Mathematical Reflective Thinking Type of Error on Newman's Error Analysis

Muntazhimah[1*],Sufyani Prabawanto[2] and Turmudi[3]

[1]Faculty of Teacher Training and Education, Universitas Muhammadiyah Prof. Dr. Hamka,
Jl. Tanah Merdeka, Jakarta Timur. DKI Jakarta, 13830, Indonesia
muntazhimah@uhamka.ac.id

[2 3]Department of Mathematics Education, University of Education Indonesia,
Jl. Dr. Setibudhi No 229, Kota Bandung, Jawa Barat, 40154, Indonesia.
sufyani@upi.edu
turmudi@upi.edu
*Corresponding Author

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Abstract: A body of literature has revealed that Mathematical reflective thinking could improve student achievement and success in mathematics learning. However many students make errors in completing mathematics reflective thinking problems. Hence, this qualitative study sought to explore the errors of pre-service mathematics teachers in completing the reflective thinking ability test using Newman's Error Analysis (NEA). A total of twenty-four of the 4th-semester students of mathematics education or pre-service mathematics teachers participated in the study. The methods used in this study were tests, interviews, and triangulation. The results showed that pre-service mathematics teachers made technical, conceptual, procedural, and interpretation errors. In terms of NEA, the error started from the comprehension stage, transformation, and process skills to encoding. Further research should design learning that can stimulate and improve the mathematical reflective thinking ability of pre-service mathematics teachers in order to reduce errors of mathematical reflective thinking processes.

Keywords: Mathematical error, Newman's Error Analysis, Reflective thinking

1. Introduction

In the 21st century, teaching and learning are getting more challenging (Hoon et al., 2022). One of the main domains in the 21st century is the ability to think (Dalim et al., 2022), which has become a life skill that needs to be developed through the educational process (Muntazhimah et al., 2021a). Reflective thinking is one of the advanced thinking abilities categorized as a Higher Order Thinking Skill (HOTS) (Kapranos, 2007). Rahmadhani et al. (2020) stated that reflective thinking is a crucial ability to facilitate human development. while Howlett et al., (2016) revealed that reflective thinking supports pupils in making meaning out of experiences at the highest critical level. Thus, reflective thinking ability is pivotal in the learning process.

Reflective thinking is a systematic and precise way to create meaning from experiences and relate them to new problems (Clarà, 2015; Muin et al., 2018). Ghanizadeh (2017) mentioned that reflective thinking could improve student achievement and success in mathematics learning because they will be stimulated to develop strategies to solve complex problems. Through this ability, students actively, thoughtfully, and carefully use the knowledge gained from a given mathematical problem. Furthermore, it is also revealed that reflective thinking can significantly improve mathematical problem-solving abilities (Demirel et al., 2015; Porntaweekul & Gianttaya, 2012). The ability of mathematical reflective thinking is beneficial to facilitating learning (Aldahmash et al., 2021; Rahmi et al., 2020).

However, mathematics learning in school nowadays is not optimal, as indicated by students' difficulties in mathematics learning (Pomalato et al., 2020; Omar et al., 2022). Many students make errors in completing mathematics problems (Haryanto & Pujiastuti, 2020). Umam & Susandi (2022) stated that errors in mathematics problems could be categorized as procedural, conceptual, interpretation, and technical. A procedural error occurs because of failure in manipulation or algorithms, even though the student has understood the concept (Lien et al., 2021). The conceptual error is indicated by failure to understand the concept of a problem (Ferrer, 2016). The interpretation error refers to translating or inferring word choices in determining variables into numbers. The interpretation tends to be about objects rather than numbers (Hitt, 2006). These errors in completing math problems require further analysis to obtain clear and detailed views on the weaknesses in completing mathematics problems.

