UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

NUMERICAL SOLUTIONS OF RICCATI EQUATIONS USING ADAM-BASHFORTH AND ADAM-MOULTON METHODS

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ABSTRACT

A differential equation can be solved analytically or numerically. In many complicated cases, it is enough to just approximate the solution if the differential equation cannot be solved analytically. Euler's method, the improved Euler's method and Runge-Kutta methods are examples of commonly used numerical techniques in approximately solved differential equations. These methods are also called as single-step methods or starting methods because they use the value from one starting step to approximate the solution of the next step. While, multistep or continuing methods such as Adam-Bashforth and Adam-Moulton methods use the values from several computed steps to approximate the value of the next step. So, in terms of minimizing the calculating time in solving differential , multistep method is recommended by previous researchers.In this project, a Riccati differential equation is solved using the two multistep methods in order to analyze the accuracy of both methods. Both methods give small errors when they are compared to the exact solution but it is identified that Adam-Bashforth method is more accurate than Adam-Moulton method.

1 INTRODUCTION

It has been shown that a solution of a differential equation exist in certain specified domain. But in many instances, it is enough to just approximate the solution if the differential equation cannot be solved analytically. Euler's method, the improved Euler's method and Runge-Kutta methods are examples of commonly used numerical techniques in approximately solved differential equations. These methods are also called as single-step methods or starting methods because they use the value from one starting step to approximate the solution of the next step.

In the other hand, multistep or continuing methods such as Adam-Bashforth and Adam-Moulton methods use the values from several computed steps to approximate the value of the next step. Since linear multistep methods need several starting values to compute the next value, it is necessary to use a one-step method to compute enough its' starting values of the solution in order to be used in the multistep method.

First-order numerical procedure for solving ordinary differential equations (ODEs) like Euler method with a given initial value. Simplest Runge–Kutta method is the custom of basic explicit method for numerical integration in an ordinary differential equations. Euler method refers to only one previous point and its derivative to determine the current value. A simple modification of the Euler method which eliminates the stability problems is the backward Euler method. This modification leads to a family of Runge-Kutta.

Runge–Kutta methods are a family of implicit and explicit iterative methods, which includes the well-known routine called the Euler Method. The most popular and widely used is RK4 because its less computational requirement and high accuracy. This RK4 is an example of one-step method in numerical, Petzoldf (1986). Development of modified this RK4 leads from one-step to multi-step method,like Adam's methods.

Adam-Bashforth method and Adam-Moulton methods are the families of linear multistep method that commonly used. Adam-Bashforth methods is an example of explicit methods of