implicit Differentiation

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PREFACE

This e-book, “Implicit Differentiation” aimed to help students in Calculus subject. Targeted users for this e-book are students who take Calculus course in pre-university. Calculus is more than numbers and equations – it’s a way of thinking that unlocks the mysteries of the universe. This ebook is your gateway to exploring patterns, solving problems, and discovering the logic that shapes our world. Whether you’re new or experienced, Calculus is connected to everything. With clear examples, this ebook invites you to see Calculus not as a task, but as an adventure. Welcome to the world of Calculus – let’s explore !

Technique of Differentiation

1) Power Function

$$f(x) = x^n$$

$$\frac{d}{dx}x^n = nx^{n-1}$$

2) Constant Multiple Rule

$$\frac{d}{dx}[c\,f(x)] = c\frac{d}{dx}f(x)$$

3) The Sum Rule

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

4) The Differrence Rule

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

5) The Product Rule

$$uv' + vu'$$

OR

$$u\frac{dv}{dx} + v\frac{du}{dx}$$

6) The Quotient Rule

$$\frac{vu' - uv'}{v^2}$$

OR

$$\frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

IMPLICIT DIFFERENTIATION

1. $2x^2 - 4y^2 = 7xy$
2. $3xy^2 + 5x^3 = 6e^y$
3. $\sin(5x + 5y) = -3x^2 + 6xy$
4. $6ye^{1-2x} + 7\sin(2y) = 4x^4 + 8x$
5. $-4xy^2 - 6x^3 = 8\tan(2x + y) - e^{-3y}$
6. $\sin y + x^{-3} + 5y = \cos x$
7. $2x - \cos x^2 + \frac{y^2}{x} + 4x^6 = 3x^2$

