UNIVERSITI TEKNOLOGI MARA

TECHNICAL REPORT

SOLUTIONS OF KORTEWEG-de VRIES (KdV) EQUATION

FAIZATUL ASYEKIN BINTI YUSRI
2013890096 K15/09
RUSYA IRYANTI BINTI YAHAYA
2013478888 K15/09
NURHIDAYAH BINTI MAT RAMLI
2013405794 K15/09

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IN THE NAME OF ALLAH, THE MOST GRACIOUS, THE MOST MERCIFUL

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ABSTRACT

In the present work, the derivation of Korteweg-de Vries (KdV) equation and its single-soliton solution are shown step by step to give better understanding to people. This project is only devoted to shallow water waves where the derivation of KdV equation by using Euler equation in (1+1) dimensions is proposed. In the derivation, a set of governing equations is used and it follows the assumption of irrotational two-dimensional motion of an incompressible inviscid fluid that is bounded above by a free surface and below by a rigid horizontal plane. Then, scaling and substitution method are implemented. In addition, the far-field variables for wave that propagate to the right are also utilised. After solving the KdV equation, single-soliton solution for standard KdV equation is obtained from traveling wave solution which is one of the D'Alembert equation method. This project is then completed by sketching and plotting the graphs of solitons using Maple. These graphs illustrated the propagation of single-soliton solution to the right, left and both side of the graph.

1 INTRODUCTION

1.1 Introduction

Korteweg-de Vries equation has long and colourful history. It all started with the discovery of solitary wave made by John Scott Russell(1808-1882) in 1834 on the Edinburgh-Glasgow canal. As stated by Johnson & Drazin (1989), the solitary wave got its name because it often occurs as single entity and is localised. John Scott Russell then called it as the 'great wave of translation'. He reported his observations to the British Association in his 1844 'Report on Waves'. According to Palais (2000), Stokes and Airy made an early attempt to model these waves mathematically. However, it seemed that such waves could not be stable and their existence were doubtful. Later, Boussinesq and Lord Rayleigh corrected the errors in the earlier theory and finally in 1895, Korteweg and de Vries solved the matter by giving a convincing mathematical argument that wave motion in a shallow canal is governed by KdV.

According to Meyers (2011) in mathematics, the Korteweg-de Vries (KdV) equation is a mathematical model of waves on shallow water surfaces. It is particularly famous as the prototypical example of an exactly solvable model, that is, a nonlinear partial differential equation whose solutions can be exactly and precisely specified.

The equation can be expressed as non-linear 3rd Order of Partial Differential Equation (PDE) which contain an unknown function of more than one variable and some derivatives function respect to the differentiable independent variable. This PDE consider the simplest case in only one space variable x. The decision has been made which is by using function u depending on the variables x and t that is $u(x,t)$ which explain the elongation of the wave at the place x at time, t. The standard KdV equation can be formulated as:

$$u_t(x,t) + 6u(x,t)u_x(x,t) + u_{xxx}(x,t) = 0$$