembilan Branch, Kuala Pilah Campus, 72000 Kuala Pilah, Negeri Sembilan, Malaysia

ARTICLE INFO

Article history:
Received 28 March 2025
Revised 11 April 2025
Accepted 25 April 2025
Online first
Published 30 April 2025

Keywords:
q-Rung Orthopair Fuzzy Set
Heronian Mean
Dombi
MCDM

DOI:
10.24191/mij.v6i1.4630

ABSTRACT

This paper presents a set of Dombi generalized Heronian mean operators for q-rung orthopair fuzzy numbers (qROFNs) and proposes a multi-criteria decision-making (MCDM) method based on these operators. The operational rules of qROFNs, grounded in the Dombi t-conorm and t-norm, are presented. A q-rung orthopair fuzzy Dombi generalized Heronian mean (qROFDGHM) operator is formulated based on these principles. A method for addressing MCDM problems utilizing qROFNs and the developed operators is proposed. A comprehensive mathematical proof of the operator's idempotency condition, as well as its monotonicity and boundedness properties, validates its theoretical basis. When addressing MCDM challenges, the proposed operator generates effective outcome solutions by leveraging membership and non-membership degree interactions. The qROFDGHM operator has a better ability to handle imprecise, uncertain, and conflicting data when compared with existing methods. This study contributes to fuzzy decision-making advancement through its development of a flexible theoretical aggregation operator.

1. INTRODUCTION

Multiple Criteria Decision Making (MCDM) is a technique that uses multiple criteria to arrive at the final decision. In intricate conditions, decision-makers are integrated to assess the values of many characteristics of all candidates in a systematic way. The rationale for the present endeavor is correspondingly two-fold; that is, to explicate the attribute values and to underscore the potential of fuzzy sets in this regard. Fuzzy

[1*] Corresponding author. *E-mail address*: zahari@uitm.edu.my

https://doi.org/10.24191//mij.v6i1.4630

set (FS) is one of the basic types of fuzzy sets which exploits the degree of membership to determine satisfaction level based on numerical ratio of its membership (Zadeh, 1965). Atanassov (1986) introduced the intuitionistic fuzzy set (IFS) to address this limitation by incorporating a non-membership degree; consequently, the hesitancy function can be represented as one minus the sum of the membership and non-membership degrees. It is noteworthy that IFS is an extension of the FS, with additional degree of non-membership of an element in a universe, both confined to the interval of 0 to 1 (Voskoglou, 2023). Moreover, it can articulate more intricate fuzzy information than FS, leading to various research topics, including the analysis of intuitionistic fuzzy sets in risk-based inspection (Sadeghi et al., 2024), the concept of intuitionistic fuzzy rough power aggregation operators (Khan et al., 2025), functions for intuitionistic fuzzy set (Ngan, 2024), rules of intuitionistic fuzzy calculus (Lei & Xu, 2016), and on some knowledge measures of intuitionistic fuzzy sets of type two with application to MCDM (Singh et al., 2020).While IFSs demonstrate significant potential in MCDM, their applicability is constrained by their capacity to represent fuzzy information, specifically that the sum of membership and non-membership degrees must fall within the interval of 0 to 1.

To address this limitation, Yager (2014) introduced the Pythagorean fuzzy sets (PFSs), which consider the sum of the squares of both membership and non-membership degrees to be within the range of 0 to 1. PFSs have attracted substantial interest from researchers due to their enhanced expressiveness compared to IFSs. Pythagorean fuzzy sets are among the more advanced fuzzy sets that can tackle the prevalent vagueness in decision-making. In cases where conventional mathematical techniques may fail to generate a close solution due to the present of uncertainties and lack of information, PFSs can be helpful to address this problem. On the other hand, other research areas where PFS can be found include goal programming, transport problems, and healthcare to increase the veracity of decisions. A new goal programming approach using PFS has been suggested which extends several traditional intuitionistic approaches in supply chain management, integrating a score function that improves the modelling of multiple objectives (Seal et al., 2024). Some applications in PFS are strategy selection for IOT based sustainable supply chain system (Alkan & Kahraman, 2024), application to hospital siting for COVID-19 patients (Rahman et al., 2023), and application selection in optimal treatment for depression and anxiety (Rahim et al., 2024). Hemalatha & Venkateswarlu (2023) proposed a new way of ranking for Pythagorean fuzzy sets and mean squares approach for transportation problems. All these past studies have shown that PFS could overcome uncertainty in decision making.

To enhance the expressiveness of PFS, Yager (2017) introduced the q-rung orthopair fuzzy set (qROFS) through the concept of the generalized orthopair fuzzy set. Within this framework, the degrees of membership and non-membership adhere to the condition that the sum of their $q-th$ powers remain within the interval of 0 to 1. IFSs and PFSs can be considered specific instances of qROFSs when $q=1$ and $q=2$. This characteristic enhances the expressiveness of qROFSs compared to IFSs and PFSs, as it facilitates the assignment of an appropriate value to q. For example, a decision-maker assessing an alternative in uncertain conditions may assign a membership degree of 0.6 and a non-membership degree of 0.9, represented as (0.6, 0.9). The application of IFS or PFS is impractical in this context, as the following inequalities hold for IFS, $0.6+0.9>1$, and for PFS, $0.6^2+0.9^2>1$. In the case of qROFS, this issue is addressed by selecting a suitable value for q, specifically $q \geq 3$, which enables the attribute values to meet the condition $0.6^q + 0.9^q \leq 1$. As the parameter q increases, the space of allowable orthopairs expands, allowing a wider range of values to meet the bounding constraint. This enhances qROFSs' flexibility and effectiveness in representing fuzzy information through the dynamic adjustment of the parameter q. The field of q-rung orthopair fuzzy sets (qROFSs) has drawn significant attention from researchers. They have explored various aspects, like score functions of qROFNs (Peng et al., 2018; Peng & Dai, 2019), distance measures of qROFNs (Wang et al., 2024), ranking products based on qROFS theory (Yin et al., 2023), correlation and correlation coefficients of qROFSs (Ch et al., 2025), hesitant q-rung orthopair fuzzy sets (Ghoushchi et al., 2024), and extensions of qROFSs (Wang et al., 2019). Indeed, the

https://doi.org/10.24191//mij.v6i1.4630

qROFS plays a crucial role in handling uncertainty, while aggregation operators offer a process for converting uncertain information into summary formats that may be analyzed and used to make decisions.