Newmann (1990) proposed the Newman Error Analysis (NEA), a straightforward method for diagnosing mistakes in resolving mathematical problems. NEA is also applicable to identify error made by students in completing higher order thinking tasks in mathematics (Prakitipong & Nakamura, 2006; Zakaria et al., 2010). NEA is a framework with simple diagnostic procedures, including reading, comprehension, transformation, process skills, and encoding (Chusnul et al., 2017). Seng (2020) argued that errors could be easily identified through NEA. Error in reading questions or primary information makes students unable to use the information to solve problems. Comprehension error occurs when the students fail to identify given and asked problems. Transformation error occurs when the students fail to turn the problem into a mathematical model such as an equation, figure, graph, or table. Process skill error occurs when students fail in choosing rules/procedures or have used the correct procedures/rules, but an error occurs in the calculation or computing. Encoding error occurs when the answer is already confirmed but cannot show the authenticity of the answer or does not write down the conclusion of the answer.

Numerous studies have been conducted to better understand of mathematical reflective thinking ability. Aldahmash et al. (2021) stated that teachers' reflective thinking in mathematics was just moderate. Whereas for students, Muzaimah & Noer (2019) explained that students' mathematical reflective thinking was lacking. Thus, further studies are required. In line with this study, research on pre-service mathematics teachers concluded that their mathematical reflective thinking is inadequate (Abidin et al., 2021). Thus, some activities that can accommodate the development of the reflective thinking ability of pre-service mathematics teachers must be conducted (Kholid et al., 2021).

In contrast, Sa'dijah et al. (2020) revealed that pre-service mathematics teacher with high ability (proficiency) has consistency and fulfill all indicators of mathematical reflective thinking. It is due to the pre-service teacher's ability, so the prior knowledge from the prerequisite topic will affect their performance (Muntazhimah et al., 2021a). Furthermore research on error analysis in mathematical reflective thinking has just been conducted by involving high-achieving students as research participants (Ratnaningsih & Hidayat, 2020; Sari et al., 2019; Winarso et al., 2022). Thus, it is necessary to conduct the research on the analysis of errors in mathematical reflective thinking carried out by students with other level of abilities that will be studied in this research by Newman's Error Analysis (NEA).

This study aims to identify pre-service mathematics teacher errors in completing the test of reflective thinking ability in mathematics with NEA. The research results explained that pre-service mathematics teachers' reflective thinking was not optimally developed (Abidin et al., 2021; Aldahmash et al., 2021; Choy et al., 2017; Muzaimah & Noer, 2019). However, research on items contributing to pre-service mathematics teachers' reflective thinking is limited. Thus, this study describes and identifies the errors of pre-service teachers in solving problems requiring reflective thinking ability with Newman Error Analysis on the topic of group theory. The research questions include: what kind of error that pre service mathematics teacher made in resolving mathematical reflective thinking test? And based on Newman's Error Analysis, what type of error that pre service mathematics teacher made in completing mathematical reflective thinking test?. Furthermore, this study is crucial as initial research in developing the reflective thinking ability of pre-service mathematics teachers. Based on this research result, a learning model can be designed to improve mathematical reflective thinking ability.

2. Method

2.1 Research design

This research analyzes the errors of pre-service mathematics teachers in completing mathematical reflective thinking abilities with NEA. A qualitative study is utilized in this research based on the method suggested by Kumar. R & James (2015). The approach used in this study is a explorative approach. This approach allows to explore the data of participants expression in completing mathematical reflective thinking test. The participants would have the opportunity to respond using their answers or language (Alwadai, 2014).

2.2 Participants

Participants in this study were twenty-four of the 4th-semester students of the Mathematics Education study program at one of private university in Indonesia or also called as pre-service mathematics teachers. In order to effectiveness and time efficiency then the participants were selected using a convenience sampling technique. All answer sheets were collected and checked based on mathematics reflective thinking indicators. Only pre-service mathematics teachers with errors in their answers were chosen as participants in this study. The ability of pre-service mathematics teachers to communicate their answers also becomes another consideration in determining research participants so that the data can be explored easily.

