Solving Unit Commitment Using Lagrangian Relaxation Method

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Abstract— **Unit commitment (UC) is one of the important analyses required in the scheduling and dispatch of power system .The unit commitment problem consists of determining the schedules of the power generating units and the generating level of each unit. The decisions concern to which units to commit during each time period and at what level to generate power to meet the electricity demand. This paper presents the application of a Lagrangian Relaxation method for solving the unit commitment problem. One of the most obvious advantages of the Lagrangian relaxation method is its quantitative measure of the solution quality since the cost of the dual function is a lower bound on the cost of the primal problem. The proposed method has been tested on a 3 generation unit system using Matlab programming and the result give the minimization of generating cost and at the same time optimize the operation of the system base on the load demand for the given day**

Keywords-unit commitment; lagrangian relaxation.

I. INTRODUCTION

Unit commitment problem (UCP) is defined as the problem of how to schedule generators economically in a power system in order to meet the requirements of load and spinning reserve. The idea is to determine the combination of available generating units and scheduling their respective outputs to satisfy the forecasted demand with the minimum total production cost under the operating constraints enforced by the system for a specified period that usually varies from 24 hours to one week [1]. The resultant schedule should minimize the system production cost during the period while simultaneously satisfying the load demand, spinning reserve, physical and operational constraints of the individual unit.

Since the problem was introduced, several solution methods using some sort of approximation and simplification have been developed. However, they differ in the solution quality, computational efficiency and the size of the problem they can solve. The available approaches for solving unit commitment problem can usually be classified into Heuristic methods (with priority ordering) and mathematical programming methods such as (Dynamic Programming (DP)[2], Branch and Bound (BB)[3], Langarian Relaxation (LR)[4], Bender decomposition method and mixed integer programming). For moderately sized production systems, mathematical methods can be used to solve the UCP, successfully. For larger systems, mathematical methods fail because the size of the solution space increases exponentially with the number of time periods and units in the system. As a result, the computation time of exact methods becomes impractical. In these cases meta-heuristic methods (evolutionary programming (EP), Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithms (GA), etc) can be used to produce near optimal solutions in a reasonable computation time but the main shortcoming of heuristic methods is that they cannot guarantee the optimal solutions or even furnish an estimate of the magnitude of their sub-optimality[5].

The application of LR in the scheduling of power generations was proposed in the late 1970s. In Lagrangian relaxation approaches, unit commitment problem is formulated in terms of a cost function, that is the sum of terms each involving a single unit, a set of constraints involving a single unit, and a set of coupling constraints (the generation and reserve requirements), one for each hour in the study period, involving all units[6]. An approximate solution to this problem can be obtained by adjoining the coupling constraints onto the cost using Lagrangian multipliers. The cost function (primal objective function) of the unit commitment problem is relaxed to the power balance and the generating constraints via two sets of Lagrangian multipliers to form a Lagrangian dual function. The dual problem is then decoupled into small sub problems which are solved separately with the remaining constraints. Meanwhile, the dual function is maximized with respect to the Lagrangian multipliers, usually by a series of iteration [7][4].

Lagrangian relaxation method is more advantageous due to its flexibility in dealing with different types of constraints. It is relatively easy to add unit constraints. Lagrangian relaxation is flexible to incorporating additional coupling constraints that have not been considered so far. The only requirement is that constraints must be additively separable in units. Such constraints could be area reserve constraint, area interchange constraint, etc [8]. To incorporate such constraint into the framework of Lagrangian relaxation, a Lagrangian multiplier is defined for each constraint for each time period and the constraints are adjoined into the objective function of the relaxed problem. The amount of computation varies linearly with the number of units. Hence, it is computationally much more attractive for large systems [9].

In this paper, a method for unit commitment based on the Langarian Relaxation (LR) technique is proposed .The LR method is developed using Matlab programming to successful solve the unit commitment problem and minimize the generating cost and at the same time optimize the operation of the system base on the load demand for the given day while fulfilling all the related to the delivery constrain of electrical energy from the generating unit to the load demands.