MCDM problems are typically addressed using two main categories of methods: conventional approaches such as TOPSIS, VIKOR, and ELECTRE, and those that utilize aggregation operators. Aggregation operators offer a more effective solution to MCDM problems than traditional methods, as they provide comprehensive values for ranking alternatives, unlike conventional approaches that only generate rankings without the comprehensive data needed for decision-making. The operational rules and functions defining aggregation operators are crucial in determining their capabilities and properties for effectively solving complex MCDM problems. In operational rules, certain aggregation operators represent specific instances of elements within the t-norm (TN) and t-conorm (TC) families. The Archimedean t-norm and t-conorm serve as generalizations for numerous TNs and TCs. Various operators and operational rules of q-rung orthopair fuzzy sets (qROFSs) are associated with particular (TNs) and (TCs), such as Frank operational rules (Du et al., 2022), Archimedean aggregation operators (Seikh & Mandal, 2023), and Hamacher operational rules (Khan et al., 2023).

The relationships among aggregated arguments have been examined through various aggregation operators of q-rung orthopair fuzzy sets (qROFSs). These include weighted averaging (WA) and weighted geometric (WG) operators (Liu & Wang, 2018), Bonferroni mean (BM) and geometric Bonferroni mean (GBM) operators (Liu & Liu, 2018), normalized weighted Bonferroni mean (Rodzi et al., 2023), weighted point operators (Xing et al., 2019), fuzzy-valued neutrosophic sets (Al-Quran et al., 2024), Heronian mean (HM) and geometric HM operators (Wei et al., 2018). Yu & Wu (2012) provided a detailed explanation of the advantages of HM operators compared to BM operators in the existing literature. Both aggregation operators can account for interrelationships among aggregated parameters; however, they are limited to addressing decision-making problems involving interrelationships among attributes within the same partition, excluding those between different partitions. Liu et al. (2018) introduced HM operators utilizing qROFSs to address this limitation.

The q-rung orthopair fuzzy Dombi generalized Heronian mean (qROFDGHM) is an advanced aggregation operator that combines the principles of q-rung orthopair fuzzy sets (q-ROFS), Dombi operations, and Heronian means (HMs) to enhance multi-criteria decision-making (MCDM) processes (Yaacob et al., 2024). This aggregation operator effectively encapsulates the interrelationships among the various decision attributes, while also addressing the inherent uncertainties present within the decision-making environment (Sarkar et al., 2023). The qROFDGHM operator effectively mitigates the impact of extreme values while seamlessly integrating the interactions between membership and non-membership degrees of qROFSs, yielding a more balanced and nuanced aggregation method. The flexibility of Dombi operations enables the qROFDGHM to be readily tailored to diverse decision-making contexts, thereby serving as an effective tool for managing complex aggregation tasks (Hussain et al., 2022). Overall, the qROFDGHM operator represents a significant advancement in fuzzy decision-making methodologies, as it provides an innovative and versatile aggregation strategy that effectively addresses the challenges posed by uncertain, imprecise, or conflicting information in multi-criteria decision-making scenarios. This operator's flexibility and ability to seamlessly integrate the interrelationships among decision attributes, while also accounting for the inherent uncertainties, make it a valuable tool for navigating the complexities of real-world decision-making processes.

The paper is structured as follows. Section 2 provides a brief overview of key concepts related to q-rung orthopair fuzzy sets, the DTT, the GHM operator, and the operational rules of qROFNs derived from the DTT. Section 3 introduces a proposed aggregation operator, namely qROFDGHM. The final section summarizes the paper.

https://doi.org/10.24191//mij.v6i1.4630

2. PRELIMINARIES

This section provides an overview of the key concepts and definitions related to q-Rung orthopair fuzzy sets, Dombi operators, and Heronian mean operators.

2.1 q-Rung Orthopair Fuzzy Sets

Definition 1. A qROFS E in a finite universe of discourse X is (Yager, 2017):

$$E = \{\langle x, \mu_E(x), v_E(x)\rangle | x \in X\} \tag{1}$$

where $\mu_E: X \to [0,1]$ denotes the degree of membership of the element $x \in X$ to the set E and $v_E: X \to [0,1]$ denotes the degree of non-membership of the element $x \in X$ to the set E, with the condition that $0 \leq (\mu_E(x)^q + v_E(x)^q) \leq 1$ $(q = 1,2,3, \dots, n)$. The degree of hesitancy (indeterminacy) of the element $x \in X$ to the set E is:

$$\pi_E(x) = \left(1 - \left(\mu_E(x)\right)^q - \left(v_E(x)\right)^q\right)^{\frac{1}{q}} \tag{2}$$

For convenience, a pair $\left(\mu_E(x), v_E(x)\right)$ is called a qROFN and denoted $\sigma = (\mu, v)$. To compare two qROFNs, their score and accuracy must be calculated. The following provides the definitions of the score and accuracy of a qROFN.