2.3 Research Instrument

The instruments employed in this study were tests and interviews. The test was utilized to observe various errors and contained five problems to measure mathematics reflective thinking ability. The Indicators of mathematical reflective thinking test can be seen in table 1. Hopefully, the students will be able to look for essential and correct information before solving the given problem. All tests have met the indicators of pre-service mathematics teachers' mathematical reflective thinking ability (Muntazhimah & Wahyuni, 2022).

Table 1. Definitions and Indicators of Mathematics Reflective Thinking Test

Definition	Indicators
Think thoughtfully by applying mathematical knowledge and experience obtained in order to analyze, evaluate and get deep meaning in solving mathematical problems.	1. Analyzing the truth of the question/solution/analogy or generalization of mathematics, 2. Identifying mathematical concepts or formulas used in complex math problems, 3. Distinguishing between relevant and irrelevant data, 4. Evaluating the validity of arguments based on the concepts or properties used, 5. Finding various strategies in solving math problems

The researcher confirmed the participants about their test result, so the interview questions were based on students' answer sheets. In addition, the interview aims to obtain additional information from students' answer sheets so that the research result could be more sharp and clear. The interview formed an unstructured interview, where the researcher did not plan the questions to be asked to the participants. The interview process was conducted comfortably to encourage informal, comfortable, and stressless communication.

2.4 Data Analysis

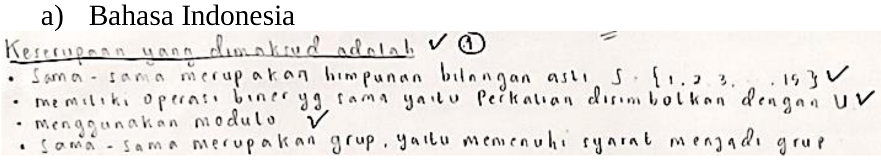
The twenty-four pre-service mathematics teachers were asked to take a test containing five problems about their mathematical reflective thinking. In the next step, the researcher chosed some paticipants' answer based on the assumption of errors made by them, which will be explained deeply in the discussion part of this study. After the research participants (pre-service mathematics teachers) were selected, interviews were held to confirm the answers. Interviews were recorded and transcribed for the last 45 to 60 minutes. Interviews for this study were conducted using NEA to find out the mistakes made by students when completing the test of reflective thinking in mathematics. The next step was the triangulation of data by comparing the results of the written test with the interview result. The researcher ended the data analysis process marked by narrowing the conclusions from the research findings based on the triangulation process.

3. Results

Following are the test results of mathematics reflective thinking ability of pre-service mathematics teachers presented based on indicators of mathematical reflective thinking ability defined in this research which mentioned in research instrument. Table 1 shows the pre-service mathematics teacher's answer sheet for indicator one: clarification, solution, analogy, or generalization of mathematics. Table 1a) presents the pre-service teacher's answers in Bahasa Indonesia, Table 1b) shows the answers in English, and Table 1c) shows error identification. This problem requires pre-service teachers to use their reflective thinking ability to clarify the similarity of the examples set in the problem. For example, $U_{(12)}$, which consists of $\{1,5,7,11\}$ with the set $U_{(15)} = \{x \in \mathrm{R\ I}\ FPB\ (x, 15) = 1\}$, as well as a link between group concepts in multiplication. After error identification was obtained, interviews were conducted to ensure students' errors in completing the mathematical reflective thinking using NEA**.**

Table 2a. Student's answer for indicator 1.

