

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

NUMERICAL SOLUTION OF NONLOCAL  
PARTIAL DIFFERENTIAL EQUATION USING  
BERNSTEIN POLYNOMIAL METHOD

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## ABSTRACT

This research discusses about numerical method for solving one-dimensional parabolic partial differential equation (heat equation) subject to non classical conditions based on Bernstein polynomial method with its properties. Some continuous function with higher even order cannot be solved directly with a common methods such as least square methods, Euler's method, Bisection method and eigenfunction expansion method. Thus, Bernstein polynomial can be used to solve the continuous function at higher even-order due to their special properties. In addition, the heat equation will be solved with different degree of Bernstein polynomials by using *Matlab* and the result will be presented through the illustrative graph.

## 1 INTRODUCTION

Equation that relates the derivatives of a function depend on one or more variable is called Partial Differential Equation (PDE) (Olver, 2013). Some examples of linear PDE can be found in heat equation, wave equation, Laplace's equation and Poisson's equation. The study of PDE with nonlocal boundary conditions has increased the area of scientific interest. The parabolic PDE is a type of second order PDE which is in the form of:

$$au_{xx} + bu_{xy} + cu_{yy} + du_x + eu_y + fu = 0$$

and this satisfied the condition

$$b^2 - 4ac = 0$$

where  $a$ ,  $b$  and  $c$  cannot all be zero for the equation to be of second order.

Traditional PDE is a relation between the values of an unknown function and its derivatives of different orders. Thus, nonlocal equation is a relation for which opposite happens due to the involving of integral operators (Imbert, 2008).

However, mathematicians will investigate the non classical boundary conditions in PDE subject, but to get more accurate solutions, some improvements should be done with the existing method as stated by Karimi et al. (2016). PDE can be classified into 3 types which are elliptic (Laplace equations), parabolic (heat conduction equation) and hyperbolic (wave equation). There are many researches that dealt with non classical conditions. Finite Difference Method (FDM), Galerkin method and collocation method are the usual numerical method that subject to the non classical conditions.

FDM can approximate the forward, backward and central of the derivatives as they are easy to use compared with the other methods and they are also a discretization method. The process of transferring continuous functions, models, variables and equations into discrete counterparts