Definition 2. Let $\sigma_1 = (\mu_1, v_1)$ and $\sigma_2 = (\mu_2, v_2)$ be a q-ROFNs. Then (Liu & Wang, 2018):

1. $\bar{\sigma_1} = \langle v_1, \mu_1\rangle$
2. $\sigma_1 \vee \sigma_2 = \langle max\{\mu_1, v_1\}, min\{\mu_2, v_2\}\rangle$
3. $\sigma_1 \wedge \sigma_2 = \langle min\{\mu_1, v_1\}, max\{\mu_2, v_2\}\rangle$
4. $\sigma_1 \oplus \sigma_2 = \langle \left(\mu_1^q + \mu_2^q - \mu_1^q \mu_2^q\right)^{1/q}, v_1 v_2\rangle$
5. $\sigma_1 \otimes \sigma_2 = \langle \mu_1 \mu_2, \left(v_1^q + v_2^q - v_1^q v_2^q\right)^{1/q}\rangle$
6. $\lambda\sigma_1 = \langle \left(1 - \left(1 - \mu_1^q\right)^\lambda\right)^{1/q}, v_1^\lambda\rangle$
7. $\sigma_1^\lambda = \langle \mu_1^\lambda, \left(1 - \left(1 - v_1^q\right)^\lambda\right)^{1/q}\rangle$

Definition 3. Let $\sigma = (\mu, v)$ be a qROFN. Then, the score of σ is (Yager, 2017):

$$S(\sigma) = \mu^q - v^q \tag{3}$$

where $-1 \leq S(\sigma) \leq 1$.

Definition 4. Let $\sigma = (\mu, v)$ be a qROFN. Then, the accuracy of σ is (Yager, 2017):

$$A(\sigma) = \mu^q + v^q \tag{4}$$

where $-1 \leq S(\sigma) \leq 1$.

https://doi.org/10.24191//mij.v6i1.4630

Definition 5. Let $\sigma_1 = (\mu_1, v_1)$ and $\sigma_2 = (\mu_2, v_2)$ be two arbitrary qROFNs; let $S(\sigma_1)$ and $S(\sigma_2)$ be the score of σ_1 and σ_2, respectively; and let $A(\sigma_1)$ and $A(\sigma_2)$ be the accuracies of σ_1 and σ_2, respectively. Then (Yager, 2017):

1. If $S(\sigma_1) > S(\sigma_2)$, then $\sigma_1 > \sigma_2$;
2. If $S(\sigma_1) = S(\sigma_2)$, then

(1) If $A(\sigma_1) > A(\sigma_2)$, then $\sigma_1 > \sigma_2$;

(2) If $A(\sigma_1) = A(\sigma_2)$, then $\sigma_1 = \sigma_2$.

2.2 Heronian Mean (HM) Operators

This section then introduces the generalized Heronian mean and generalized weighted Heronian mean operators.

Definition 6 (Generalized Heronian mean (GHM)). Let $x_i (i = 1,2, \dots, n)$ be a set of non-negative real numbers, and $a, b \geq 0$. Then (Sýkora, 2009):

$$GHM^{a,b}(x_1, x_2, \dots, x_n) = \left[\frac{2}{n(n+1)}\left(\sum_{i=1}^{n}\sum_{j=i}^{n} x_i^a x_j^b\right)\right]^{\frac{1}{a+b}} \tag{5}$$

Definition 7 (Generalized weighted Heronian mean (GWHM)). Let $w_i = (w_1, w_2, \dots, w_n)$ be the weight vector of a collection of non-negative numbers $x_i (i = 1,2, \dots, n)$ and satisfying $a, b \geq 0$, $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$. Then (Liu & Pei, 2012):

$$GWHM^{a,b}(x_1, x_2, \dots, x_n) = \left[\frac{2}{n(n+1)}\left(\sum_{i=1}^{n}\sum_{j=i}^{n} (w_i x_i)^a \otimes \left(w_j x_j\right)^b\right)\right]^{\frac{1}{a+b}} \tag{6}$$

2.3 Dombi t-Norm and t-Conorm

The Dombi operations, which comprise Dombi t-norms and t-conorms, were introduced as flexible operators for aggregating fuzzy information. These operations excel at capturing the intricate interactions between membership and non-membership values, rendering them well-suited for applications where conventional operations, such as algebraic t-norms and Einstein t-norms, prove inadequate.

In the following, a new operational rule for qROFNs is presented, which is based on the DTT (Dombi, 1982) to generate a t-norm (TN) and t-conorm (TC):

$$D(x, y) = \delta^{-1}\big(\delta(x) + \delta(y)\big) \tag{7}$$

(1) If $f(x)$ is a monotonically increasing function such that:

$$f(x) : \big(0,1\big] \to R^+; f^{-1}(x) : R^+ \to \big(0,1\big]; \lim_{x\to\infty} f(x)^{-1} = 0\ ;\ f^{-1}(0) = 1,$$

then the TN T can be defined as $T(x, y) = f^{-1}\big(f(x) + f(y)\big)$.

https://doi.org/10.24191//mij.v6i1.4630

(2) If $g(x)$ is a monotonically decreasing function such that:

$$g(x):\left(0,1\right] \to R^{+}; g^{-1}(x): R^{+} \to \left(0,1\right]; \lim_{x\to\infty} g(x)^{-1} = 1\,; g^{-1}(0) = 0,$$

then the TC S can be defined as $S(x,y) = g^{-1}\big(g(x)+g(y)\big)$. According to Beliakov et al. (2016), the relationship of $f(x)$ and $g(x)$ is $f(x) = g(1-x)$.