a) Bahasa Indonesia


Keserupaan yang dimaksud adalah ✓ ①
• Sama-sama merupakan himpunan bilangan asli S · {1, 2, 3, . . . 15} ✓
• memiliki operasi biner yg sama yaitu perkalian disimbolkan dengan U ✓
• menggunakan modulo ✓
• Sama-sama merupakan grup, yaitu memenuhi syarat menjadi grup

b) English

The similarities are:Both are the subsets of natural number

- Having the same binary operation, namely multiplication, as symbolized U
- Using Modulo
- Both are groups, fulfilling the requirement as a group

c) Error identifications

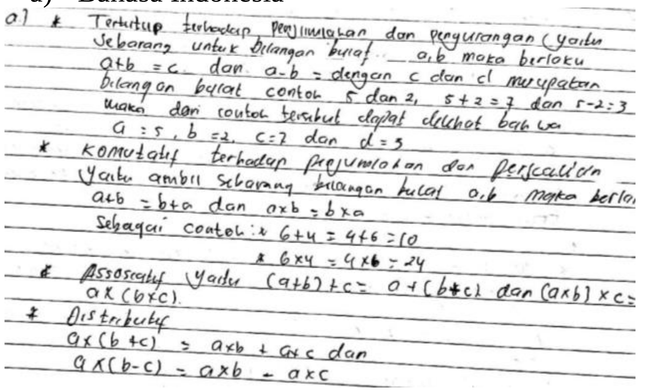
- Only mention Real Number from 1 to 15
- U symbol was interpreted as a binary operation symbol, not as a notation of a set.
- Did not write the number of modulo
- Did not describe the group of axioms

Pre-service teachers are capable of receiving information from problems, can understand the meaning required by the question, and can determine the step to solve the problem by writing the similarities known. However, the pre-service teacher conducted various technical errors, such as being unable to write the mathematical sentence correctly, not completing writing notation, and writing axiom group not systematically, which led to failure in the conclusion. Thus, based on NEA, students were on process skills and encoding errors. Interview results confirmed error identifications that can be seen in table 1c.

This problem required students to train their reflective thinking ability to use concepts or formulas on the group axioms to prove that a set is a group with binary operations known in the problem. Figure 2 is the student's answer for indicator 2: to identify, select and utilize mathematical concepts or formulas. Table 2 a) shows students' answers in Bahasa Indonesia, and Table 2 b) is in the English version, as well as Table 1c) error identifications of students' solutions. Based on the error identifications, interviews were conducted based on NEA to confirm the student's error.

Table 2b. Student's Answer for Indicator 2

a) Bahasa Indonesia


a) * Tertutup terhadap penjumlahan dan pengurangan (yaitu sebarang untuk bilangan bulat a,b maka berlaku a+b = c dan a-b = d dengan c dan d merupakan bilangan bulat contoh 5 dan 2, 5+2 = 7 dan 5-2 = 3 maka dari contoh tersebut dapat dilihat bahwa a = 5, b = 2, c = 7 dan d = 3

* komutatif terhadap penjumlahan dan perkalian yaitu ambil sebarang bilangan bulat a,b maka berlaku a+b = b+a dan axb = bxa
Sebagai contoh: * 6+4 = 4+6 = 10
* 6x4 = 4x6 = 24

* Assosiatif yaitu (a+b)+c = a+(b+c) dan (axb)xc = ax(bxc).

* Distributif
ax(b+c) = axb + axc dan
ax(b-c) = axb - axc

b) English

- It is closed for addition and multiplication, namely given any integer, a,b so a+b=c and a-b=c and c is an integer. For example 5 and 2, 5+2=7 dan 5-2 =3. Based on the example a=5, b=2, c=7, d=3.
- Comutative for addition and multiplication. Take any integer a,b so a+b=b+a dan axb =bxa. For example: 6+4 =4+6=10 and 6x4 =4x6 =24
- Associative, namely (a+b) +c =a+(b+c) dan (axb)xc=ax(bxc).
- Distributive: ax(b+c)= axb+a+c and ax(b-c)=axb=axc

c) Error Identifications

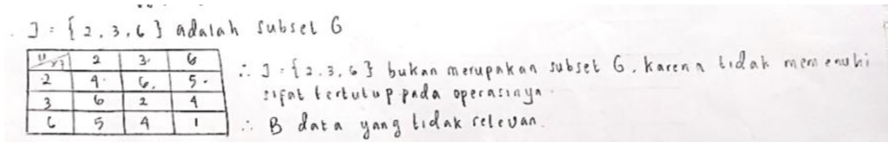
- Did not use correct group axioms
- Used different binary operations on one group of prove

