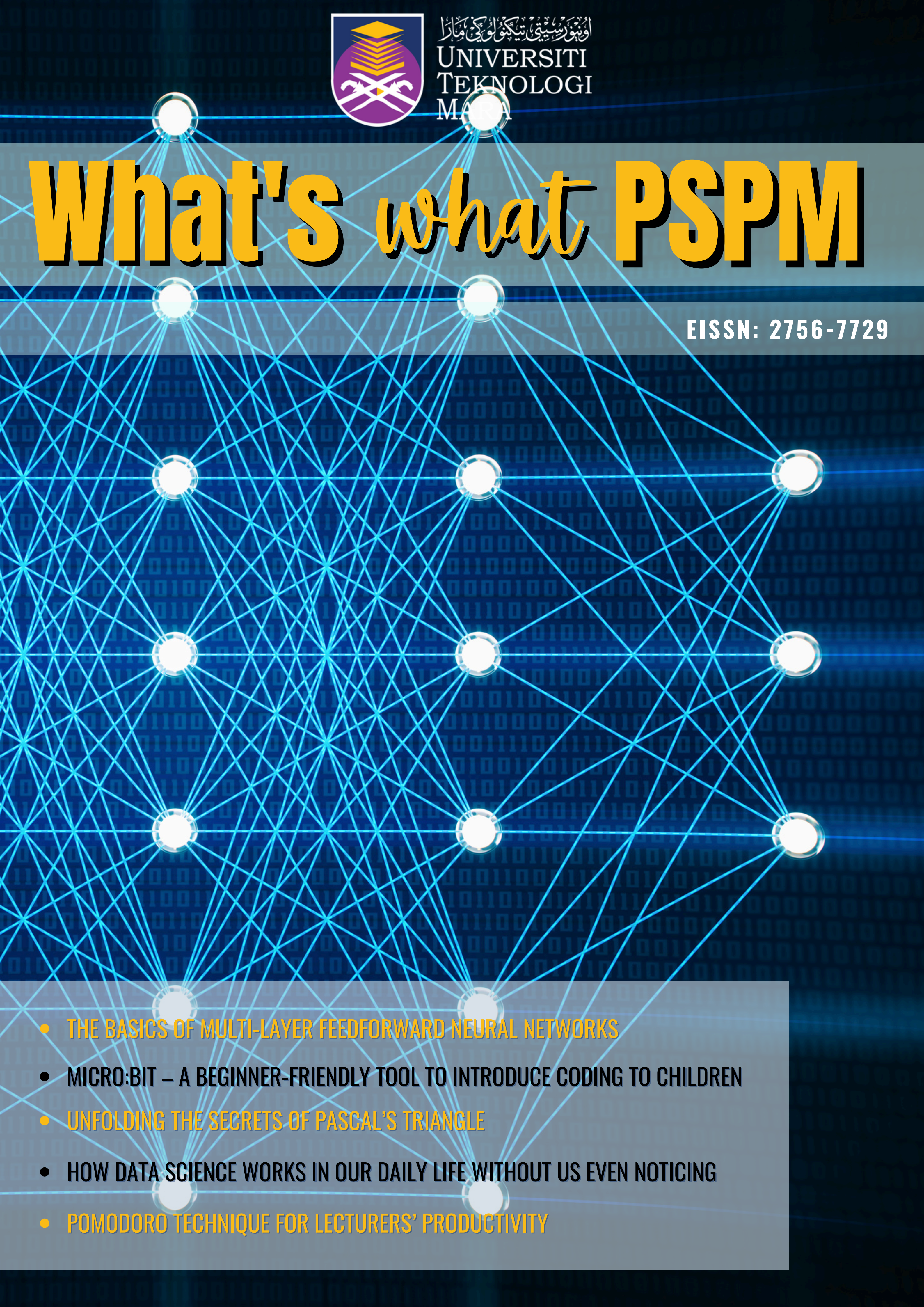

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- THE BASICS OF MULTI-LAYER FEEDFORWARD NEURAL NETWORKS
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- UNFOLDING THE SECRETS OF PASCAL'S TRIANGLE
- HOW DATA SCIENCE WORKS IN OUR DAILY LIFE WITHOUT US EVEN NOTICING
- POMODORO TECHNIQUE FOR LECTURERS' PRODUCTIVITY

