

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

APPROXIMATE SOLUTION OF LINEAR AND  
NONLINEAR  
VOLTERRA INTEGRAL EQUATION SINGULARLY  
PERTURBED  
PROBLEMS USING DIFFERENTIAL TRANSFORM  
METHOD(DTM)

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL.

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## **ABSTRACT**

In this project, singularly perturbed Volterra integral equations are solved by using Differential Transformed Method (DTM) for both linear and nonlinear equations. To show the efficiency and accuracy of the method, the singularly perturbed Volterra integral equation is calculated to obtain the approximate series solution and compared with exact solution. Those results are shown in tables and represented as graphs by using Maple software. The absolute errors for both linear and nonlinear singularly perturbed Volterra equation also shown in table. Based on the errors, DTM is very effective especially for linear singularly perturbed Integral Volterra equation for solving a large number of singularly perturbed problems.

# 1 INTRODUCTION

## 1.1 Research Background

Theory of perturbation is the review of the effects of the small disturbances in the equation to the solution of the equation. According to Kautchen (1997), there are two perturbation problems which are singular and regular. Singular perturbation problems occur when parameter  $\varepsilon$  is small that may not be approximated by setting the parameter value,  $\varepsilon = 0$ . One particular typically obtains a convergent expansion of the solution with regard to  $\varepsilon$ . A regular perturbation problem is one for which the perturbed problem for small, nonzero values of  $\varepsilon$  is qualitatively the same as the unperturbed problem for  $\varepsilon = 0$ . One typically obtains a divergent expansion of the solution with respect to  $\varepsilon$ .

In mathematics, the Volterra integral equations are a special type of integral equations. They are divided into two groups which are first and the second kind. According to Odibat (2008), the Volterra integral equation of the first kind is

$$f(x) = \int_a^x K(x,t)u(t)dt, \quad (1)$$

where  $f(x)$  be the function to be solved for,  $u(t)$  is a given known function and  $K(x,t)$  a known integral kernel. The Volterra equation of the second kind is

$$u(x) = f(x) + \int_a^x K(x,t)u(t)dt, \quad (2)$$

where  $u(x)$  be the function to be solved for,  $f(x)$  and  $u(t)$  are the given functions and  $K(x,t)$  is a known integral kernel.

The Volterra integral equations were introduced by Vito Volterra and then studied by Traian Lalescu in his 1908 thesis, *Sur les equations de Volterra*, written under the direction of Émile Picard. In 1911, Lalescu wrote the first book ever on integral equations. Volterra integral