Economic Power Dispatch Solution with Non-Smooth Cost Functions Using Differential Evolution

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Abstract- **This paper proposes a solution for Economic Dispatch problems with non-smooth cost functions using Differential Evolution (DE) algorithm. This technique is to minimize system cost by properly allocating the real power demand amongst the online generating units considering the valve point loadings. A 3-unit generator system is used to test the proposed method. The results obtained show that DE can solve Economic Dispatch with non-smooth cost function successfully. The program for this algorithm has been developed in Matlab 7.6 platform.**

Key words: Economic Dispatch, Differential Evolution, Valve Point Loading Effect.

I. INTRODUCTION

For the purpose of economic dispatch studies, generators are represented by functions that relate their production cost to their power output. Typically, smooth quadratic functions are used to model generators in order to simplify the mathematical formulation of the problem and to allow many of the conventional optimization techniques to be used. An important goal in the economic dispatch area is the utilization of improved models for the generator production cost curves, with the ability of capturing a better cost-power output relationship[1].

This type of cost function simplifies greatly the economic dispatch problem and increases the number of techniques that can be applied for its solution. However, there are cases where quadratic curves are no longer proper representations of the generators, thus requiring more accurate models to provide better results in the solution of the economic dispatch problem [2]. More accurate models usually result in highly nonlinear, non-smooth and non-convex cost functions. These types of cost functions may arise due to valve point loading effects, fuel switching, and prohibited operating regions.[1-3]

Generally, large power generators have multiple steam admission valves that are used to control the power output of the unit [4]. When a steam admission valve starts to open, a sudden increase in losses occurs, which results in ripples in the unit's cost function ,valve-points are those output levels at which a new admission valve is opened. When valve point effects are considered, the ED problem becomes extremely difficult to solve via conventional gradient based techniques, due to the abrupt changes and discontinuities present in the corresponding incremental cost functions [5].

II. PROBLEM FORMULATION

The main objective of the Economic Dispatch (ED) is to minimize the total fuel cost at thermal power plants subjected to the operating constraints of power system. Therefore, it can be formulated mathematically with an objective function and two constraints are equality and inequality constraints.

$$Minimize\ F_T = \sum_{i=1}^{n} F_i(P_{G,i}) \tag{1}$$

Where,

F_T =The Total Generation Cost (\$/h),
$F_i(P_{G,i})$ =The Fuel Cost Function of Unit i (\$/h),
$P_{G,i}$ =The Total Generation of Unit i (MW), and
n =The Number of Generators connected to the power system

Each generator cost function has the relationship between the power injected to the power system by the generator and the incurred costs to load the machine to that capacity. Figure 1 illustrates typical fuel cost function of a thermal generation unit. Typically, generators modeled by smooth quadratic functions can be formulated as equation (2) to simplify the optimization problem and facilitate the application of classical techniques.

$$F_i(P_{G,i}) = a_i + b_i P_{G_i} + c_i P_{G_1}^2 \tag{2}$$

Where,

a_i, b_i, c_i = The cost coefficients of the generator i

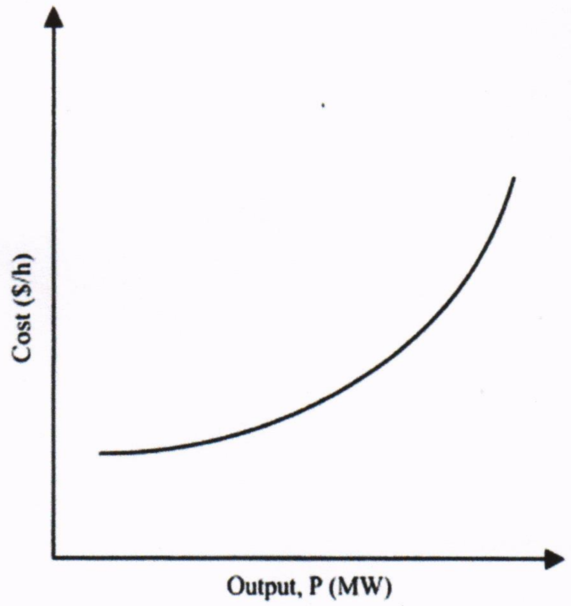

Fig 1 : Typical fuel cost function of a thermal generating unit[3]