UNFOLDING THE SECRETS OF PASCAL'S TRIANGLE: PATTERNS, APPLICATIONS AND MATHEMATICAL BEAUTY

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One of mathematics' most fascinating and useful structures is Pascal's Triangle. Although it appears to be a straightforward triangle of integers at first glance, it actually has a surprisingly large number of mathematical properties and practical uses. From probability and algebra to computer science and the natural world, Pascal's Triangle patterns can be found in unexpected areas. This article explores the depth, beauty, and practical uses of this remarkable mathematical structure.

HISTORY

Blaise Pascal (as shown in Figure 1), born on June 19, 1623, in Clermont-Ferrand, France, was a brilliant mathematician and inventor [1]. In 1653, he wrote the Treatise on the Arithmetical Triangle (Figure 3), now known as Pascal's Triangle. Although named after Blaise Pascal, the triangle was studied earlier by Omar Khayyam, Yang Hui (Figure 2), and Indian mathematicians [2]. Despite this, Pascal identified most of its key properties and applications. Beyond this, he made major contributions to geometry, laid the foundations of probability and calculus, and invented the Pascaline calculator, but he remains best known for his work on Pascal's Triangle [1].

Figure 1: Blaise Pascal

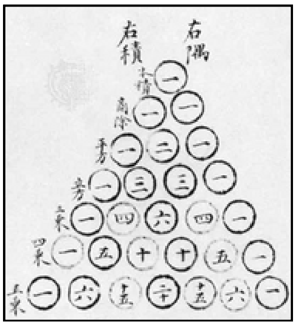

Figure 2: Yang Hui's triangle

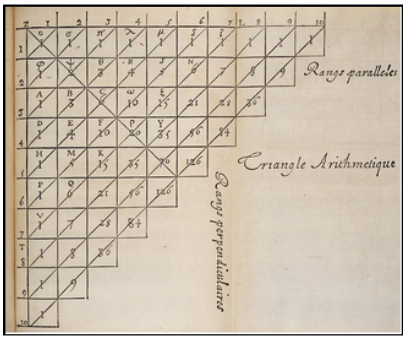

Figure 3: Blaise Pascal's illustration of the Arithmetic Triangle, now known as Pascal's Triangle

$$
\begin{array}{cccccccccccc}
 & & & & & \binom{0}{0} & & & & & \\
 & & & & \binom{1}{0} & & \binom{1}{1} & & & & \\
 & & & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & & & \\
 & & \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} & & \\
 & \binom{4}{0} & & \binom{4}{1} & & \binom{4}{2} & & \binom{4}{3} & & \binom{4}{4} & \\
\binom{5}{0} & & \binom{5}{1} & & \binom{5}{2} & & \binom{5}{3} & & \binom{5}{4} & & \binom{5}{5}
\end{array}
$$

Figure 4: The first six rows of Pascal's triangle as binomial coefficients

CONSTRUCTION OF PASCAL'S TRIANGLE

Pascal's Triangle is built using a simple recursive rule: each number is the sum of the two numbers directly above it [3]. It starts with a single 1 at the top and each row grows symmetrically. The first row, gives the coefficient for the expansion of $(a + b)^0 = 1$, the second row gives the coefficients for $(a + b)^1 = a + b$, the third row gives the coefficients for $(a + b)^2 = a^2 + 2ab + b^2$ and so on [2].