After the interview, information can be drawn that the research participant knows what was given in the problem and could mention the information in the problem. Participants could also mention that the problem required them to prove whether the information was in the same group or not. However, the participant failed to prove it. This fact was because of the minimum knowledge of students related to group axioms. As a result, the participant did not prove groups with groups axioms or theorems of existing groups. Concept errors of group proof also become reasons for errors identified from the solution. Participants also used different operations in each stage of written proof. In the first step, the operation used was addition and subtraction, while in the second step, the operation was addition and multiplication. So as for the next step, the students conducted errors in the stages of transformation, process skills, and encoding.

The research results can be seen in Table 3, namely the description of students' answers to question three, derivative from indicator 3, differentiating between relevant and irrelevant data. Table 1a) describes students' answers in Bahasa Indonesia, Table 1b) shows students' answers in English, and Table 1c) shows error identifications in the answer. This problem facilitates pre-service teachers' reflective thinking ability to identify relevant or irrelevant data from a given data in the problem. Because of the assumption that students made errors in solving this problem, an interview based on NEA was conducted. The interview was to identify students' errors in achieving mathematical reflective thinking ability.

Table 3. Student's Answer for Indicator 3

a) Bahasa Indonesia



b) English
$j = \{2,3,6\}$ is a subset G because it does not satisfy the close properties of the operation
B: data is not relevant

c) Error Identifications
- Prove subsets by using Cayley's table
- There was no explanation for irrelevant data

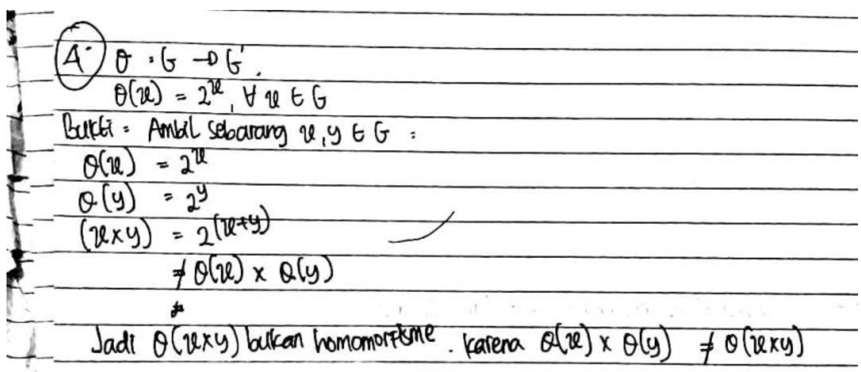
Interview results showed that pre-service teachers misinterpreted the word subset. Because of the problem discussed group, students interpreted that subset as a subgroup. However, the subset and subgroup are two different things, so the procedure to prove is also different. This fact means that students already made errors since the comprehension stages, so the following stages, like process skills and encoding, were also errors.

Table 4 shows students' answers for the indicator of checking or evaluating the argument's validity based on the concept of properties used. Table 4a) explains the student's solution in Bahasa Indonesia, Table 4b) shows the answer in English, and Table 4c) shows error assumptions obtained.

Interviews were conducted based on NEA to validate these errors. The problem served as proof of the homomorphism group, so the intended evaluation in this problem is whether the proof on the question was already right, whether there was an incorrect step, why it is incorrect, and how the correct proof was. In completing this question, the student will use reflective thinking to evaluate proof of the homomorphism group.