Definition 8. Let λ be a positive real number and $x, y \in [0,1]$. The DTT and their additive generators are described below (Dombi, 1982):

$$T_{D,\lambda}(x,y) = f^{-1}\big(f(x)+f(y)\big) = \left(\frac{1}{1+\left(\left(\frac{1-x^q}{x^q}\right)^{\lambda}+\left(\frac{1-y^q}{y^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}} \tag{8}$$

$$S_{D,\lambda}(x,y) = g^{-1}\big(g(x)+g(y)\big) = \left(1-\frac{1}{1+\left(\left(\frac{x^q}{1-x^q}\right)^{\lambda}+\left(\frac{y^q}{1-y^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}} \tag{9}$$

$$f(t) = \left(\frac{1-t^q}{t^q}\right)^{\lambda},\ g(t) = \left(\frac{t^q}{1-t^q}\right)^{\lambda} \tag{10}$$

Then, the following can be obtained:

$$f^{-1}(t) = \left(\frac{1}{1+t^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\ g^{-1}(t) = \left(\frac{t^{\frac{1}{\lambda}}}{1+t^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}} \tag{11}$$

Based on DTT (Zhong et al., 2019), a set of operational rules of qROFNs can be established as follows:

Definition 9. Let $\sigma = (\mu, v)$, $\sigma_1 = (\mu_1, v_1)$ and $\sigma_2 = (\mu_2, v_2)$ be three arbitrary qROFNs, and let φ and τ be two arbitrary positive real numbers. Then, the sum, product, multiplication and power operations between qROFNs based on $T_{D,\lambda}(x,y) = f^{-1}\big(f(x)+f(y)\big)$ and $S_{D,\lambda}(x,y) = g^{-1}\big(g(x)+g(y)\big)$ can be defined as follows, respectively (Zhong et al., 2019):

$$\sigma_1 \oplus \sigma_2 = \Big(g^{-1}\big(g(\mu_1)+g(\mu_2)\big), f^{-1}\big(f(v_1)+f(v_2)\big)\Big)$$

$$= \left(\left(1-\frac{1}{1+\left(\left(\frac{\mu_1^q}{1-\mu_1^q}\right)^{\lambda}+\left(\frac{\mu_2^q}{1-\mu_2^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \left(\frac{1}{1+\left(\left(\frac{1-v_1^q}{v_1^q}\right)^{\lambda}+\left(\frac{1-v_2^q}{v_2^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right) \tag{12}$$

https://doi.org/10.24191//mij.v6i1.4630

$$\sigma_1 \otimes \sigma_2 = \left(f^{-1}\left(f(\mu_1)+f(\mu_2)\right), g^{-1}\left(g(v_1)+g(v_2)\right)\right)$$

$$= \left(\left(\frac{1}{1+\left(\left(\frac{1-\mu_1^q}{\mu_1^q}\right)^{\lambda}+\left(\frac{1-\mu_2^q}{\mu_2^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \left(1-\frac{1}{1+\left(\left(\frac{v_1^q}{1-v_1^q}\right)^{\lambda}+\left(\frac{v_2^q}{1-v_2^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right) \tag{13}$$

$$\varphi\sigma = \left(g^{-1}\left(\varphi g(\mu)\right), f^{-1}\left(\varphi f(v)\right)\right)$$

$$= \left(\left(1-\frac{1}{1+\left(\varphi\left(\frac{\mu^q}{1-\mu^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \left(\frac{1}{1+\left(\varphi\left(\frac{1-v^q}{v^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right) \tag{14}$$

$$\sigma^{\tau} = \left(f^{-1}\left(\tau f(\mu)\right), g^{-1}\left(\tau g(v)\right)\right)$$

$$= \left(\left(\frac{1}{1+\left(\tau\left(\frac{1-\mu^q}{\mu^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \left(1-\frac{1}{1+\left(\tau\left(\frac{v^q}{1-v^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right) \tag{15}$$

3. PROPOSED q-RUNG ORTHOPAIR FUZZY DOMBI GENERALIZED HERONIAN MEAN

This section discusses the extended GHM to the q-rung orthopair fuzzy environment, and a q-rung orthopair fuzzy Dombi generalized Heronian mean (qROFDGHM) operator are presented. Their properties are explored. Figure 1 shows the flowchart of the research methodology:

https://doi.org/10.24191//mij.v6i1.4630

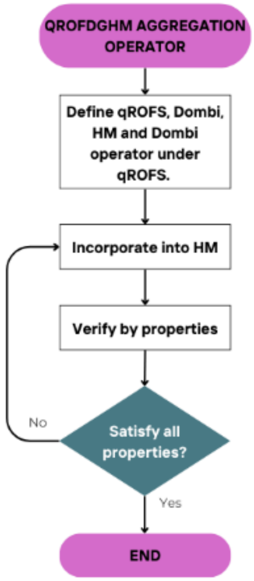

Fig. 1. Flowchart of the research methodology

3.1 qROFDGHM Operator

Definition 10. Let $\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ $\left(where\ \sigma_i = (\mu_i, v_i)(i = 1,2, \dots, n)\right)$ be a collection of qROFNs $(q = 1,2, \dots, n)$. For any two real numbers a and b such that $a, b \geq 0$ but a and b are not zero simultaneously, the qROFDGHM is defined as follows:

$$qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = \left[\frac{2}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \left((\sigma_i)^a \otimes \left(\sigma_j\right)^b\right)\right]^{\frac{1}{a+b}} \tag{16}$$

Theorem 1. Let $\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ $\left(where\ \sigma_i = (\mu_i, v_i)(i = 1,2, \dots, n)\right)$ be a collection of qROFNs $(q = 1,2, \dots, n)$. Let a and b such that $a, b \geq 0$ but a and b are not zero simultaneously and let λ be a positive real number. Then, the aggregated value produced by qROFDGHM is still a qROFN, and $qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n)$.

$$= \left(\left(\frac{1}{1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i) + bf(\mu_j)\right)}}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}, \left(1 - \frac{1}{1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(ag(v_i) + bg(v_j)\right)}}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right) \tag{17}$$

https://doi.org/10.24191//mij.v6i1.4630

where $f(\mu_i)=\left(\frac{1-\mu_i^q}{\mu_i^q}\right)^{\lambda}, f(\mu_j)=\left(\frac{1-\mu_j^q}{\mu_j^q}\right)^{\lambda}, g(v_i)=\left(\frac{v_i^q}{1-v_i^q}\right)^{\lambda}, g(v_j)=\left(\frac{v_j^q}{1-v_j^q}\right)^{\lambda}$.