There are also applications of Pascal's triangle in the calculation of combinations and probabilities. Mathematically, the element in row n and position k is given by the binomial coefficient [2]:

$$\mathbf{C}(n,k) = \mathbf{C}_k^n = {}_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

For example: If you flip a coin 4 times, the number of ways to get exactly 2 heads is:

$$\binom{4}{2} = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = \frac{4\times3\times2\times1}{2\times1\times2\times1} = 6$$

Hence, row 4 position 2 in Pascal's Triangle as illustrated in Figure 4 is 6.

PATTERNS AND PROPERTIES OF PASCAL'S TRIANGLE

It has several interesting patterns and properties that arise from its structure, which have been widely studied in mathematics. Here are some of the key patterns and properties of Pascal's Triangle:

Binomial Coefficients :

Each row in Figure 5 represents the coefficients in the expansion of $(a + b)^n$.

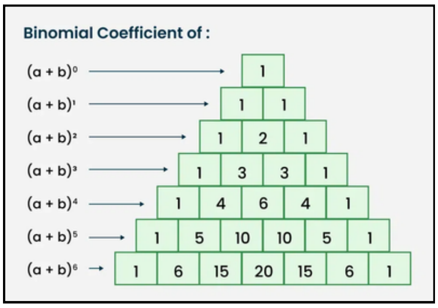

Figure 5: Binomial coefficient

Symmetry :

The triangle is symmetrical. The numbers on the left side as shown in figure 6 have identical matching numbers on the right side [5].

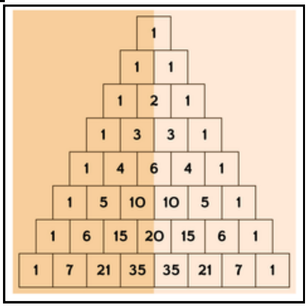

Figure 6: Symmetrical properties

Diagonal Patterns:

Natural Numbers:

The second diagonal contains counting numbers: 1, 2, 3, 4...

Triangular Numbers:

The third diagonal represents triangular numbers: 1, 3, 6, 10...

Tetrahedral Numbers:

The fourth diagonal represents tetrahedral numbers: 1, 4, 10, 20..

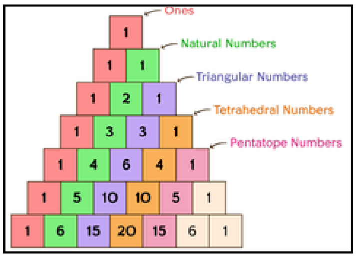

Figure 7: Diagonal Patterns Within the Pascal's Triangle

Fibonacci Sequence:

Summing numbers along slanted diagonals in Figure 9 produces the Fibonacci sequence [6].

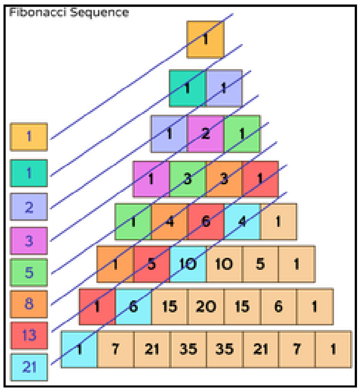

Figure 8: Fibonacci Sequence

Exponents of 11:

Each line in Figure 9 is also the powers (exponents) of 11 [5].

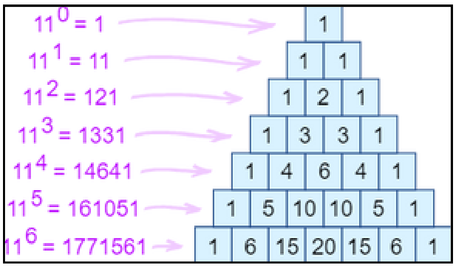

Figure 9: Exponents of 11

Power of 2:

Summing the numbers of each row (Figure 10) produces the series of Powers of 2.