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IMPLICIT DIFFERENTIATION

6. $\sin y + x^{-3} + 5y = \cos x$

7. $2x - \cos x^2 + \frac{y^2}{x} + 4x^6 = 3x^2$

8. $y\sqrt{x} + x\sqrt{y} = 16$

9. $x^2y + y^2x = 2$

10. $y = \sin xy$

1 $2x^2 - 4y^2 = 7xy$

$$u = 7x \qquad v = y$$

$$u' = 7 \qquad v' = \frac{dy}{dx}$$

$$uv' + vu' = (7x)\left(\frac{dy}{dx}\right) + (y)(7)$$

$$= 7x\frac{dy}{dx} + 7y$$

1 $2x^2 - 4y^2 = 7xy$

$$\frac{d}{dx}(2x^2) - \frac{d}{dx}(4y^2) = \frac{d}{dx}(7xy)$$

$$4x - 8y\frac{dy}{dx} = 7x\frac{dy}{dx} + 7y$$

$$-8y\frac{dy}{dx} - 7x\frac{dy}{dx} = 7y - 4x$$

$$\frac{dy}{dx}(-8y - 7x) = 7y - 4x$$

$$\frac{dy}{dx} = \frac{7y - 4x}{-8y - 7x}$$

2 $3xy^2 + 5x^3 = 6e^y$

$$u = 3x \qquad v = y^2$$

$$u' = 3 \qquad v' = 2y\frac{dy}{dx}$$

$$uv' + vu' = (3x)\left(2y\frac{dy}{dx}\right) + (y^2)(3)$$

$$= 6xy\frac{dy}{dx} + 3y^2$$

2 $3xy^2 + 5x^3 = 6e^y$

$$\frac{d}{dx}(3xy^2) + \frac{d}{dx}5x^3 = \frac{d}{dx}6e^y$$

$$6xy\frac{dy}{dx} + 3y^2 + 15x^2 = 6e^y\frac{dy}{dx}$$

$$6xy\frac{dy}{dx} - 6e^y\frac{dy}{dx} = -3y^2 - 15x^2$$

$$\frac{dy}{dx}(6xy - 6e^y) = -3y^2 - 15x^2$$

$$\frac{dy}{dx} = \frac{-3y^2 - 15x^2}{6xy - 6e^y}$$

3

$$\sin(5x + 5y) = -3x^2 + 6xy$$

$$\frac{d}{dx}(\sin(5x+5y)) = \frac{d}{dx}(-3x^2) + \frac{d}{dx}(6xy)$$

$$\cos(5x+5y)\frac{d}{dx}(5x+5y) = -6x + \left(6x\frac{dy}{dx} + 6y\right)$$

$$5\cos(5x+5y) + 5\frac{dy}{dx}\cos(5x+5y) = -6x + 6x\frac{dy}{dx} + 6y$$

$$5\frac{dy}{dx}\cos(5x+5y) - 6x\frac{dy}{dx} = -6x + 6y - 5\cos(5x+5y)$$

$$\frac{dy}{dx}(5\cos(5x+5y) - 6x) = -6x + 6y - 5\cos(5x+5y)$$

$$\frac{dy}{dx} = \frac{-6x + 6y - 5\cos(5x+5y)}{5\cos(5x+5y) - 6x}$$

3 $\sin(5x + 5y) = -3x^2 + 6xy$

$$u = 6x \qquad v = y$$

$$u' = 6 \qquad v' = \frac{dy}{dx}$$

$$uv' + vu' = (6x)\left(\frac{dy}{dx}\right) + (y)(6)$$

$$= 6x\frac{dy}{dx} + 6y$$

4 $6ye^{1-2x} + 7\sin(2y) = 4x^4 + 8x$

$$\frac{d}{dx}(6ye^{1-2x}) + \frac{d}{dx}(7\sin(2y)) = \frac{d}{dx}(4x^4) + \frac{d}{dx}(8x)$$

$$-12y(e^{1-2x}) + 6e^{1-2x}\frac{dy}{dx} + [7\cos(2y) \times \frac{d}{dx}(2y)] = 16x^3 + 8$$

$$-12y(e^{1-2x}) + 6e^{1-2x}\frac{dy}{dx} + 14\cos(2y)\frac{dy}{dx} = 16x^3 + 8$$

$$6e^{1-2x}\frac{dy}{dx} + 14\cos(2y)\frac{dy}{dx} = 16x^3 + 8 + 12y(e^{1-2x})$$

$$\frac{dy}{dx}(6e^{1-2x} + 14\cos(2y)) = 16x^3 + 8 + 12y(e^{1-2x})$$

$$\frac{dy}{dx} = \frac{16x^3 + 8 + 12y(e^{1-2x})}{6e^{1-2x} + 14\cos(2y)}$$

4

$$6ye^{1-2x} + 7\sin(2y) = 4x^4 + 8x$$

$$u = 6y \qquad v = e^{1-2x}$$

$$u' = 6\frac{dy}{dx} \qquad v' = e^{1-2x} \times \frac{d}{dx}(1-2x)$$

$$= -2e^{1-2x}$$

$$uv' + vu' = (6y)(-2e^{1-2x}) + (e^{1-2x})(6\frac{dy}{dx})$$

$$= -12y(e^{1-2x}) + 6e^{1-2x}\frac{dy}{dx}$$

5 $-4xy^2 - 6x^3 = 8\tan(2x + y) - e^{-3y}$

$$u = -4x \qquad v = y^2$$

$$u' = -4 \qquad v' = 2y\frac{dy}{dx}$$

$$uv' + vu' = (-4x)\left(2y\frac{dy}{dx}\right) + (y^2)(-4)$$

$$= -8xy\frac{dy}{dx} - 4y^2$$

5 $-4xy^2 - 6x^3 = 8\tan(2x+y) - e^{-3y}$

$$\frac{d}{dx}(-4xy^2) - \frac{d}{dx}(6x^3) = \frac{d}{dx}(8\tan(2x+y)) - \frac{d}{dx}(e^{-3y})$$

$$-8xy\frac{dy}{dx} - 4y^2 - 18x^2 = \left[8\,sec^2(2x+y) \times \frac{d}{dx}(2x+y)\right] - \left[e^{-3y} \times \frac{d}{dx}(-3y)\right]$$

$$-8xy\frac{dy}{dx} - 4y^2 - 18x^2 = 16\,sec^2(2x+y) + 8\,sec^2(2x+y)\frac{dy}{dx} + 3e^{-3y}\frac{dy}{dx}$$

$$-8xy\frac{dy}{dx} - 8\,sec^2(2x+y)\frac{dy}{dx} - 3e^{-3y}\frac{dy}{dx} = 16\,sec^2(2x+y) + 4y^2 + 18x^2$$

$$\frac{dy}{dx}(-8xy - 8\,sec^2(2x+y) - 3e^{-3y}) = 16\,sec^2(2x+y) + 4y^2 + 18x^2$$

$$\frac{dy}{dx} = \frac{16\,sec^2(2x+y) + 4y^2 + 18x^2}{-8xy - 8\,sec^2(2x+y) - 3e^{-3y}}$$