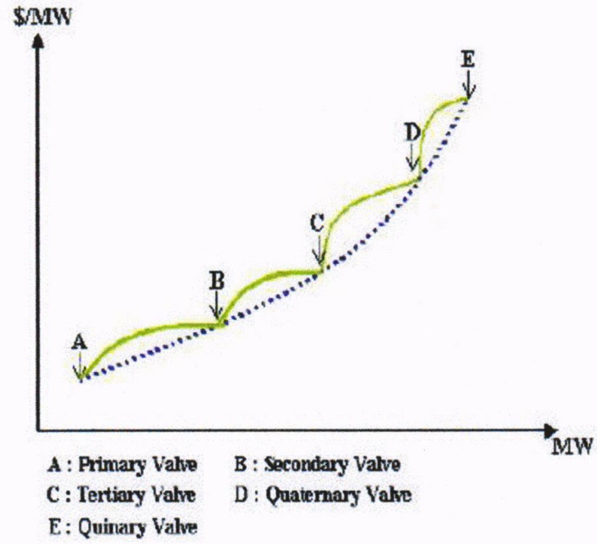

Fig 2 : The fuel cost function with 5 valves of a thermal generation unit[4].

Practically, the objective function of an Economic Dispatch problem has non-differentiable points according to valve-point effects and change of fuels. Therefore, the objective function should be composed of a set of non-smooth cost functions [6].

The generator with multi-valve steam turbines has very different input-output curve compared with the smooth cost function [7]. Typically, the valve point results in, as each steam valve starts to open, the ripples like in Figure 2 illustrates that the number of local optima is increased and then to take account for the valve-point effects, therefore, the equation (2) should be replaced as the equation (3). The fuel cost function with valve point loadings of the generators can be formulated as follow:

$$F_i(P_{G_i}) = \mathrm{a_i} + \mathrm{b_i}P_{G_i} + \mathrm{c_i}{P_{G_i}}^2 + \left|d_i \sin\left(e_i({P_{G_i}}^{min} - P_{G_i})\right)\right| \quad (3)$$

Where $\mathrm{a_i}$, $\mathrm{b_i}$, $\mathrm{c_i}$, d_i and e_i are the cost coefficients of unit i.

A. Equality Constraints

The power balance constraint is an equality constraint that reduces the power system to a basic principle of equilibrium between total system generation and total system loads. Equilibrium will be met when the total system generation ($\Sigma P_{G,i}$) equals the total system load (P_D) plus system losses (P_L) as stated in equation (4).

$$\sum_{i=1}^{n} P_{G,i} = P_D + P_L \quad (4)$$

Systems losses can be determined by solving the power flow problem. One approach to estimate losses is by modeling them as a function of the system generators outputs using Kron's loss formula. Other ways to model losses are with use of penalty factors or considering losses as constant [8].

$$P_L = \sum_{i=1}^{n}\sum_{j=1}^{n} P_{G,i}B_{ij}P_{G,i} + \sum_{j=1}^{n} P_{G,i}B_{i0} + B_{00} \quad (5)$$

Where,
B_{ij} ,B_{i0} ,B_{00} are known as the loss or B coefficients.
For the purposes of this study, transmission loss is not considered. Therefore, the equation (5) becomes equation (6).

$$\sum_{i=1}^{n} P_{G,i} = P_D \quad (6)$$

B. *Inequality Constraints*

The ED planning must perform the optimal generation dispatch among the operating units to satisfy the power balance constraint and subjected to the generation limit. Generating units have lower ($P_{G,i}^{min}$) and upper ($P_{G,i}^{max}$) production limits which can be defined as a pair of inequality constraints. Generating units have ($P_{G,i}^{min}$) and upper ($P_{G,i}^{max}$) production limits that are based on the machine design. This inequality constraint is shown in equation (7).

$$P_{G,i}^{min} \leq P_{G,i} \leq P_{G,i}^{max} \ for\ i = 1, \dots, NP \qquad (7)$$

III. METHODOLOGY

Differential Evolution (DE) has three essential control parameters; these are the scaling factor (F), the crossover constant (C_R) and the population size (N_P).The scaling factor is a value in the range [0,2] that controls the amount of perturbation in the mutation process. The constant of crossover reflect the probability with which the trial individual inherits the actual individual's genes. Moreover, small values of C_R, increase the diversity of the population[9].

The size of Population N_P is very important factor. It should not be too small in order to avoid stagnation and to provide sufficient exploration. The increase the number of N_P induces the increase of a number of function evaluations. Furthermore, the correlation between NP and F may be observed. It is intuitively clear that a large NP required a small F. It means that, the large the size of a population is , the more densely the individuals fill the search space, so less amplitude of their movements is needed[10].