Table 4. Student's Answer for Indicator 4

a) Bahasa Indonesia



b) English
$\theta = G \to G'$
$\theta(x) = \theta^x, \forall x \in G$
Proof, take any $x, y \in G$
$\theta(x) = 2^x$
$\theta(y) = 2^y$
$(x.y) = 2^{(x+y)}$
$\neq \theta(x)\ x\ Q(y).$
So, $\theta(x.y) is\ not\ homoformisme\ because\ \theta(x)\ x\ \theta(y) \neq \theta(x.y)$

c) Error Identifications
- Misspelled step

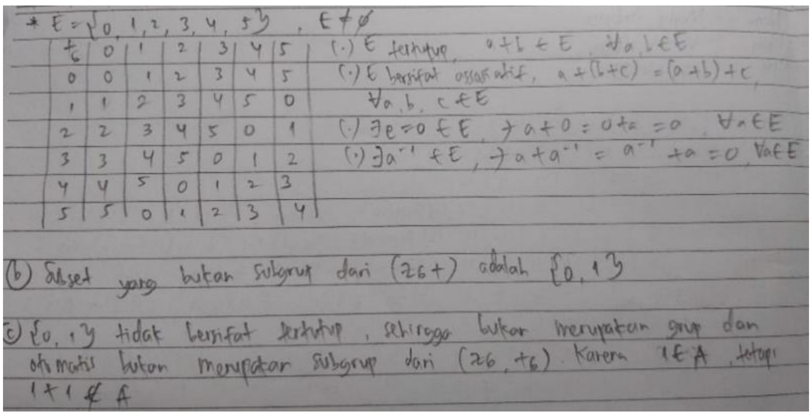
Data was obtained through the interview that the pre-service teacher conducted errors in the third line of proof. There was an incorrect step in writing technics. The student could understand the information given by the problem and determine the steps to solve it. However, there was an incorrect notation written. The student could also write the stages or order of the proof. This fact means that the student conducted errors at the process skills stage, which would also result in the wrong encoding step.

Table 5 shows that students also made errors for a problem with the indicator of finding solutions to mathematical problems using various strategies. This problem would explore how far the ability of the student to prove a group by using various existing theorem. Table 5 (a) shows the solution performed by the student in Bahasa Indonesia, Table 5 (b) shows the completion in English, as well as Table 5(c) which is the error identifications. An interview using Newman's error analysis was conducted to determine the errors made by students when solving the problems to confirm the presumption they had made. The students could not satisfy the reflective thinking ability overview indicator.

It was found that pre-service teachers did not bravely take risks by conducting proof through existing theorem. Students knew the theorems, yet they were worried about making a procedural error in proving them. The student utilized the most straightforward way, such as proofing the group using group axioms. Students could not follow the proving steps, although the proof stages were shorter than the group axioms. This fact means that the error in the last indicator was at the transformation stage. The following stages, namely process skills and encoding, could not be correct if this transformation stage were not right.

Table 5. Students' Answer for Question 5

a) Bahasa Indonesia



* E = {0, 1, 2, 3, 4, 5} , E ≠ ∅

$+_6$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

(·) E tertutup, a+b ∈ E ∀a, b ∈ E
(·) E bersifat assosiatif, a+(b+c) = (a+b)+c ∀a, b, c ∈ E
(·) ∃e = 0 ∈ E ∋ a+0 = 0+a = a ∀a ∈ E
(·) ∃a⁻¹ ∈ E, ∋ a+a⁻¹ = a⁻¹+a = 0 ∀a ∈ E

(b) Subset yang bukan subgrup dari (Z6+) adalah {0, 1}

(c) {0, 1} tidak bersifat tertutup, sehingga bukan merupakan grup dan otomatis bukan merupakan subgrup dari (Z6, +6). Karena 1 ∈ A, tetapi 1+1 ∉ A

b) English
E is close, $a + b \in E \ \forall\ a, b \in E$
E is associative, a+(b+c) = (a+b)+c $\forall a. b, c \in E$
$\exists e = 0 \in E \dashv a + 0 = 0 + a = a \ \forall a \in E$

$$\exists a^{-1} + E, \dashv a + a^{-1} = a^{-1} + a = 0 \ \forall a \in E$$

b. Subset that is not sub-group from (Z6+) is $\{0,1\}$

c. $\{0,1\}$ is not close so it is not group and automatically it is not sub-group (z6, +6) because $1 \in A \ 1 + 1 \notin A$

c) Error Identifications

- Notation writing for neutral element axiom and inverse of the proof group
- Did not explore solution strategies