Proof.

According to Definition 8:

$$(\sigma_i)^a=\left(\left(\frac{1}{1+\left(a\left(\frac{1-\mu_i^q}{\mu_i^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+\left(a\left(\frac{v_i^q}{1-v_i^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right)=\left(\left(\frac{1}{1+a^{\frac{1}{\lambda}}\left(\frac{1-\mu_i^q}{\mu_i^q}\right)}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+a^{\frac{1}{\lambda}}\left(\frac{v_i^q}{1-v_i^q}\right)}\right)^{\frac{1}{q}}\right)$$

$$(\sigma_j)^b=\left(\left(\frac{1}{1+\left(b\left(\frac{1-\mu_j^q}{\mu_j^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+\left(b\left(\frac{v_j^q}{1-v_j^q}\right)^{\lambda}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right)=\left(\left(\frac{1}{1+b^{\frac{1}{\lambda}}\left(\frac{1-\mu_j^q}{\mu_j^q}\right)}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+b^{\frac{1}{\lambda}}\left(\frac{v_j^q}{1-v_j^q}\right)}\right)^{\frac{1}{q}}\right)$$

Let $\frac{1-\mu_i^q}{\mu_i^q}=f(\mu_i)^{\frac{1}{\lambda}}, \frac{1-\mu_j^q}{\mu_j^q}=f(\mu_j)^{\frac{1}{\lambda}}, \frac{v_i^q}{1-v_i^q}=g(v_i)^{\frac{1}{\lambda}}, \frac{v_j^q}{1-v_j^q}=g(v_j)^{\frac{1}{\lambda}}$; then

$$(\sigma_i)^a=\left(\left(\frac{1}{1+\left(af(\mu_i)\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+\left(ag(v_i)\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right)$$

$$(\sigma_j)^b=\left(\left(\frac{1}{1+\left(bf(\mu_j)\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+\left(bg(v_j)\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right)$$

$$(\sigma_i)^a\otimes(\sigma_j)^b=\left(\left(\frac{1}{1+\left(af(\mu_i)+bf(\mu_j)\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\left(1-\frac{1}{1+\left(ag(v_i)+bg(v_j)\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right)$$

$$\sum_{i=1}^{n}\sum_{j=i}^{n}(\sigma_i)^a\otimes(\sigma_j)^b=\left(\left(1-\frac{1}{1+\left(\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(af(\mu_i)+bf(\mu_j)\right)}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}},\left(\frac{1}{1+\left(\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(ag(v_i)+bg(v_j)\right)}\right)^{\frac{1}{\lambda}}}\right)^{\frac{1}{q}}\right)$$

$$\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}(\sigma_i)^a\otimes(\sigma_j)^b$$

https://doi.org/10.24191//mij.v6i1.4630

$$= \left(\left(1 - \frac{1}{1+\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(af(\mu_i)+bf(\mu_j)\right)}\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \left(\frac{1}{1+\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(ag(v_i)+bg(v_j)\right)}\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}} \right)$$

$$\left(\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}(\sigma_i)^a \otimes (\sigma_j)^b\right)^{\frac{1}{a+b}}$$

$$= \left(\left(\frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(af(\mu_i)+bf(\mu_j)\right)}}\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}, \left(1 - \frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(ag(v_i)+bg(v_j)\right)}}\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}} \right)$$

It follows that $qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = \left[\frac{2}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\left((\sigma_i)^a \otimes (\sigma_j)^b\right)\right]^{\frac{1}{a+b}}$

Thus, the proof of Theorem 1 is completed.

Theorem 2 (Idempotency). Let $\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ where $\sigma_i = (\mu_i, v_i)$ $(i = 1,2, \dots, n)$ be a collection of qROFNs $(q = 1,2, \dots)$, and let a and b be two real numbers such that $a, b \geq 0$. However, a and b are not zero simultaneously. If $\sigma_i = \sigma = (\mu, v)$ for all $i = 1,2, \dots, n$, then

$$qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = \sigma \qquad (18)$$

Proof.

Let $qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = (\mu_\alpha, v_\alpha)$. It shows that

$$qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = (\mu, v)$$

Since $\sigma_i = \sigma = (\mu, v)$ and $\sigma_j = \sigma = (\mu, v)$, then

$$\mu_\alpha = \left(\frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)}\sum_{i=1}^{n}\sum_{j=i}^{n}\frac{1}{\left(af(\mu_i)+bf(\mu_j)\right)}}\right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}$$

https://doi.org/10.24191//mij.v6i1.4630

$$= \left(\frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{(af(\mu)+bf(\mu))}} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}$$

$$= \left(\frac{1}{1+\left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{((a+b)f(\mu))}} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}$$

$$= \left(\frac{1}{1+\left(\frac{1}{\frac{1}{(f(\mu))}} \right)^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}$$

$$= \left(\frac{1}{1+(f(\mu))^{\frac{1}{\lambda}}} \right)^{\frac{1}{q}}$$

$$= \left(\frac{1}{1+\left(\frac{1-\mu^q}{\mu^q} \right)} \right)^{\frac{1}{q}}$$

$$= (\mu^q)^{\frac{1}{q}}$$

$$= \mu$$

where $f(\mu_i) = \left(\frac{1-\mu_i^q}{\mu_i^q} \right)^{\lambda}, f(\mu_j) = \left(\frac{1-\mu_j^q}{\mu_j^q} \right)^{\lambda}, g(v_i) = \left(\frac{v_i^q}{1-v_i^q} \right)^{\lambda}, g(v_j) = \left(\frac{v_j^q}{1-v_j^q} \right)^{\lambda}$.

Therefore, it can also show that $v_\alpha = v$. Hence

$$qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = (\mu_\alpha, v_\alpha) = (\mu, v)$$

which completes the proof for Theorem 2.