II. METHODOLOGY

A. Unit Commitment Problem Formulation

The basic goal of the optimal UC is to properly schedule the On/Off states of all unit in the system plus to minimize the system operating costs, which is the sum of production and startup costs of all units over the entire study time span (e.g., 24 h), under the generator operational and spinning reserve constraints. To formulate the problem mathematically, the following notation is introduced.

J, Q: Primal and dual solution of the LR based UC algorithm;

N: total number of generating unit;

T: total number of scheduling periods;

P_{it}: Output power of unit *i* at period *t* (MW)

$F_i\ (P_{it})$: Fuel cost of unit *i* when its output power is P_{it}($)

S_{it}: Start-up price of unit *i* at period *t* ($)

u_{it}: commitment state of unit *i* at period *t* (u_{it}=1: unit is on-line and u_{it}=0: unit is off-line)

a_i, b_i, c_i: cost coefficient of generating unit *i*

λ_t , μ_t: Lagrangian multipliers at hour *t* ($/MWh)

D_t: customer demand in time interval *i*

R_t: spinning reserve requirements

Mathematically, the objective function or the total operating cost of the systems can be written as follow:

$$J = \min_{P_{it}, u_{it}} f(P_{it}, u_{it}) = \min_{P_{it}, u_{it}} \left[\sum_{t=1}^{T} \quad \sum_{i=1}^{N} u_{it} \left[F_i\ (P_{it}) + S_{it}(1 - u_{i,t-1})\right]\right] \tag{1}$$

Fuel cost function $F_i\ (P_{it})$ is frequently represented by the polynomial function:

$$F_i\ (P_{it}) = a_i + b_i P_{it} + c_i P_{it}^2 \tag{2}$$

Due to the operational requirements, the minimization of the objective function is subjected to the following constraints:

(a) Power balance

$$\sum_{i=1}^{N} u_{it}\, P_{it} = D_t \quad t = 1, \ldots, T \tag{3}$$

(b) Generating Limit

$$u_{it}\, P_{i,min} \leq P_{it} \leq u_{it}\, P_{i,max} \tag{4}$$

The output of an operating unit must be within the maximum and minimum operating capacity[10].

B. Lagrangian Relaxion for Unit Commitment Problem

In the Lagrangian relaxation approach, the system operating cost function of (1) of the unit-commitment problem is related to the power balance and the spinning reserve constraints via two sets of Lagrangian multipliers to form a Lagrangian dual function

$$L(P, u, \lambda, \mu) = f(P, u) + \sum_{t=1}^{T} \lambda_t \left[D_t - \sum_{i=1}^{N} u_{it}\, P_{it}\right]$$
$$+ \sum_{t=1}^{T} \mu_t \left[D_t + R_t - \sum_{i=1}^{N} u_{it} P_{i\,max}\right] \tag{5}$$

The LR procedure solves the UC problem through the dual problem optimization procedure attempting to reach the constrained optimum[11].

The dual procedure attempts to maximize the Lagrangian with respect to the Lagrangian multipliers λ_t and μ_t, while minimizing it with respect to the other variables P_t, u_t .The dual problem is thus the search of the dual solution (Q) expressed as below

$$Q = \max_{\lambda_t, \mu_t} \left(\min_{P_{it}, u_{it}} L(P, u, \lambda, \mu)\right)$$
$$\lambda_t \geq 0 \ and\ \mu_t \geq \tag{6}$$

The Langarian function of (6) is written as

$$L(P, u, \lambda, \mu) = f(P, u) - \sum_{t=1}^{T} \lambda_t \sum_{i=1}^{N} u_{it} P_{it} -$$
$$\sum_{t=1}^{T} \mu_t \sum_{i=1}^{N} u_{it} P_{i\,max} + \sum_{t=1}^{T} \lambda_t D_t + \sum_{t=1}^{T} \mu_t\,(D_t + R_t) \tag{7}$$

