# UNIVERSITI TEKNOLOGI MARA

## TECHNICAL REPORT

## SOLVING NONLINEAR EQUATION BY USING HOMOTOPY ANALYSIS METHOD (HAM)

### NUR SAADAH MOHTAR 2014949213 D1CS2496A SYARIFAH NUR AININA SAID ABDUL KARIM 2014354227 D1CS2496A

Report submitted in partial fulfillment of the requirement for the degree of Bachelor of Science (Hons.) Mathematics Center of Mathematics Studies Faculty of Computer and Mathematical Sciences

JANUARY 2018

#### ACKNOWLEDGEMENTS

In the name of Allah, The Most Gracious and The Most Merciful

Firstly, we are grateful to God for the good health and well being that made it necessary for us to complete this project successfully. We wish to express our sincere thanks to our supervisor, Madam Rahaidah Muhamad for sharing expertise, sincere and valuable guidance and encouragement extended to us. Without her dedicated involvement throughout the process, this paper would have never been completed.

We also want to thank our Academic Advisor Madam Siti Farah Haryatie Mohd Kanafiah for her encouragement, support and attention. Beside that, we take this opportunity to express gratitude to all the lecturers at Department of Mathematics for their feedback, cooperation and continuous motivation.

Last but not least, we would like to thank our friends for accepting nothing less than excellent from us. Also million thanks to our family for supporting us spiritually throughout the writing of this project and our life in general. Lastly, we also place on record, our sense of gratitude to one and all, who directly or indirectly have lent their hands in our journey to success.

### TABLE OF CONTENTS

ACKNOWLEDGEMENTS			ii	
			iii	
			v	
			ABSTRACT	
1	INTRODUCTION		1	
	1.1	Research Background	3	
	1.2	Problem Statement	4	
	1.3	Research Objective	4	
	1.4	Significant Of Project	5	
	1.5	Scope Of Project	5	
2	LITE	LITERATURE REVIEW 6		
3	MET	METHODOLOGY 11		
4	IMPI	IMPLEMENTATION 16		
	4.1	Couple Burgers Equation	16	
	4.2	Fornberg-Whitham Equation	20	
5	RESULTS AND DISCUSSION		25	
	5.1	Results from Applying Adomian's Decomposition Method	25	
	5.2	Comparison and Discussion	28	
6	CON	CONCLUSIONS AND RECOMMENDATIONS 33		

#### ABSTRACT

In this research, the Homotopy Analysis Method (HAM) was studied and had been employed to obtain the approximate analytical solution of non-linear of Couple-Burgers Equation and Fornberg-Whitman Equation. When applied to non-linear equation the numerical result revealed that this method was more accurate, reliable and easy to implement. This was because of some limitations that appeared in other methods. Some other perturbation methods could only be solved based on small convergence region and was only valid for small parameter. By using Homotopy Analysis Method (HAM) we could choose the auxiliary parameter so that the zero-order and higher-order deformation equation could be obtained. Then two examples which are Couple-Burger Equation and Fornberg-Whitham Equation were applied. The final result that was reported by HAM is compared with the exact solution and Adomian Decomposition Method (ADM). The solution then was graphed using Maple software. Corresponding to it, it was found that the Homotopy Analysis Method was more accurate and effective compare to Adomian Decomposition Method that obtained by previously published work for validation.

#### **1 INTRODUCTION**

All this while we have found various method to solve linear and nonlinear equation, such as substitution, elimination, Perturbations, Adomian Decomposition (ADM), Homotopy Analysis and others. Like other nonlinear analytical method, ADM is restricted by its own limitation because this method is only valid for small parameter (Wahab et al., 2016). Hence, new analytical technique known as Homotopy Analysis Method (HAM) has proposed in by Liao in 1992 to overcome this limitation.

In the last two decades, linear and nonlinear differential equation has been involved in many scientific, engineering and even finance sector. Due to this, Homotopy Analysis Method has been applied. Liao (2004) proposed that HAM is an approximation method for highly nonlinear problem. The work was extended by Turkyilmazoglu (2010) where the procedure to choose the suitable base function, initial approximation, auxiliary linear operator and parameter which are important to approximate the solution's convergency was found. Through this method, the limitation of the other perturbation and non perturbation method can be eliminated which can only be solved based on small region of convergence that is mostly valid for small parameter. Besides, the complicated equation can be transformed into simpler ones. This is one of the advantages of this technique for solving the linear and nonlinear equation.

Briefly speaking, the Homotopy Analysis Method (HAM) does not depend on the existing of small or large parameters. Thus, it is one of the advantages for this method. Even if the nonlinear equation does not have any parameter, Homotopy Analysis Method is still applicable. Second, Homotopy Analysis Method (HAM) provides a suitable way to adjust and control the convergence region. Thus, rate of approximation of nonlinear equation are more accurate and reliable.