Theorem 3 (Monotonicity). Let $\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ where $\sigma_i = (\mu_i, v_i)$ $(i = 1,2, \dots, n)$ and $\{\sigma_1', \sigma_2', \dots, \sigma_n'\}$ (where $\sigma_i' = (\mu_i', v_i')$) $(i = 1,2, \dots, n)$ be two collections of qROFNs $(q = 1,2, \dots)$, and let a and b be two real numbers such that $a, b \geq 0$. However, a and b are not zero simultaneously. If $\mu_i \leq \mu_i'$ and $v_i \leq v_i'$ for all $i = 1,2, \dots, n$, then

$$qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) \leq qROFDGHM^{a,b}(\sigma_1', \sigma_2', \dots, \sigma_n') \quad (19)$$

Proof.

Let

$$qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = (\mu_\alpha, v_\alpha)$$

https://doi.org/10.24191//mij.v6i1.4630

$$qROFDGHM^{a,b}(\sigma_1', \sigma_2', \dots, \sigma_n') = (\mu', v')$$

Since $\mu_i \leq \mu_i'$ and $\mu_j \leq \mu_j'$, $f(t)$, $f^{-1}(t)$ are monotonically decreasing, and $g(t)$, $g^{-1}(t)$ are monotonically increasing, it follows that

$$\frac{1-\mu_i^q}{\mu_i^q} \geq \frac{1-\mu_i'^q}{\mu_i'^q}, \frac{1-\mu_j^q}{\mu_j^q} \geq \frac{1-\mu_j'^q}{\mu_j'^q}$$

Thus,

$$\left(af(\mu_i) + bf(\mu_j)\right) \geq \left(af(\mu_i') + bf(\mu_j')\right)$$

$$\left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i) + bf(\mu_j)\right)}\right) \leq \left(\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i') + bf(\mu_j')\right)}\right)$$

and

$$\left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i) + bf(\mu_j)\right)}}\right) \leq \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i') + bf(\mu_j')\right)}}\right)$$

$$\left(1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i) + bf(\mu_j)\right)}}\right)\right) \leq \left(1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i') + bf(\mu_j')\right)}}\right)\right)$$

$$\left(\frac{1}{\left(1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i) + bf(\mu_j)\right)}}\right)\right)}\right) \leq \left(\frac{1}{\left(1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i') + bf(\mu_j')\right)}}\right)\right)}\right)$$

Then

$$\mu_\alpha = \left(\frac{1}{\left(1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i) + bf(\mu_j)\right)}}\right)\right)}\right)^{\frac{1}{q}} \leq \left(\frac{1}{\left(1 + \left(\frac{1}{\frac{2(a+b)}{n(n+1)} \sum_{i=1}^{n} \sum_{j=i}^{n} \frac{1}{\left(af(\mu_i') + bf(\mu_j')\right)}}\right)\right)}\right)^{\frac{1}{q}} = \mu$$

Hence, $\mu_\alpha \leq \mu'$. Similarly, it can be proved that $v_\alpha \leq v'$.

https://doi.org/10.24191//mij.v6i1.4630

Thus, $qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = (\mu_\alpha, v_\alpha) \leq qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) = (\mu, v)$, which completes the proof for Theorem 3.

Theorem 4 (Boundedness). Let $\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ where $\sigma_i = (\mu_i, v_i)$ $(i = 1,2, \dots, n)$ be a collection of qROFNs $(q = 1,2, \dots)$, and let a and b be two real numbers such that $a, b \geq 0$. However, a and b are not zero simultaneously. If $\sigma_s = \left(max(\mu_i), min(v_i)\right)$ and $\sigma_I = \left(min(\mu_i), max(v_i)\right)$, then

$$\sigma_I \leq qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) \leq \sigma_s \tag{20}$$

Proof.

From Theorem 2:

$$qROFDGHM^{a,b}(\sigma_I, \sigma_I, \dots, \sigma_I) = \sigma_I, qROFDGHM^{a,b}(\sigma_S, \sigma_S, \dots, \sigma_S) = \sigma_s$$

From Theorem 3:

$$qROFDGHM^{a,b}(\sigma_I, \sigma_I, \dots, \sigma_I) \leq qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) \leq qROFDGHM^{a,b}(\sigma_S, \sigma_S, \dots, \sigma_S)$$

Therefore, it follows that $\sigma_I \leq qROFDGHM^{a,b}(\sigma_1, \sigma_2, \dots, \sigma_n) \leq \sigma_s$, which completes the proof for Theorem 4.

4. CONCLUSION

The study introduces qROFDGHM, a novel operator that improves the efficiency of MCDM in facing situations of uncertainty. In the qROFS setting, the operator uses the generalized Heronian mean to combine the Dombi t-norm and t-conorm. Verification of idempotency, monotonicity, and boundedness properties provides its theoretical underpinnings. The proposed method offers a novel, adaptable, and reliable aggregation operator that enhances the ability to identify fuzzy parameter relationships. Due to the absence of experimental evaluation, the current research is limited to theoretical evaluation. Future studies will focus on evaluating the operator's efficacy by applying it to real-world case studies, implementing it, and comparing it to existing aggregation operators.

5. ACKNOWLEDGEMENTS/FUNDING

The authors gratefully acknowledge the Universiti Teknologi MARA (UiTM), Negeri Sembilan branch, for their support in this research.

6. CONFLICT OF INTEREST STATEMENT

The authors declare there is no conflict of interest in the subject matter or materials discussed in this manuscript.