When the Lagrangian multipliers $\lambda_{t(k)}$ and $\mu_{t(k)}$ are fixed for iteration *k*, the last two terms of the Lagrangian in (7) are constant and can be dropped from the minimization problem. Hence, the system (coupling) constrains can be relaxed and the search for the dual solution can be done through the minimization of the Lagrangian as:

$$\min_{p_{it} u_{it}} L\,(P, u, \lambda_k, \mu_k) = \min_{p_{it} u_{it}} \sum_{t=1}^{T} \sum_{i=1}^{N} u_{it} \{F_i(P_{it})$$
$$+ S_{it}(1 - U_{it-1}) - \lambda_{t(K)} P_{it} - \mu_{t(k)} P_{i\,max}\} \tag{8}$$

Then, the minimum of the Lagrangian function is solved for each generating unit over the time horizon, that is:

$$\min_{p_{it}u_{it}} L\,(P,u,\lambda_k,\mu_k) = \sum_{i=1}^{N} \min_{p_{it}u_{it}} \sum_{t=1}^{T} u_t\,\{F_i(P_{it}) + S_{it}(1-U_{it-1}) - \lambda_{t(K)}P_{it} - \mu_{t(k)}P_{i\,max}\} \tag{9}$$

The dual solution is obtained for each unit separately. When the state $u_{it} = 0$, the value of the function to be minimized is equal zero (the unit is off-line). When the state $u_{it} = 1$, the value to be minimized is:

$$F_i(P_{it)} - \lambda_{t(k)}\,P_{it} \tag{10}$$

The startup cost and the last term in (9) are dropped since the minimization is with respect to P_{it}.When the units' fuel cost functions are represented as polynomial functions as in (2), the minimum of (10) can be found by taking its first derivative [5].

$$\frac{d\left(F_i(P_{it)} - \lambda_t\,P_{it}\right)}{dP_{it}} \tag{11}$$

$$= \frac{dF_i(P_{it)}}{dP_{it}} - \lambda_t = 0 \tag{12}$$

Hence, $P_{it\,(k)} = (\lambda_{t(k)} - b_i)/2c_i$ (13)

The lagrange multiplier at hour t gives the increases of the total operating cost per unit increases in load during hour t:

$$\lambda_i = \frac{dF_i(P_{it)}}{dP_{it}} \tag{14}$$

The computational flow of the LR to solve UC are illustrate in Figure 1:

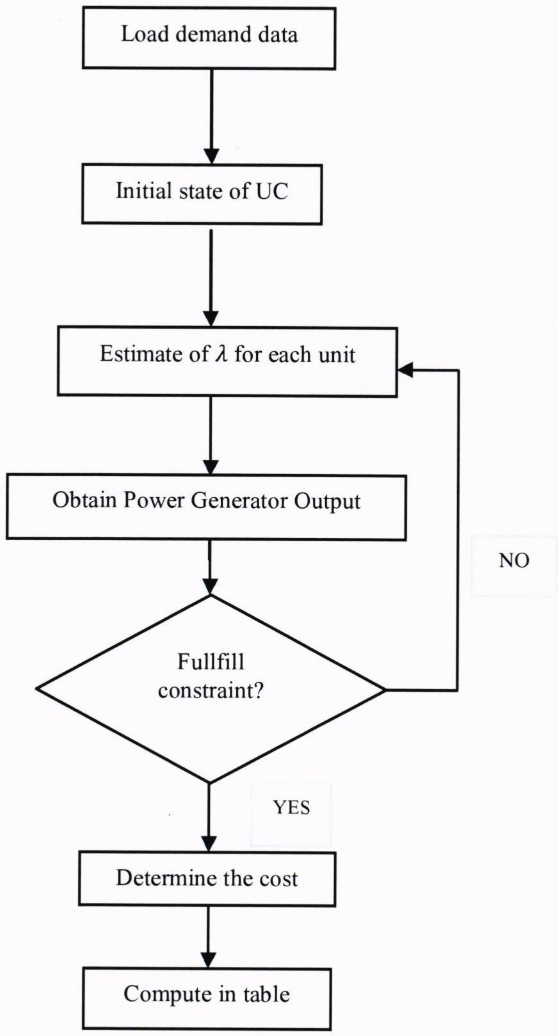

Fig1: Flowchart of the LR unit commitment problem.