This study presents a solution to solve Economic Dispatch (ED) problem using Differential Evolution (DE) algorithm to search for optimal or near optimal of generation output of each generator. The DE algorithm was utilized mainly to determine the optimal generation output of each generator that was submitted to operation at the specific period, thus minimizing the total generation cost. The procedures of the proposed DE method shows in the following steps.

Step 1: Initialization of the DE parameter

Initialize all the DE parameter, such as N_P, F, C_R, D, L, H, n, (a,b,c,d,e,f), (P_{max},P_{min}), PD

Step 2: Initialization population of N_P vectors

The algorithm starts by creating an initial population of N_P vectors. Random values are assigned to each decision parameter in every vector according to:

$$X_{j,i}^{(0)} = X_j^{min} + \eta_j(X_j^{max} - X_j^{min}) \qquad (8)$$

where i=l,..., N_P and j=l,...,D ; and X_j^{max} are the lower and upper bounds of the j[th] decision parameter; and η_j, is a uniformly distributed random number within [0, 1] generated anew for each value of j. $X_{j,i}^{(0)}$is the j[th] parameter of the i[th] individual of the initial population.

Step 3: Mutation Operation

The mutation operator creates mutant vectors X_j' by perturbing a randomly selected vector (X_a) with the difference of two other randomly selected vectors (X_b and X_c),

$$X_j'^{(G)} = X_a^{(G)} + F(X_b^{(G)} + X_c^{(G)}) \qquad (9)$$

where; i= 1,….,N_P where X_a, X_b and X_c, are randomly chosen vectors $\epsilon\{1..\ .N_P\}$ and $a \neq b \neq c \neq i$. X_a, X_b and X_c, are selected anew for each parent vector. The scaling constant (F) is an algorithm control parameter used to adjust the perturbation size in the mutation operator and improve algorithm convergence.

Step 4: Crossover Operation

The crossover operation generates trial vectors (X_i'') by mixing the parameters of the mutant vectors (X_i') with the target vectors (X_i) according to a selected probability distribution;

$$X_{j,i}''^{(G)} = \begin{cases} X_{i,j}'^{(G)}, if\ \eta_j' \leq C_R\ or\ j = q\ ; \\ X_{j,i}^{(G)}\ , otherwise \end{cases} \qquad (10)$$

Where i=l,….,N_P , j=l,...,D , q is a randomly chosen index between l,..., N_P, that guarantees that the trial vector gets at least one parameter from the mutant vector, η_j' is a uniformly distributed random number within [0, 1] generated anew for each value of j. The crossover constant C_R is an algorithm parameter that controls the diversity of the population and aids the algorithm to escape from local optima. $X_{j,i}^{(G)}$,$X_{i,j}'^{(G)}$, and $X_{j,i}''^{(G)}$ are the j[th] parameter of the i[th] target vector, mutant vector, and trial vector at generation G, respectively[11].

Step 5: Selection Operation

The selection operator forms the population by choosing between the trial vectors and their predecessors (target vectors) those individuals that present a better fitness or are more optimal according to (11).

$$X_i^{(G+1)} = \begin{cases} X_i''^{(G)}, if\ f(X_i''^{(G)}) \leq f(X_i^{(G)}) \\ X_i^{(G)}, otherwise \end{cases} \quad (11)$$

Where
i= 1,....,N_P

The optimization process is repeated for several generations, allowing individuals to improve their fitness as they explore the solution space in search for optimal values.

Step 6: Random the Power initial for three generators

The random of the power initial for the three generator is within the limits in equation (7). If the power initial value not within the limits, go to step 2.

Step 7: Evaluate and calculate the total power

Evaluate and calculate the total power generation is within the limit. If the total power generation value not within the limits, go step 2.

Step 8 and 9: Evaluate and calculate the cost function

Calculate the cost function for the system using equation (3) and if the cost function is not converge, go step 2.

The above steps can be illustrated as shown in Figure 3.