7. AUTHORS' CONTRIBUTIONS

Muhammad Mukhlis Kamarul Zaman contributed to the development of the methodology, conducted the data analysis. Zahari Md Rodzi was responsible for reviewing and editing the manuscript, ensuring its quality and coherence. Yusrina Andu assisted in the validation process and provided valuable feedback through review and editing, contributing to the overall refinement of the work.

https://doi.org/10.24191//mij.v6i1.4630

REFERENCES

Alkan, N., & Kahraman, C. (2024). CODAS extension using novel decomposed Pythagorean fuzzy sets: Strategy selection for IOT based sustainable supply chain system. *Expert Systems with Applications*, *237*, 121534. https://doi.org/10.1016/j.eswa.2023.121534

Al-Quran, A., Al-Sharqi, F., Rahman, A. U., & Rodzi, Z. M. (2024). The q-rung orthopair fuzzy-valued neutrosophic sets: Axiomatic properties, aggregation operators and applications. *AIMS Mathematics*, *9*(2), 5038–5070. https://doi.org/10.3934/math.2024245

Atanassov, K. T. (1986). Intuitionistic fuzzy sets. *Fuzzy Sets and Systems*, *20*(1), 87–96. https://doi.org/10.1016/S0165-0114(86)80034-3

Beliakov, G., Bustince Sola, H., & Calvo Sánchez, T. (2016). Review of aggregation functions. *Studies in Fuzziness and Soft Computing*, *329*, 1–53. https://doi.org/10.1007/978-3-319-24753-3_1

Ch, T. H., Abid, M., Naeem, K., & Zulqarnain, R. M. (2025). Correlation measures for q-rung orthopair m-polar fuzzy sets with application to pattern recognition. *Decision Making Advances*, *3*(1), 139–163. https://doi.org/10.31181/dma31202560

Dombi, J. (1982). A general class of fuzzy operators, the DeMorgan class of fuzzy operators and fuzziness measures induced by fuzzy operators. *Fuzzy Sets and Systems*, *8*(2), 149–163. https://doi.org/10.1016/0165-0114(82)90005-7

Du, Y., Du, X., Li, Y., Cui, J. X., & Hou, F. (2022). Complex q-rung orthopair fuzzy Frank aggregation operators and their application to multi-attribute decision making. *Soft Computing*, *26*(22), 11973–12008. https://doi.org/10.1007/s00500-022-07465-2

Ghoushchi, S. J., Vahabzadeh, S., & Pamucar, D. (2024). Applying hesitant q-rung orthopair fuzzy sets to evaluate uncertainty in subsidence causes factors. *Heliyon*, *10*(8), e29415. https://doi.org/10.1016/j.heliyon.2024.e29415

Hemalatha, K., & Venkateswarlu, B. (2023). Pythagorean fuzzy transportation problem: New way of ranking for Pythagorean fuzzy sets and mean square approach. *Heliyon*, *9*(10), e20775. https://doi.org/10.1016/j.heliyon.2023.e20775

Hussain, A., Mahmood, T., Ali, M. I., & Iampan, A. (2022). Q-rung orthopair fuzzy soft aggregation operators based on Dombi t-norm and t-conorm with their applications in decision making. *Journal of Intelligent & Fuzzy Systems*, *43*(5), 5685–5702. https://doi.org/10.3233/JIFS-212921

Khan, M. R., Ullah, K., Raza, A., Ali, Z., Senapati, T., Esztergár-Kiss, D., & Moslem, S. (2025). Evaluating safety in Dublin's bike-sharing system using the concept of intuitionistic fuzzy rough power aggregation operators. *Measurement*, *253*, 117553. https://doi.org/10.1016/j.measurement.2025.117553

Khan, S., Gulistan, M., Kausar, N., Pamucar, D., Ozbilge, E., & El-Kanj, N. (2023). Q-rung orthopair fuzzy hypersoft ordered aggregation operators and their application towards green supplier. *Frontiers in Environmental Science*, *10*, 1048019. https://doi.org/10.3389/fenvs.2022.1048019

https://doi.org/10.24191//mij.v6i1.4630

Lei, Q., & Xu, Z. (2016). Chain and substitution rules of intuitionistic fuzzy calculus. *IEEE Transactions on Fuzzy Systems*, *24*(3), 519–529. https://doi.org/10.1109/TFUZZ.2015.2450832

Liu, H. Z., & Pei, D. W. (2012). HOWA operator and its application to multi-attribute decision making. *Journal of Zhejiang Sci-Tech University*, *25*, 138–142.

Liu, P., & Liu, J. (2018). Some q-rung orthopai fuzzy Bonferroni mean operators and their application to multi-attribute group decision making. *International Journal of Intelligent Systems*, *33*(2), 315–347. https://doi.org/10.1002/int.21933

Liu, P., & Wang, P. (2018). Some q-rung orthopair fuzzy aggregation operators and their applications to multiple-attribute decision making. *International Journal of Intelligent Systems*, *33*(2), 259–280. https://doi.org/10.1002/int.21927

Liu, Z., Wang, S., & Liu, P. (2018). Multiple attribute group decision making based on q-rung orthopair fuzzy Heronian mean operators. *International Journal of Intelligent Systems*, *33*(12), 2341–2363. https://doi.org/10.1002/int.22032

Ngan, S. C. (2024). An extension framework for creating operators and functions for intuitionistic fuzzy sets. *Information Sciences*, *666*, 120336. https://doi.org/10.1016/j.ins.2024.120336

Peng, X., & Dai, J. (2019). Research on the assessment of classroom teaching quality with q-rung orthopair fuzzy information based on multiparametric similarity measure and combinative distance-based assessment. *International Journal of Intelligent Systems*, *34*(7), 1588–1630. https://doi.org/10.1002/int.22109

Peng, X., Dai, J., & Garg, H. (2018). Exponential operation and aggregation operator for q-rung orthopair fuzzy set and their decision-making method with a new score function. *International Journal of Intelligent Systems*, *33*(11), 2255–2282. https://doi.org/10.1002/int.22028

Rahim, M., Amin, F., Shah, K., Abdeljawad, T., & Ahmad, S. (2024). Some distance measures for Pythagorean cubic fuzzy sets: Application selection in optimal treatment for depression and anxiety. *MethodsX*, *12*, 102678. https://doi.org/10.1016/j.mex.2024.102678