III. RESULT AND DISCUSSION

The method proposed in this paper has been tested for a 3 generator unit. System data and load demand are given in the Table 1 and Table 2 respectively. The program is developed using MATLAB 7.8(R2009a).

TABLE 1.Unit data of the 3 unit 24 hour test system

	UNIT 1	UNIT 2	UNIT 3
a($/h)	500	400	200
b($/MWh)	5.3	5.5	5.8
c($/MW^{2}h)	0.004	0.006	0.009

TABLE 2.Demand of 3 unit 24 hour test system

STAGE	LOAD DEMAND
1	400
2	600
3	1100
4	950
5	750
6	650

TABLE 3.Unit Commitment schedule for 3 unit generator system over 6 stages of 24 hours

	P1	P2	P3
STAGE 1	0	1	0
STAGE 2	1	1	0
STAGE 3	1	1	1
STAGE 4	1	0	1
STAGE 5	1	0	1
STAGE 6	1	1	0

TABLE 4. Solution of 3unit test system over 6 stages using the proposed LR method.

STAGE	1	2	3	4	5	6
P1(MW)	0	305.26	542.12	471.06	376.32	328.94
P2(MW)	400	294.74	344.73	0	0	447.36
P3(MW)	0	0	213.15	328.94	323.68	0
SCHEDULING OPERATION	010	110	111	101	101	110
TOTAL POWER (MW)	400	600	1000	800	700	650
DEMAND	400	600	1000	800	700	650
TOTAL COST($)	3860	5058	9403	8000	6264	5450
LAMBDA ($/MWh)	8.501	7.742	9.636	9.002	8.31	7.936

Table 3 and Table 4 show the results for the unit commitment problem for test system. As shown in Table 4 each of the six stages has different demand. During the first stage of the scheduling horizon, only unit 2 is on-line due to the lower demand. As the demand increases, the number of unit generator that start operate is increasing and so the cost. Unit 2 and unit 1 works as based unit. It means these 2 unit runs for 16 to 20 hours per day to satisfy the load demand capacity. The operating time of the unit generator scheduled is based on the most minimize cost and at the same time optimized the operation of the system while fulfilling the load demand. It also noted that, the third stage have the highest load demand compared to all stage and at this time all 3 unit need to operate to fulfill load demand capacity. During the fourth stage to the sixth stage, the demand start to decrease and the number of generating unit that operate are also decrease.

Moreover, from table 4, it is proved that the total power obtained for the solution also satisfies the demand. For scheduling operation, the value of the output power indicate the scheduling unit whether to be ON or OFF. This means when the output power for each unit draws some values, the generating units is in ON condition while the units which not indicate any value (0 MW) it is in OFF condition.

IV. CONCLUSION

The proposed LR is effectively implemented to solve the UC problem .An initialization procedure intends to create a high quality feasible schedule in the first iteration is proposed, based on unit and time interval classification. The Lagrangian relaxation methodology generates easy sub problems for deciding commitment and generation schedules for single units over the planning horizon, independent of the commitment of other units. A pricing mechanism links such decisions on individual units together, so that the system constraints are satisfied. An iterative approach usually ensures convergence to a near-optimal solution and the generation of bounds gives an indication of the proximity of the solution to the theoretical optimal solution. The LR method is successful solve the unit commitment problem and minimize the generating cost and at the same time optimize the operation of the system base on the load demand for the given day while fulfilling all the related to the delivery constrain of electrical energy from the generating unit to the load demands. As for future development, this method is particularly attractive in large-scale systems.

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