6

$$\sin y + x^{-3} + 5y = \cos x$$

$$\frac{d}{dx}(\sin y) + \frac{d}{dx}(x^{-3}) + \frac{d}{dx}(5y) = \frac{d}{dx}(\cos x)$$

$$\cos y \frac{d}{dx}(y) + (-3x^{-4}) + 5y\frac{dy}{dx} = -\sin x$$

$$\cos y \frac{dy}{dx} - 3x^{-4} + 5y\frac{dy}{dx} = -\sin x$$

$$\cos y \frac{dy}{dx} + 5y\frac{dy}{dx} = -\sin x + 3x^{-4}$$

$$\frac{dy}{dx}(\cos y + 5y) = -\sin x + 3x^{-4}$$

$$\frac{dy}{dx} = \frac{-\sin x + 3x^{-4}}{\cos y + 5y}$$

7 $2x - \cos x^2 + \frac{y^2}{x} + 4x^6 = 3x^2$

$$u = y^2 \qquad v = x$$

$$u' = 2y\frac{dy}{dx} \qquad v' = 1$$

$$\frac{vu' - uv'}{v^2} = \frac{(x)\left(2y\frac{dy}{dx}\right) - (y^2)(1)}{(x)^2}$$

$$= \frac{2y}{x}\frac{dy}{dx} - \frac{y^2}{x^2}$$

7

$$2x - \cos x^2 + \frac{y^2}{x} + 4x^6 = 3x^2$$

$$\frac{d}{dx}(2x) - \frac{d}{dx}(\cos x^2) + \frac{d}{dx}\left(\frac{y^2}{x}\right) + \frac{d}{dx}(4x^6) = \frac{d}{dx}(3x^2)$$

$$2 - (-\sin x^2 \frac{d}{dx}(x^2)) + (\frac{2y}{x}\frac{dy}{dx} - \frac{y^2}{x^2}) + 24x^5 = 6x$$

$$2 + 2x \sin x^2 + \frac{2y}{x}\frac{dy}{dx} - (\frac{y}{x})^2 + 24x^5 = 6x$$

$$\frac{2y}{x}\frac{dy}{dx} = 6x - 2 - 2x \sin x^2 + (\frac{y}{x})^2 - 24x^5$$

$$\frac{dy}{dx} = \frac{6x - 2 - 2x \sin x^2 + (\frac{y}{x})^2 - 24x^5}{\frac{2y}{x}}$$

$$= 6x - 2 - 2x \sin x^2 + (\frac{y}{x})^2 - 24x^5 \times \frac{x}{2y}$$

$$= \frac{6x^2 - 2x - 2x^2 \sin x^2 + \frac{y^2}{x} + 24x^6}{2y}$$

8 $y\sqrt{x} + x\sqrt{y} = 16$

$$u = y \qquad v = x^{\frac{1}{2}}$$

$$u' = \frac{dy}{dx} \qquad v' = \frac{1}{2}x^{-\frac{1}{2}}$$

$$uv' + vu' = (y)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + \left(x^{\frac{1}{2}}\right)\left(\frac{dy}{dx}\right)$$

$$= \frac{1}{2}xy^{-\frac{1}{2}} + x^{\frac{1}{2}}\frac{dy}{dx}$$

8 $y\sqrt{x} + x\sqrt{y} = 16$

$$u = x \qquad v = y^{\frac{1}{2}}$$

$$u' = 1 \qquad v' = \frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}$$

$$uv' + vu' = (x)\left(\frac{1}{2}y^{-\frac{1}{2}}\frac{dy}{dx}\right) + \left(y^{\frac{1}{2}}\right)(1)$$

$$= \frac{1}{2}xy^{-\frac{1}{2}}\frac{dy}{dx} + y^{\frac{1}{2}}$$

8 $y\sqrt{x} + x\sqrt{y} = 16$

$$\frac{d}{dx}(y\sqrt{x}) + \frac{d}{dx}(x\sqrt{y}) = \frac{d}{dx}(16)$$

$$\frac{d}{dx}\left(y(x)^{\frac{1}{2}}\right) + \frac{d}{dx}\left(x(y)^{\frac{1}{2}}\right) = \frac{d}{dx}\ (16)$$

$$\left(\frac{1}{2}xy^{-\frac{1}{2}} + x^{\frac{1}{2}}\frac{dy}{dx}\right) + \left(\frac{1}{2}xy^{-\frac{1}{2}}\frac{dy}{dx} + y^{\frac{1}{2}}\right) = 0$$