4. Discussion

Students with low reflective thinking abilities in mathematics were selected to participate in the research. According to the indicators of mathematical reflective thinking, certain students cannot complete the five problems provided. Due to the student's lack of experience with these problems, they could not solve mathematical reflective thinking problems. The types of questions that students usually use are non-facilitating problems for developing mathematical reflective thinking ability. It is recommended that students train themselves to solve questions or problems that can explore their mathematical reflective thinking ability (Etkina et al., 2010; Fitriati et al., 2020; Lee, 2005).

Schön and Mezirow explained that mathematical reflective thinking ability could be triggered by unfamiliar and confusing or complicated situation as well as need guidance circumstances to understand (Schön, 1992). The experts revealed that the learning activities must be well designed to create innovative learning to facilitate the process of mathematical reflective thinking in pre-service mathematics teachers. This condition means that the learning must be designed so that the pre-service teachers involve in non-routine learning activities. The learning should consist of confusion or deadlock so that students need feedback or help from friends or lecturers. This condition is also expected to stimulate students to think of alternative solutions based on their prior knowledge. This is becomes an initial ability for students to have a better mathematical reflective thinking ability.

Due to the mathematical reflective thinking ability is not optimally facilitated, pre-service mathematics teachers' ability to solve problems related to mathematics thinking ability is also not optimal (Dünda, 2015). There were four common errors by pre-service mathematics teachers in exploring the ability of mathematical reflective thinking: technical, concept, interpretation, and procedural error. A technical error occurred; the students could write a mathematical sentence correctly, incorrect and did not complete it in writing notation, was not systematic in writing group axioms, and conducted error in answer writing technic. Rong & Mononen (2022) declared that technical error did not only consist of numbers but also facts about the definitions of another basic mathematics concept, including properties of numbers, quantities, geometric properties, and notation.

The conceptual error found in this study was that students did not prove group with group axioms or group theorem that has been studied previously and used different operations in each stage of proof writing. The conceptual error was caused by the student's knowledge about group axioms is still very low. Whereas the mathematical concepts and skills that do not fully master will cause difficulties and errors in solving mathematics problems. This statement is in line with the study of Cemalettin and Tuğrul (2012). They stated that using language or suitable concepts with the problem is pivotal in understanding mathematics concepts. Research by Pomalato et al. (2020) also mentioned that one difficulty experienced by students in solving mathematics problems is a lack of understanding of the concepts and problems that will be solved.

On the other hand, the interpretation error found in this study is that students misinterpret the word subset. Students considered a subset in the problem as a subgroup. However, the subset and subgroup are two different things, so proof requires different ways. Furthermore, procedural errors were found as proof of the group concept. Students understood the theorems used in the proving process. However, there were incorrect stages in the procedure in stages of the proof. According to Fei Lai (2012), procedural errors generally do not occur due to inherent misunderstanding yet because of memory deficits, impulsivity, or visual-motor integration problems. Thus, according to Loibl & Rummel (2014), this procedural error could be minimized if the students realize their knowledge gap.

According to the findings and NEA, pre-service teachers' errors in mathematical reflective thinking are in the stages of comprehension, transformation, process skills, and encoding. Errors in comprehension can be interpreted as errors in understanding the question and then writing down what is being asked (Zakariyya et al., 2018) and can be seen in students' understanding of related symbols, expressions, and the problem given in the question (Zakariyya et al., 2018). In this research, pre-service teachers are at the stage of comprehension error because they do not understand the problem sentence. This fact is in line with Alhassora et al. (2017) reporting that this mistake generally occurs because students do not understand the sentence of the problem. There was a subset of the problem, but the students understood that the subset was a subgroup. Moreover, Martyrs et al. (2017) mentioned that the comprehension phase is one of the most significant contributions, as errors cause students to fail to provide and explain the proper final answer.