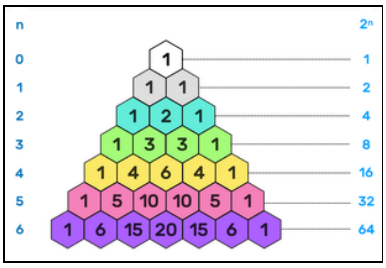

Figure 10: Power of 2

REAL-WORLD APPLICATIONS OF PASCAL'S TRIANGLE

Pascal's Triangle isn't just for fun; it's practical too! It has many surprising and practical real-world applications across math, science, and even daily life. Here are some examples in various fields:

Binomial Expansion

Pascal's Triangle provides the coefficients for expanding binomials of the form $(a + b)^n$ as shown in figure 11.

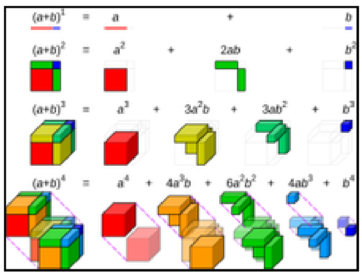

Figure 11: Binomial Expansion

Probability and Combinations

The triangle's numbers represent binomial coefficients, which also calculate combinations.

Electrical Engineering
It is used in designing certain circuit layouts and error-correcting codes.

Geometry and Fractals
When you color only the odd numbers in Pascal's Triangle, it forms the Sierpinski Triangle (Figure 12) a famous fractal. This demonstrates the deep connection between number theory and geometry.

Physics
It is used in probability distributions, wave and quantum mechanics and statistical mechanics.

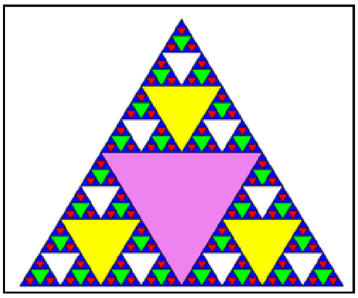

Figure 12: The Sierpinski Triangle

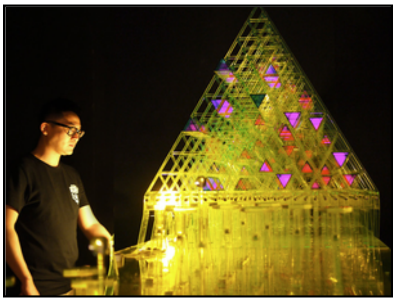

Figure 13: Architectural sculpture 'lost in pascal's triangle' by super nature design

Computer Science
It is used in algorithms for various applications including cryptography, data compression and image processing.

Coding and Error Correction
In digital communication, Pascal's Triangle is used in binomial coding systems and error-detecting algorithms like Hamming codes.

Art and Design
The triangle's patterns have inspired artists, architects and designers as depicted in Figure 13 and Figure 14 in creating aesthetically pleasing and mathematically harmonious compositions.

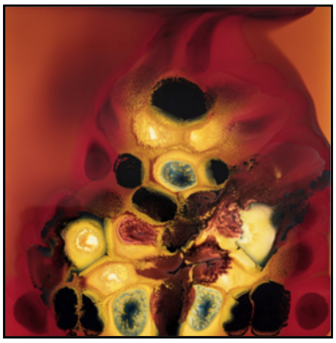

Figure 14: Pascal's Triangle Nature Painting

Natural Phenomena
Pascal's Triangle patterns appear in natural growth processes and biological structures (Figure 15) like the branching of trees, arrangement of petals, and spirals in shells often reflecting binomial and Fibonacci relationships.

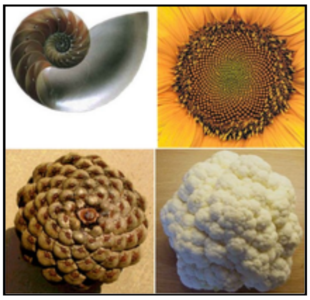

Figure 15: Interesting spiral patterns that are found in nature

CONCLUSION

Pascal's Triangle is far more than a simple array of numbers. It provides a starting point for comprehending fundamental mathematical ideas and actual occurrences. It is fundamental to both pure and applied mathematics due to its beauty, complexity, and broad range of applications.

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