$$\frac{1}{2}xy^{-\frac{1}{2}} + x^{\frac{1}{2}}\frac{dy}{dx} + \frac{1}{2}xy^{-\frac{1}{2}}\frac{dy}{dx} + y^{\frac{1}{2}} = 0$$

$$x^{\frac{1}{2}}\frac{dy}{dx} + \frac{1}{2}xy^{-\frac{1}{2}}\frac{dy}{dx} = -\frac{1}{2}xy^{-\frac{1}{2}} - y^{\frac{1}{2}}$$

$$\frac{dy}{dx}\left(x^{\frac{1}{2}} + \frac{1}{2}xy^{-\frac{1}{2}}\right) = -\frac{1}{2}xy^{-\frac{1}{2}} - y^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2}xy^{-\frac{1}{2}} - y^{\frac{1}{2}}}{x^{\frac{1}{2}} + \frac{1}{2}xy^{-\frac{1}{2}}}$$

$$= \frac{-\frac{1}{2x\sqrt{y}} - \sqrt{y}}{\sqrt{x} + \frac{1}{2x\sqrt{y}}}$$

9

$$x^2y + y^2x = 2$$

$$u = x^2 \qquad v = y$$

$$u' = 2x \qquad v' = \frac{dy}{dx}$$

$$uv' + vu' = (x^2)\left(\frac{dy}{dx}\right) + (y)(2x)$$

$$= x^2\frac{dy}{dx} + 2xy$$

9

$x^2y + y^2x = 2$

$$u = y^2 \qquad v = x$$

$$u' = 2y\frac{dy}{dx} \qquad v' = 1$$

$$uv' + vu' = (y^2)(1) + (x)\left(2y\frac{dy}{dx}\right)$$

$$= y^2 + 2xy\frac{dy}{dx}$$

9

$$x^2y + y^2x = 2$$

$$\frac{d}{dx}(x^2y) + \frac{d}{dx}(y^2x) = \frac{d}{dx}(-2)$$

$$\left(x^2\frac{dy}{dx} + 2xy\right) + \left(y^2 + 2xy\frac{dy}{dx}\right) = 0$$

$$x^2\frac{dy}{dx} + 2xy + y^2 + 2xy\frac{dy}{dx} = 0$$

$$x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} = -2xy - y^2$$

$$\frac{dy}{dx}(x^2 + 2xy) = -2xy - y^2$$

$$\frac{dy}{dx} = \frac{-2xy - y^2}{x^2 + 2xy}$$

10 $y = \sin xy$

$$u = x \qquad v = y$$

$$u' = 1 \qquad v' = \frac{dy}{dx}$$

$$uv' + vu' = (x)\left(\frac{dy}{dx}\right) + (y)(1)$$

$$= x\frac{dy}{dx} + y$$

LET'S DO EXERCISE ~

- "It always seems impossible until it's done." - Nelson Mandela

LET'S TRY OUT !

1. $16x^2 + 25y^2 = 400$

2. $3x^2y - 2xy^3 = 1$

3. $2xy - y^2 = 3x$

4. $\sqrt{x} + \sqrt{y} = 1$

5. $(x-1)y^2 = x + 1$

6. $\sin(x+y) = xy^2$

7. $e^{x-y^2} = 5 - y$

8. $\sin x + 2\cos 2y = 1$

ANSWER

1. $$\frac{dy}{dx} = -\frac{16x}{25y}$$

2. $$\frac{dy}{dx} = \frac{-6xy + 2y^3}{3x^2 - 6xy^2}$$

3. $$\frac{dy}{dx} = \frac{3 - 2y}{2x - 2y}$$

4. $$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$$

5. $$\frac{dy}{dx} = \frac{1 - y^2}{2xy - 2y}$$

6. $$\frac{dy}{dx} = \frac{y^2 - \cos(x + y)}{\cos(x + y) - 2xy}$$

7. $$\frac{dy}{dx} = \frac{-e^{x-y^2}}{-2y\left(e^{x-y^2}\right) + 1}$$

8. $$\frac{dy}{dx} = \frac{\cos x}{4 \sin 2y}$$

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"UNCLOCK THE POWER OF MATHEMATICS – DISCOVER PATTERNS,
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AND SEE THE WORLD IN A WHOLE NEW WAY!"

thankyou

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