

UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

SOLVING HEAT EQUATION BY LIE ALGEBRA

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ABSTRACT

Differential equations are important in variety scientific fields. Usually, they are quite challenging to solve. In this project, a more universal method for solving differential equation is obtained compared to the familiar methods in solving ordinary and partial differential equation, (PDE). In this research, a method developed by Sophus Lie which uses groups of symmetries, called Lie Groups is applied in order to solve differential equation. The main problem in solving PDE is when it is in higher order and contains more than two independent variables involved which will contribute to high levels of difficulty. The purpose of this project is to obtain one-parameter infinitesimal transformation group and used it to transform heat equation. Lastly, the infinitesimal transformation will be apply to find the exact solution of heat equation. The calculation of the derivatives that will be used in finding the exact solution of heat equation will be explain in detailed. In future, Lie algebra can be apply to solve higher order partial differential equation that contains more than two independent variables.

1 INTRODUCTION

In this thesis, a systematic method in finding exact solution to a system of partial differential equations (PDE) is applied to the heat equation. A few exact solutions are known today since these equation are originated a century and a half ago.

One-parameter infinitesimal transformation groups are been used in finding these new solutions. A system of PDE with two independent variables is applied with one-parameter infinitesimal transformation group to it which is easier to solve than the original system. Ovsjannikov & Bluman (1962) who studied the nonlinear diffusion equation is the first person that began to exploit systematically and used the most general infinitesimal transformation groups to find invariance solution of PDE.

The higher the order of PDE, the harder it is to solve the equation. Therefore, the study of infinitesimal transformation that may be successfully to find exact solution of an equation is needed.

The infinitesimal transformation group that connected with heat equation is obtained by using classical method of Lie algebra. By solving the PDE, an exact solution of heat equation is obtained. Theories of Lie groups and Lie algebras have a broad application in various areas of mathematics and physics (Woodard & Ames, 1971). This research project focuses on solving heat equation by using Lie algebra manually.