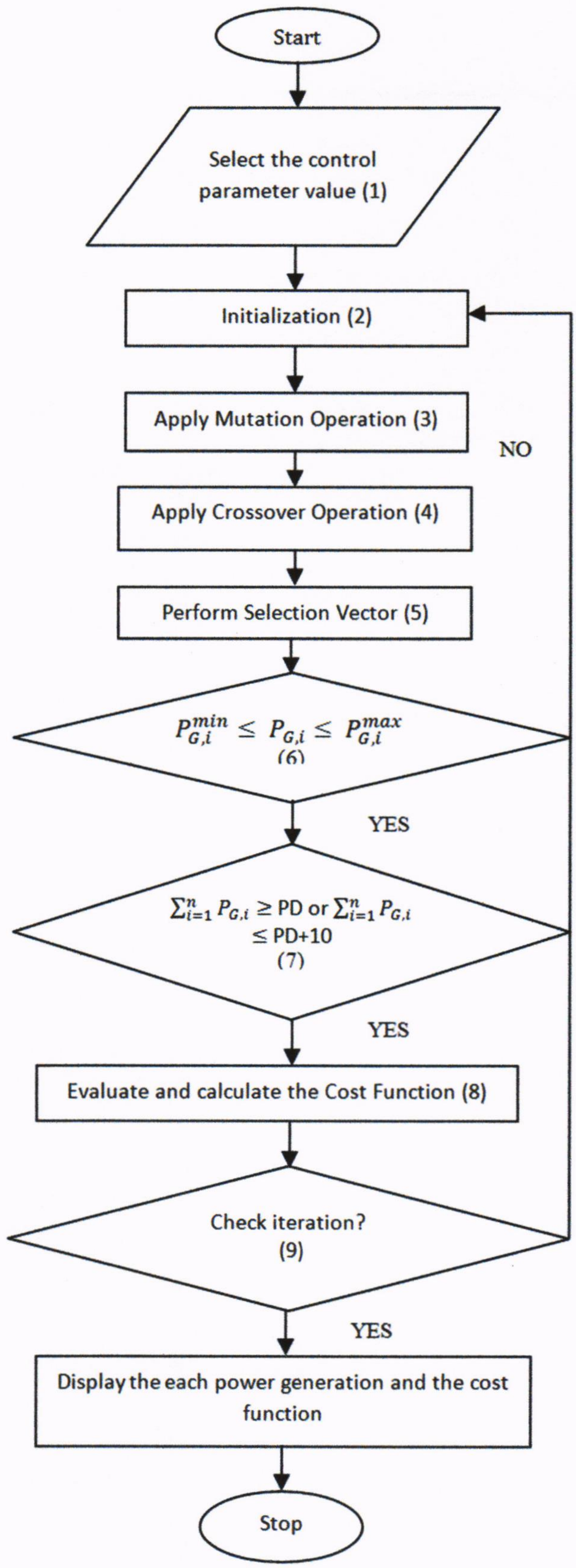

Fig 3: Flow chart of Differential Evolution (DE)

IV. RESULT AND DISCUSSION

The control parameters used for the case with the three-generation unit system are:
$F = 0.95$, $C_R = 0.1$, $N_P = 10000$, $D=1$, $L=-0.2048$, $H=0.2048$. This system comprises of 3 thermal units with valve-point effects and the input data are given in Table 1. The expected power demand to be met by all the 3 generating units is 850MW.

Table 1: The Unit Data

Parameter	Unit 1	Unit 2	Unit 3
Maximum(MW)	600	400	200
Minimum(MW)	100	100	50
Input-output curve:			
a ($/h)	561	310	78
b ($/MWh)	7.92	7.85	7.97
c ($/MW^{2}h)	0.001562	0.00194	0.00482
Input-output (valve-point) curve:			
d ($/h)	300	200	150
e ($/MWh)	0.0315	0.042	0.063

The Table 2 show the results obtained by using the proposed method.

Table 2: The Result for 3 unit generation system

Runs	P_{G1} (MW)	P_{G2} (MW)	P_{G3} (MW)	Total Cost ($/h)
1	336.75	356.68	161.24	8821.91
2	282.36	394.67	178.34`	8622.57
3	305.39	370.42	181.06	8669.06
4	468.99	278.70	106.65	8743.16
5	420.67	250.08	182.90	8582.62
6	390.80	289.08	179.78	8722.70
7	563.87	148.58	146.41	8867.25
8	562.39	129.04	160.68	8903.47
9	571.62	186.00	92.96	8675.97
10	304.86	377.10	176.32	8659.89

The results in Table 2 show the generation output of each generator and the corresponding cost by first run to ten run. As seen from Table 2, the best result is obtained at run 5 where cost function is 8582.62 $/h and the generation outputs of unit 1, 2 and 3 are 420.67 MW, 250.08 MW, 182.90 MW and respectively. Table 2 also shows that all the generation output is within the upper and lower limits of each generator.

V. CONCLUSION

This paper presents the solution ED problem using DE method by considering valve point loading effect. A test system consisting of three generator units has been used in this study. From results obtained, it can be concluded that the developed program had successfully solved to minimize system cost by properly allocating the real consuming the non-smooth cost function. In summary, the results obtained by using Differential Evolution method satisfy the power demand at the minimum total generator fuel cost while satisfying the equality and inequality constraints.

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