Next, the transformation stage refers to students' ability to choose the right formula or method to solve a given problem (Alhassora et al., 2017). Moreover, Santoso et al. (2019) explained that transformation errors in solving mathematics problems could cause students to fail to work on the problem carefully and proceed to the problem-solving procedure. As a result, students write incorrect answers. In this research, the pre-service teachers conducted errors in determining steps of group proof and proving methods.

The pre-service teachers did not conduct reading errors in NEA. This fact is in line with Abdullah et al. (2015) and Suseelan et al. (2022) research, which mentioned that students do not commonly make errors in the reading stage. Several studies explained this condition because the mathematics pre-service teacher naturally has no problem at the reading problem stage, both in words and mathematical symbols. Besides that, the instrument also has a pass validation process to ensure that the language in the problem can be easily understood by research participants (Ganuza & Hedman, 2017).

On the other hand, according to Suseelan et al. (2022) study, most students started to conduct errors in the transformation stage. The pre-service teacher in this research also mainly started the error was on the transformation stage. This finding differed from research by Jiang et al. (2020). However, these findings contradict Raduan's (2010) research, which showed that comprehension errors are the most common mistakes students make in solving mathematics problems. This condition was caused by a different context of the problem used by the complainant, which is a routine problem and involves participants with low cognitive levels.

Error analysis can help provide a richer learning experience that leads to a deeper understanding of long-term knowledge (Rushton, 2018). Because of that, error analysis is crucial to getting used to mathematics learning. Furthermore, many studies mention things that can be conducted to reduce or resolve students' errors in learning, for example, open discussion with a colleague, teacher, or lecturer (Kim et al., 2016; Rushton, 2018) by using a particular model or approach, for example, metacognitive learning (Muntazhimah et al., 2020). Another example is sufficient mental effort and optimal thinking operations that activate prior knowledge and experience in solving specific problems.

Thinking activities involving knowledge or experience are often called reflective thinking, which in the mathematical context is mathematical reflective thinking (Muntazhimah et al., 2021b). Therefore, students' ability to mathematical reflective thinking must be improved to minimize errors in mathematics learning. By facilitating the mathematical reflective thinking ability, the pre-service teacher will carry out the looking back process that will sharpen the process of finding rules, concepts, or formulas used in a complete mathematics problem.

5. Conclusions

This study concluded that pre-service mathematics teachers who conducted a mathematical reflective thinking test made errors in all Newman Error Analysis (NEA) stages except the reading stage. Several errors found are i) technical errors which consist of not writing mathematical sentences correctly, incorrect and incomplete notation, unsystematic writing of group axioms, and errors in answering writing technics. The student could understand the information given in problems as well as was capable of determining steps of solution. However, there was an error in writing the notation; ii) errors in axiom concept of group proof that resulted that they could not prove group with group axiom nor existing group theorem; iii) interpretation error: student interpreted subsets as subgroups. However, the subset and subgroup are two different things, so the prove procedure was also different; iv) procedural error: Students could not identify the false proof procedure.

This study provides several suggestions for further research, for example, developing a new learning model to minimize pre-service teachers' errors in practicing their critical thinking in mathematics. Furthermore, another research relevant to this study is about the process of reflective thinking in mathematics by pre-service teachers. This study will be beneficial for teachers in improving the learning process in the following meeting. On the other hand, this study has limitations in generalizations that could not be conducted because this study is qualitative research. This research is limited to a few research participants that met the criteria. There is a method and sample development for further study to produce a more general study.

6. Co-Author Contribution

The authors declare that they have no competing interests surrounding this article's research, authorship, or publishing. The authors are liabiled for conducting the research, collecting data from respondents, analysing the data, and writing the whole article. Concurrently, the three authors simultaneously collaborated in conducting a comprehensive discussion to answer the research questions stated in this research.

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