Rahman, K., Garg, H., Ali, R., Alfalqi, S. H., & Lamoudan, T. (2023). Algorithms for decision-making process using complex Pythagorean fuzzy set and its application to hospital siting for COVID-19 patients. *Engineering Applications of Artificial Intelligence*, *126*, 107153. https://doi.org/10.1016/j.engappai.2023.107153

Rodzi, Z. M., Amin, F. A. M., Jamiatun, N., Qaiyyum, A., Al-Sharqi, F., Zaharudin, Z. A., & Khair, M. H. M. (2023). Integrated single-valued neutrosophic normalized weighted Bonferroni mean (SVNNWBM)-DEMATEL for analyzing the key barriers to halal certification adoption in Malaysia. *International Journal of Neutrosophic Science*, *21*(3), 106–114. https://doi.org/10.54216/IJNS.210310

Sadeghi, J., Sarvari, H., Zangeneh, S., Fatemi, A., & Edwards, D. J. (2024). An analysis of intuitionistic fuzzy sets in risk-based inspection: A case study of hydrogen crack damage in steel tanks under gas pressure. *Cleaner Engineering and Technology*, *23*, 100855. https://doi.org/10.1016/j.clet.2024.100855

https://doi.org/10.24191//mij.v6i1.4630

Sarkar, A., Moslem, S., Esztergár-Kiss, D., Akram, M., Jin, L. S., & Senapati, T. (2023). A hybrid approach based on dual hesitant q-rung orthopair fuzzy Frank power partitioned Heronian mean aggregation operators for estimating sustainable urban transport solutions. *Engineering Applications of Artificial Intelligence*, *124*, 106505. https://doi.org/10.1016/j.engappai.2023.106505

Seal, S., Ray, P., Deb, S. C., & Dey, A. (2024). A goal programming approach based on score function of the Pythagorean fuzzy sets. *Research Square*. https://doi.org/10.21203/rs.3.rs-5261491/v1

Seikh, M. R., & Mandal, U. (2023). Q-rung orthopair fuzzy Archimedean aggregation operators: Application in the site selection for software operating units. *Symmetry*, *15*(9), 1680. https://doi.org/10.3390/sym15091680

Singh, S., Lalotra, S., & Ganie, A. H. (2020). On some knowledge measures of intuitionistic fuzzy sets of type two with application to MCDM. *Cybernetics and Information Technologies*, *20*(1), 3–20. https://doi.org/10.2478/cait-2020-0001

Sýkora, S. (2009). Mathematical means and averages: Generalized Heronian means. *Stan's Library: Castano Primo, Italy*. https://doi.org/10.3247/SL3Math09.002

Voskoglou, M. G. (2023). Applications of intuitionistic fuzzy sets to assessment, decision making and to topological spaces. *Journal of AppliedMath*, *1*(4), 325. https://doi.org/10.59400/jam.v1i4.325

Wang, D., Yuan, Y., Liu, Z., Zhu, S., & Sun, Z. (2024). Novel distance measures of q-rung orthopair fuzzy sets and their applications. *Symmetry*, *16*(5), 574. https://doi.org/10.3390/sym16050574

Wang, H., Ju, Y., & Liu, P. (2019). Multi-attribute group decision-making methods based on q-rung orthopair fuzzy linguistic sets. *International Journal of Intelligent Systems*, *34*(6), 1129–1157. https://doi.org/10.1002/int.22089

Wei, G., Gao, H., & Wei, Y. (2018). Some q-rung orthopair fuzzy Heronian mean operators in multiple attribute decision making. *International Journal of Intelligent Systems*, *33*(7), 1426–1458. https://doi.org/10.1002/int.21985

Xing, Y., Zhang, R., Zhou, Z., & Wang, J. (2019). Some q-rung orthopair fuzzy point weighted aggregation operators for multi-attribute decision making. *Soft Computing*, *23*, 11627–11649. https://doi.org/10.1007/s00500-018-03712-7

Yaacob, S. N., Hashim, H., Awang, N. A., Sulaiman, N. H., Al-Quran, A., & Abdullah, L. (2024). Bipolar neutrosophic Dombi-based Heronian mean operators and their application in multi-criteria decision-making problems. *International Journal of Computational Intelligence Systems*, *17*, 181. https://doi.org/10.1007/s44196-024-00544-2

Yager, R. R. (2014). Pythagorean membership grades in multicriteria decision making. *IEEE Transactions on Fuzzy Systems*, *22*(4), 958–965. https://doi.org/10.1109/TFUZZ.2013.2278989

Yager, R. R. (2017). Generalized orthopair fuzzy sets. *IEEE Transactions on Fuzzy Systems*, *25*(5), 1222–1230. https://doi.org/10.1109/TFUZZ.2016.2604005

https://doi.org/10.24191//mij.v6i1.4630

Yin, S., Wang, Y., & Shafiee, S. (2023). Ranking products through online reviews considering the mass assignment of features based on BERT and q-rung orthopair fuzzy set theory. *Expert Systems with Applications*, *213*, 119142. https://doi.org/10.1016/j.eswa.2022.119142

Yu, D., & Wu, Y. (2012). Interval-valued intuitionistic fuzzy Heronian mean operators and their application in multi-criteria decision making. *African Journal of Business Management*, *6*(11), 4158-4168. https://doi.org/10.5897/ajbm11.2267

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. https://doi.org/10.1016/S0019-9958(65)90241-X

Zhong, Y., Gao, H., Guo, X., Qin, Y., Huang, M., & Luo, X. (2019). Dombi power partitioned Heronian mean operators of q-rung orthopair fuzzy numbers for multiple attribute group decision making. *PLoS ONE*, *14*(10), e0222007. https://doi.org/10.1371/journal.pone.0222007

https://doi.org/10.24191//mij.v6i1.4630